Constant Proportion Debt Obligations (CPDOs)

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Constant Proportion Debt Obligations (CPDOs), launched in 2006 by ABN Amro, are structured credit derivatives indexed on a portfolio of investment grade debt which generate high coupon payments by dynamically leveraging a position in the underlying portfolio. To generate income a CPDO strategy sells default protection on the underlying investment grade portfolio through index default swaps. The CPDO strategy involves high initial leverage but this is reduced as the difference between promised liabilities and the CPDO portfolio value decreases. If, on the other hand, losses are incurred the CPDO will increase its risky exposure to regain some of the lost capital. With this leverage rule the CPDO has no upside potential but has an added ability to recover from negative positions.

Existing CPDO coupons and principal notes have received AAA from the major rating agencies. This has been somewhat surprising since the CPDO premium (around 200 bp in some examples) has been much higher than the spread on similarly rated structured credit products.

A new approach to the rating and risk analysis of CPDOs

We propose in this paper an independent analysis of the performance and risks of CPDO strategies using a top-down model which captures the essential risk factors of the CPDO while remaining parsimonious in its specification. Our analysis allows an independent assessment of the credit ratings attributed by agencies, allows to compute default probabilities, loss distributions and other tail risk measures for the CPDO strategy and allows to

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analyze the dependence of these risk measures on various parameters describing the risk factors.

The strategy

Initially, note notional is invested in a risk free asset and index swap contracts are entered. The optimal risky exposure is determined such that the CPDO will meet its obligations prior to expiry given that there are no further defaults or changes in the index spread. The actual leverage employed is piecewise linear and only adjusted to equal optimal leverage on roll dates or if actual leverage differs too much from the optimal leverage. Every releveraging will imply a mark to market settlement which is the cost or benefit of unwinding the current risky exposure. All cashflows – interest accrued, CPDO coupon payments, swap spread income, credit losses and mark to market settlements – are balanced by the risk free investment.

The CPDO will default if at some point the value of the risk free assets is below some lower bound. In this case the CPDO defaults on both the remaining coupons and the principal repayment. The CPDO also defaults if at expiry it is not able to repay the entire notional.

Risk factors

The two main risk factors for the CPDO performance are the evolution in the index swap spread and the statistical default rate of the underlying index. The level of the swap spread determines the income generated by the CPDO and the volatility of the swap spread highly influences the mark to market settlements between roll dates. The number of default events in the underlying index determines compensation paid to buyers of protection.

The index is rolled over every 6 months by replacing downgraded names by better performing ones. This affects the index swap spread around roll dates and thereby determines the sign and size of the mark to market settlements on these dates. The index roll also affects the statistical default intensity since by rolling over the index the average default rate is lowered.

A top-down modeling approach

The standard model used by rating agencies and banks for analyzing the CPDO performance has been to use a high-dimensional model for the joint transition of ratings and spreads for all names in the underlying portfolio. Rating agencies typically assess the defaults in the underlying index from an extensive rating migration model and thereafter the index spread is modelled as a stochastic process depending on the average rating of the names in the index. Such a detailed joint modeling of rating and spread movements is

not accessible to entities other than rating agencies due to lack of historical data on ratings.

We show that the main factors driving the risk and performance of a CPDO strategy can be modeled using a much more parsimonious top-down approach in which we model the overall index default intensity as a stochastic process from which we can generate default events in the underlying portfolio and compute the index swap spread. We choose to model the risk-neutral default intensity λ_t^Q as a CIR process and specify the statistical default intensity $\lambda^P = \vartheta\lambda^Q$ as being a multiple of the risk-neutral one, where ϑ may be viewed as the proportional premium for bearing default risk. This risk premium determines the spread income relative to the level of credit losses.

This setup is well suited for analyzing the CPDO performance since the cashflows involve only require knowledge of the portfolio loss. On roll dates, the replacement of downgraded names is modeled as a negative jump in the default intensity.

Risk and performance analysis

The CPDO strategy is highly path dependent and explicit computations become very involved. We have therefore analyzed the performance using Monte Carlo simulation. We compute default probabilities, VaR, expected shortfall and other tail risk measures for the CPDO strategy and analyze the dependence of these risk measures on various parameters describing the risk factors. Depending on parameter specifications, we find a default rate in the interval 0-15% which translates into a rating between AAA and BB for the rating on the repayment of principal, the ratings obtained strongly depending on the credit environment -high spread or low spread- assumed. On the other hand the probability of the CPDO defaulting on its coupon payments is found to be very small, justifying a AAA rating for coupon payments. We find that the performance is highly dependent on the aggressiveness of the leverage strategy – a more aggressive strategy will result in fewer defaults at the cost of higher losses given default. Our results also point to a heterogeneous range of tail risk measure inside a given rating category, suggesting that credit ratings for such complex leveraged strategies should be suitably complemented by other risk measures for the purpose of performance analysis.