A new approach for measuring credit contagion

Jiwook Jang

Department of Actuarial Studies, Division of Economic and Financial Studies, Macquarie University, Sydney NSW 2109, Australia, E-mail: jjang@efs.mq.edu.au

Summary

In financial industry, a shock which initially affects a couple of institutions or a particular region of the economy spreads to the rest of the financial industry and then infect the larger economy. This is called financial contagion. The stock price falling of *Countrywide Financial Corporation* and closing down of *New Century Financial Corporation* due to mismanagement of subprime mortgage in US are one of the particular and recent examples of financial contagion. The prevalence of above financial contagion has led to bankruptcy and default of mortgage lenders in US announcing their significant losses. This crisis also has led to the collapse of stock prices in worldwide and it could shake global financial markets further due to new waves of default.

Main causes of defaults are due to domestic and global business and financial links or ties between firms. All firms financial stability is universally subject to macroeconomic factors such as the price of energy and minerals, interest rates, mortgage rate and exchange rate. Also the global economic system that allows free trades for goods & services and investment makes them highly dependent each other. As a result, a shock to a business sector or one region can create a series of default locally and globally.

In this paper, we focus on credit market in financial contagion (simply we call it 'default or credit contagion'). In order to accommodate clustering of defaults in reality, we introduce a new default intensity process, i.e. multiple shot noise intensity. It consists of n component processes $\lambda_t^{(1)}$, $\lambda_t^{(2)}$, $\lambda_t^{(3)}$, $\cdots$, $\lambda_t^{(n)}$. For $i = 2, 3, \cdots n$, $\lambda_t^{(i)}$ decays with rate $\delta^{(i)}\lambda_t^{(i)}$ and additive jumps occur with rate of $\lambda_t^{(i-1)}$, i.e. each process acts a jump intensity for the next one. Jumps sizes are independent identically distributed random variables with distribution function $G_i(y)$. $\lambda_t^{(1)}$ decays with rate $\delta^{(1)}\lambda_t^{(1)}$ but its jump arrival rate is deterministic ρ. Its jump sizes have distribution function $G_1(y)$. Hence the multivariate default intensity model we consider has the following structure:

$$d\lambda_t^{(1)} = -\delta^{(1)}\lambda_t^{(1)}dt + dC_t^{(1)}, \qquad C_t^{(1)} = \sum_{j=1}^{M_t^{(1)}} Y_j^{(1)},$$

$$d\lambda_t^{(2)} = -\delta^{(2)}\lambda_t^{(2)}dt + dC_t^{(2)}, \qquad C_t^{(2)} = \sum_{k=1}^{M_t^{(2)}} Y_k^{(2)},$$

$$\vdots$$

$$d\lambda_t^{(n)} = -\delta^{(n)}\lambda_t^{(n)}dt + dC_t^{(n)}, \quad C_t^{(n)} = \sum_{l=1}^{M_t^{(n)}} Y_l^{(n)}, \tag{1.1}$$

where:

- $\left\{Y_j^{(i)}\right\}_{j=1,2,\cdots}$, $\left\{Y_k^{(i)}\right\}_{k=1,2,\cdots}$, $\cdots$, $\left\{Y_l^{(i)}\right\}_{l=1,2,\cdots}$ are sequences of independent and identically distributed random variables with distribution function $G_i(y)$ $(y > 0)$ and $i = 1, 2, \cdots, n$.
- $M_t^{(i)}$ is the total number of events up to time t.
- $\delta^{(i)}$ is the rate of exponential decay for firm $i = 1, 2, \cdots, n$.

We also make the additional assumption that the point process $M_t^{(i)}$ and the sequences $\left\{Y^{(i)}\right\}$ are independent of each other.

$M_t^{(1)}$ follows a homogeneous Poisson process with frequency ρ and $M_t^{(i)}$ for $i = 2, 3, \cdots n$, follow the Cox process with frequency $\lambda_t^{(i-1)}$ respectively. So in this model, dependence between the intensities $\lambda_t^{(i)}$ comes from the structure that each process acts a jump intensity for the next one.

The intensity $\lambda_t^{(1)}$ is triggered by primary events (or shocks) such as oil and commodity prices, the governments' fiscal and monetary policies, the release of corporate financial reports, the political and social decisions, the romours of mergers and acquisitions among firms, collapse and bankruptcy of firms, September 11 WTC catastrophe and Hurricane Katrina etc. that will result in a positive jump in intensity process. As time passes, the intensity process decreases, as the firm 1 in the market will do its best to avoid being in bankruptcy after the arrival of a primary event. This decrease continues until another event occurs which again will result in a positive jump in intensity process.

The intensity $\lambda_t^{(1)}$ is the jump arrival rate for the second firm's default intensity $\lambda_t^{(2)}$ and the intensity $\lambda_t^{(2)}$ is the jump arrival rate for the third firm's default intensity $\lambda_t^{(3)}$ and so on. Hence the intensity $\lambda_t^{(1)}$ is the prime trigger in influencing all other relative local/global firms default intensities. As time passes, the intensity processes for the firm $2, 3, \cdots$ decrease, as these firms will also do their best to avoid being in bankruptcy from the influence by the prime company's default intensity $\lambda_t^{(1)}$ that is triggered by primary events (or shocks).

We use the Cox process to model the multivariate default time and derive multivariate survival/default probabilities. As an example of pricing of credit derivatives, we calculate the market credit default swaps (CDS) rates, assuming that jump size distributions are exponentials. We also derive the pair-wise conditional default probabilities and pair-wise linear default correlations. Standard martingale theory is used to derive the multivariate Laplace transforms.