# Choice of Rating Technology and Price Formation in Imperfect Credit Markets

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**Abstract**

Accurate rating systems are of key importance for banks to price and manage their loan portfolios. In this paper we analyze the choice of the rating technology in an oligopolistic banking sector. In our model the rating system estimates the probabilities of default for the individual borrowers and therefore provides important input for the pricing of the loans. We model the technology choice and the pricing as a two-stage game. In the first stage banks choose the rating technology and in the second stage banks choose their pricing policy given the imperfect (oligopolistic) market using a risk-based pricing approach. The presented probabilistic framework and the modeling of the technology choice is novel in the banking literature and can provide important insights. In a comparative static analysis we study the implications for a market with two banks, which can employ two different rating systems (low or high accuracy). We find that in equilibrium the rating technology choice critically depends on the cost structure. If the additional costs for the high accuracy system are large both banks will have no incentive to adopt this technology. If the additional costs are low equilibrium behavior of banks results in the implementation of the accurate technology. In this case credit spreads unambiguously decrease and credit volume increases. The use of the more accurate technology, however, does not necessarily result in higher profits for the banks. Only if the costs are sufficiently small the equilibrium behavior results in a Pareto improvement. This has important implications for banking regulation which aims to provide incentives to use high accuracy rating systems (e.g. Basel II regulation).

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# 1 Introduction

The optimal allocation of loans is one of the central economic functions of banks as financial intermediaries. To achieve this objective the pricing and management of credit risk exposures is a major challenge for all banks. Thus, accurate estimates of risk parameters provided by rating systems are of key importance. This view is emphasized by the recent development in banking regulation set forth by the Basel Committee of Banking Supervision commonly known as Basel II. Under this framework particular incentives for banks are set to improve their rating technology in order to decrease their minimum regulatory capital requirements. Although rating technology which comprises rating systems, methods and processes can be regarded as one of the key success factors of the banking industry there is little academic work about the role of the rating technology for the price formation in credit markets.

It is the main purpose of this paper to analyze the choice of the rating technology by a bank where the precision or the accuracy of the rating outcome is the central decision variable. We model the technology choice and the pricing of loans in an oligopolistic environment as a two-stage game. In the first stage banks choose the rating technology which is represented by the accuracy of the estimated default probabilities. In the second stage banks choose their pricing policy in a Bertrand competition using a risk-based pricing approach. Banks use the estimated default probabilities to assess the expected costs of credit risk in their pricing decision. As differences in the accuracy of the estimated default probabilities among competing banks lead to adverse selection effects the choice of the rating technology is crucial for the maximization of expected profits.

Our analysis is based on a probabilistic framework for the rating process which is different from the usual approach followed by the academic literature. One of the most important features of our framework is the model of the inter-temporal information structure, or, in probabilistic terms, the filtration of the probability space. In our framework we assume that the information about the default indicator (i.e., whether a borrower is creditworthy or not, or, if a borrower will default or not during a given period) is revealed after the observation date, either continuously over time or at the end of a certain time period. The economic interpretation of such a framework is that the repayment capacity of the individual obligors depends on the stochastic future realizations of their undertaken investment projects. Hence, a perfect rating system is a rating system which produces optimal estimates of the expected value of the default indicator, i.e. the probability of default (PD). In our framework obligors exist with different levels of credit risk represented by different default probabilities. Therefore in our model banks apply a *risk-based lending* approach, where offered credit spreads are dependent on these risk levels or, equivalently, on the estimated PDs.

In contrast to our stochastic framework traditional models assume that at the observation date the information about the realization of the default indicator is available in general (e.g. by defining ex ante "good" and "bad" obligors or projects), though not always accessible to all agents. In the canonical case information about the default indicator is costly and at least partially available (see, e.g. Ruckes (2004)). This approach is also reflected in popular validation measures, like the Accuracy Ratio or Gini Coefficient, where the discriminatory power of rating systems is benchmarked against a 'perfect' rating system which has perfect ex-ante knowledge about the realizations of the default indicator, see Basel Commitee on Banking Supervision (2005) and Satchell and Xia (2007). In such a framework the traditional *cut-off lending* approach is adequate where all obligors deemed to be creditworthy are offered a loan at an equal rate independent of the individual obligor's risk (for a critical discussion of the cut-off lending approach see Stein (2005)). However, if a further differentiation among the 'creditworthy' obligors is possible by acquiring additional information (e.g. by screening of existing obligors, take-over of a competing bank, or rating technology in general) the situation changes fundamentally.

In such a framework the traditional cut-off lending approach is no longer optimal. In a recent paper Jankowitsch et al. (2007) showed that banks with a more accurate rating system will potentially offer loans to obligors with lower credit risk at a more attractive rate and banks who cannot differentiate between their obligors are left with the high-risk obligors. In such a risk-based lending environment adverse selection leads to an advantage for banks with a more accurate rating system. While under perfect competition among banks all banks will invest in the most accurate available rating technology by equating marginal costs and marginal profits, the situation under imperfect competition is more complex. Jankowitsch et al. (2007) analyze a similar situation by introducing an ad-hoc parametric model of obligor price elasticity which does not take into account strategic action of banks in the credit market.

Imperfect competition is a common assumption in the banking literature. Chiesa (2001), Almazan (2002), and Repullo (2004) discuss various aspects of imperfect markets. In the paper closest to ours Marquez (2002) analyzes the relation between competition and adverse selection in an entry model. Less informed entrant banks face larger adverse selection costs than incumbents providing existing banks with an significant advantage. Focus there, however, is on the potential effects of increasing competition caused by financial deregulation and not on modeling the role of the rating technology. In a related paper Hauswald and Ruckes (2003) discuss how changes in the state of the information technology and information spillovers affect financial markets. In contrast to our study there is no strategic technology decision by the banks.

The choice of the rating system, once it is made, has important implications on the pricing strategy of each bank. In this paper we assume an oligopolistic credit market and banks choose their pricing policy by taking the strategic reactions of rival suppliers into account. Banks are assumed to play Bertrand competition and to follow a sequentially rational strategy. This behavior implies that each bank is aware of the impact the rating technology has on the pricing decision and uses this when solving for both the optimal pricing and technology choice strategies. Since in a risk-based lending framework the prices of loans (i.e., the credit spreads of the loan rate defined as the difference of the loan rate to the risk-free reference rate) are the key strategic variables, it seems natural to model the loan market as a Bertrand-type oligopoly. This is, however, in contrast to the majority of studies in the field of banking competition where usually a Cournot-type competition is assumed in order to endogenously model the lending capacity, see e.g. Cetorelli and Peretto (2000). Since the focus of our analysis is on the price formation in the credit market we formulate a partial equilibrium model where restrictions of the lending capacity can only be imposed exogenously, e.g., through minimum capital requirements and restrictions on the issuance of new equity capital.

We assume Betrand competition among two banks with linear demand functions, while ruling out strategic behavior of obligors. Obviously, this simplification is likely to hold in markets with a large number of smaller obligors, like the retail market, in contrast to large corporations or sovereign obligors.

In the loan pricing game we assume that banks choose those loan prices that optimize the difference of interest income and costs, which consist of operating costs and risk costs. As the outcome of the default indicator is not known at the time of the pricing decision, banks have to build their decisions upon the expected risk costs, i.e. the *expected losses*. For simplicity we assume risk-neutral banks such that expectations are taken under the empirical measure. Note that we can easily extend our framework to risk-averse banks where all expectations have to be taken with respect to the relevant risk-adjusted pricing measure which need not be unique if the market is incomplete.

For expository reasons we assume that recovery rates are known and equal across all obligors and uncertainty solely stems from the default indicator. Under these assumptions the expected loss rate is solely driven by the PD of the obligor. We assume that banks obtain estimates of the 'true' PDs of each obligor usually with a non-zero estimation error which will be parameterized in our model. This implies that the cost parameter of the objective function in the Bertrand competition is stochastic. It is the main contribution of our paper to assume that banks can choose the accuracy of their rating systems, i.e. they can invest into more accurate rating systems which provide more accurate PD estimates at given costs.

Credit quality of the market in our model is given by a probability distribution representing the true PDs of all potential obligors, i.e. the true PD of each obligor is drawn from this distribution. Banks have perfect knowledge about this distribution of the true PDs representing the entire universe of obligors. Since information about default rates of the entire market is made publicly available by rating agencies and credit bureaus this assumption seems not to be very restrictive. Banks, however, do not observe the true PD for the individual obligor. What they have available is an estimated PD which is the outcome of their rating system. The estimated PD is a random variable depending on the accuracy of the rating system. In this sense banks cannot observe the true PD without error.

As pointed out above, the choice of the rating system is embedded in a two-stage game where banks in the first stage choose their rating technology and in the second stage choose loan prices. Assuming two available rating technologies (high accuracy-high costs and low accuracy-low costs) banks obtain their expected payoffs by integrating over the entire continuum of potential obligors and the possible rating errors. For each potential obligor the price formation in the Bertrand oligopoly takes place under the error-prone PD estimate whereas the payoffs of the second stage game are evaluated with respect to the distribution of true PDs. This structure allows us to explicitly take adverse selection into account: Assume that one bank has estimated the correct true PD. If the estimated PD of the other bank is higher (lower) than this bank will offer a higher (lower) price and receives a lower (higher) quantity. Therefore this bank grants on average a low quantity of overpriced and a high quantity of underpriced loans. This adverse selection effect can be mitigated by the rating technology choice.

In a comparative static analysis we study the implications of alternative costs associated with the different rating systems and demand specifications on the equilibrium outcome of the game. If both banks have symmetric demand it turns out that if costs for the accurate rating technology are sufficiently high, equilibrium behavior causes both players not to switch to the more accurate rating technology. Banks find it unattractive to bear the costs of switching to the new technology. When costs become sufficiently small both banks switch simultaneously to the more accurate system. In such an equilibrium loan rates decrease (because banks are able to better align obligor risk to the pricing of the loans) and given price dependent demand functions loan volumes increase for both banks. However, at this threshold the equilibrium is no longer

Pareto-optimal, i.e. banks are worse off than in the other equilibrium. If the costs of the rating system are particularly low we find that the banks profits increase. In such a case the adoption of a new rating technology leads to lower interest rates *and* higher profits for the banks at the same time. This has important implications for banking regulation which aims to provide incentives to use high accuracy rating systems via reduced capital requirements (e.g. Basel II regulation).

While the use of a two-stage game is novel in the literature on rating technology choice, it is the natural way to simultaneously study the effects of new rating systems on the pricing of the loans. When loan markets are characterized by imperfect competition, the choice of technology not only has an impact on the level of loan prices but gives rise to strategic interactions among the rivals in the loan markets. Such strategic interactions have extensively been studied in the industrial organizations literature (see Tirole (1989)) in which the impact of cost reducing investment on the product market strategy of rival firms is studied in a two-stage game. The paper by Brander and Spencer (1983) is among the first to study the impact of R&D on the product market strategy in oligopolistic industries. They also point out the commitment effect that can be associated with R&D investment of firms. An increase in R&D investment credibly shifts the firm's reaction function in the output market and hence allows the adoption of a product market strategy that otherwise would not have been possible. This commitment effect is also the driving force behind the strategic capital structure theory advanced by Brander and Lewis (1986). In this theory firms compete in a duopolistic product market with stochastic demand by setting their optimal output levels. Since part of the operation of each firm can be debt financed, the choice of the debt level determines those states of the economy in which firms do not go bankrupt. In such a setting, equity maximizing managers will choose the level of output dependent on the current level of debt. Hence, the game is a two-stage game in which through the choice of the debt level firms can credibly enforce a product market strategy that they could not otherwise. Our two-stage game in this paper has similar features although the choice of the rating technology does not imply a commitment effect for the banks. Although two-stage games are widely used in industrial organizations the class of models that we introduce here differs from the existing literature. We allow firms in the first stage to choose among two types of technologies that each result in a different distribution and not in a given level of costs.

Our paper is organized as follows. In the next section we present our two-stage game. Section 3 presents the results and a detailed discussion of their economic intuition and finally section 4 concludes the paper.

# 2 Model

We consider a loan market in which two strategically interacting banks face two types of decisions. In a first stage each bank decides on the rating technology which provides input for the pricing system. In the second stage banks choose their pricing policy. Choices are derived under the assumption that both banks are sequentially rational. This implies that they are aware that the rating technology has an impact on the pricing strategy and hence take this into account when making the technology choice. In the following we present the two-stage game, starting with a description of the first stage in which the rating technology is chosen.

The two banks can decide between two alternative rating systems ($H$ and $L$) with different accuracy (high or low), characterized by the parameters $\sigma_L$ and $\sigma_H$ with $\sigma_L > \sigma_H$. The rating system is used to estimate the individual probabilities of default (PD) of the obligors and thus provides a crucial input parameter for the pricing of the loans. These estimates, however, are not free of error and depend on the accuracy of the rating system. In our model setup the estimated PD of an individual obligor is a random variable that consists of the true PD of the obligor and a white noise error term. To be more specific the estimated PDs are modeled as follows: The true default probability $\pi$ (defined on [0,1]) is transformed to a credit "score" (defined on (-$\infty$,+$\infty$)) using the inverse of the standard normal distribution function $N^{-1}$ (probit transformation). Such a transformation is common when regression models are used for PD estimation (for a recent overview see e.g. Westgaard and van der Wijst (2001)). Note that the probit transformation is consistent with standard structural credit risk models (see e.g. Merton (1974)) where the probit transformation links the PD with the (standardized) asset value. After the transformation of the

PD to the corresponding credit score, a normally distributed error term $\epsilon$ with zero mean and standard deviation $\sigma_H$ or $\sigma_L$ is added to this credit score. This estimated credit score is finally transformed back by $N$ to the estimated default probability $\hat{\pi}$:

$$\hat{\pi} = N(N^{-1}(\pi) + \epsilon).$$

This model setup implies that the estimated credit scores are *unbiased*. It is possible to extend our model to non-zero means for the rating errors providing banks with an additional decision variable. However, in order to gain better insight from our results we restrict ourselves to the use of unbiased ratings only.

With the choice of the rating technology, a certain level of fixed costs is associated with the purchase or development of such a system, denoted by $c_L$ and $c_H$ for rating systems $L$ and $H$, respectively (with $c_L < c_H$). Banks are aware of these fixed costs and of the effect the rating system accuracy has on the PD estimates, and have to decide which rating system to employ. This technology choice is the outcome of the first stage of the game while the pricing game is solved thereafter. Since we look for a sequentially rational equilibrium we apply backward induction and solve the pricing game for each obligor first, as presented in the following.

The loan market is characterized by a distribution of default probabilities of individual obligors. While the distribution refers to the entire population of obligors, we now only concentrate on the pricing of a single (representative) borrower characterized by her default probability drawn from this distribution. For this borrower, however, banks do not observe the true but the estimated probability of default based on their rating systems (bank 1 observes $\hat{\pi}_1$ and bank 2 observes $\hat{\pi}_2$).

In the pricing game bank 1 and bank 2 decide on the credit spreads for the obligor denoted by $s_1$ and $s_2$, where this spread is defined as the charged interest rate minus the riskfree rate. In an imperfect market the demand for loan volume at bank $i$ and $j$ depends on the credit spreads charged to the individual obligors by both banks, i.e.

$$q_i = \alpha_i + \beta_i(s_j - s_i) - \gamma_i s_i, \quad i, j \in \{1, 2\}, i \neq j$$

where $q_i$ is the amount of money extended to the obligor, and $\alpha_i, \beta_i$ and $\gamma_i$ are positive parameters. The demand functions are standard linear ones in which the different parameters can be given the following interpretation. $\alpha_i$ measures all the spread independent (autonomous) loan demand of bank $i$. It captures the impact of the level of the risk free interest rate as well as all economic forces that lead to a loan decision independent of the credit spread. $\beta_i$ is a measure of substitutability between a loan offered by bank $i$ and one offered by bank $j$. If $\beta_i$ is very high small differences in the two credit spreads have significant consequences on loan demand. This implies that we can use the parameters $\beta_i$ as measures for the level of competition in the banking sector. Finally, $\gamma_i$ measures the impact of the credit spread of bank $i$ on its loan demand, i.e., it measures the absolute effect of the credit spread.

For a certain level of $s_i$, the estimated profits $\hat{R}_i$ for bank $i$ are given by

$$\hat{R}_i = (1 - \hat{\pi}_i)s_i q_i - \hat{\pi}_i \delta q_i$$

where $s_i$ represents the earned credit spread in the case of no default and $\delta$ is the loss rate in default, which is assumed to be constant and known. This implicitly assumes that in default there is zero recovery of the credit spread. It is important to point out that the estimated PD for a single obligor and hence the loan costs are taken as given when the bank makes its pricing choice. While for each obligor the bank incurs these variable costs, the rating technology has fixed costs $c_L$ or $c_H$ depending on the technology choice. These fixed costs need not be taken into account in the pricing game, because they do not depend on price and quantity of the loan. They have to be considered, however, in the stage of the technology choice.

For given estimated PDs of the borrower, banks choose their optimal credit spreads by maximizing expected profits. This implies the first order conditions

$$\frac{d\hat{R}_i}{ds_i} = 0, \quad i = 1, 2$$

which result in a system of two linear equations given by

$$\alpha_i + (\beta_i + \gamma_i)(\hat{l}_i - 2s_i) + \beta_i s_j = 0, \quad i, j \in \{1, 2\}, i \neq j$$

where

$$\hat{l}_i = \frac{\hat{\pi}_i \delta}{1 - \hat{\pi}_i}$$

The solution to this equation system determines the equilibrium credit spreads $\bar{s}_1$ and $\bar{s}_2$ given by

$$\bar{s}_i = \frac{\alpha_j \beta_i + (\beta_j + \gamma_j)(2\alpha_i + 2\beta_i \hat{l}_i + 2\gamma_i \hat{l}_i + \beta_i \hat{l}_j)}{3\beta_i \beta_j + 4\beta_j \gamma_i + 4\beta_i \gamma_j + 4\gamma_i \gamma_j}.$$

For these equilibrium spreads the respective loan volumes $\bar{q}_1$ and $\bar{q}_2$ are determined according to the demand function. After the banks solved their pricing game they now optimally choose rating technologies.

As can be seen from equilibrium credit spreads, prices and volumes are dependent on the banks own estimated PD as well as on the estimated PD of the rival bank via $\hat{l}_i$ and $\hat{l}_j$. Noisy estimates of the PD have asymmetric effects: Assume that the rival bank has estimated the correct true PD. If the own estimated PD is higher (lower) than the bank will offer a higher (lower) price and receives a lower (higher) quantity. Therefore the bank grants on average a low quantity of overpriced and a high quantity of underpriced loans. This adverse selection effect deteriorates the realized profits of the bank. Note that in the pricing game the bank cannot avoid adverse selection, because it knows that its rating system has white noise errors and therefore the estimated PD is the most accurate information which is available. However, a rating system with higher accuracy can mitigate this effect. This advantage has to be compared to the cost of the rating system. Thus, the rating technology choice requires to specify the expected profits that are the basis for this decision problem, and to quantify the adverse selection effect.

Realized profits $R_i$ of bank $i$ for a borrower are given by

$$R_i = (1 - I)s_i q_i - I\delta q_i$$

where $I$ is the indicator function that takes on the value 0 if there is no default and 1 if there is default. Choosing credit spreads and corresponding loan volumes as in the pricing game specified above, realized profits in equilibrium $\bar{R}_i$ become

$$\bar{R}_i = (1 - I)\bar{s}_i\bar{q}_i - I\delta\bar{q}_i.$$

Remember that $\bar{s}_i = \bar{s}_i(\epsilon_i, \epsilon_j, \pi)$ and $\bar{q}_i = \bar{q}_i(\epsilon_i, \epsilon_j, \pi)$, as $\bar{s}_i$ and $\bar{q}_i$ depend on $\hat{\pi}_i$ and $\hat{\pi}_j$, and thus on $\pi$ and the errors $\epsilon_i$ and $\epsilon_j$ both banks made in their default probability estimations. Realized profits therefore depend on the realization of four random variables: the estimation errors $\epsilon_i, \epsilon_j$, the true default probability $\pi$ (note that ex ante we only know the distribution of default probabilities of the obligor population) and the indicator function $I$. Turning to the expected profits, under the assumption that the four random variables are independent and due to the simplicity of the indicator function ($I = 0$ with probability $1 - \pi$ and $I = 1$ with probability $\pi$), we obtain for the expectation of profits $E(\bar{R}_i)$ for bank $i$

$$\begin{aligned} E(\bar{R}_i) &= E((1 - I)\bar{s}_i\bar{q}_i - I\delta\bar{q}_i) = E((1 - E(I))\bar{s}_i\bar{q}_i - E(I)\delta\bar{q}_i) = E((1 - \pi)\bar{s}_i\bar{q}_i - \pi\delta\bar{q}_i) \\ &= \int_0^1 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ((1 - \pi)\bar{s}_i\bar{q}_i - \pi\delta\bar{q}_i)\ f_1(\epsilon_i)f_2(\epsilon_j)f_3(\pi)d\epsilon_i\ d\epsilon_j\ d\pi, \end{aligned}$$

where $f_1, f_2$, and $f_3$ are the probability density functions of the random variables $\epsilon_i, \epsilon_j$, and $\pi$. This integral can only be solved numerically.

When banks choose their rating technology in the first stage of the game, they take expected profits as basis for their decision. Each bank has two alternative rating systems available, one with high and one with low accuracy. The level of accuracy enters the expected profits via the estimation errors $\epsilon_i$ and $\epsilon_j$. At this point, however, the fixed costs of the rating technologies have to be taken into account. For the two different levels of accuracy $H$ and $L$ we therefore obtain a matrix of expected payoffs for both banks given by

$$M = \begin{pmatrix} E(\bar{R}_1^{H,H}) - c_H,\ E(\bar{R}_2^{H,H}) - c_H & E(\bar{R}_1^{H,L}) - c_H,\ E(\bar{R}_2^{H,L}) - c_L \\ E(\bar{R}_1^{L,H}) - c_L,\ E(\bar{R}_2^{L,H}) - c_H & E(\bar{R}_1^{L,L}) - c_L,\ E(\bar{R}_2^{L,L}) - c_L \end{pmatrix},$$

where $E(\bar{R}_i^{X,Y})$ is the expected profit for bank $i$ whenever bank 1 uses rating system $X$ and bank 2 uses $Y$. In the first stage of the game, banks play a game based on this payoff matrix.

As we have applied backward induction, the Nash equilibria of this game, consequently, are the equilibria of the two-stage game for sequentially rational players.

# 3 Results

This section contains numerical examples based on the model presented in Section 2. To illustrate the decision of banks whether to invest into the more accurate rating technology we present various scenarios based on the oligopolistic market structure.

To derive equilibrium technology choices we calculate the Nash equilibria for the matrix game $M$ and analyze how equilibria change as we change the composition of parameter values. Depending on parameter specifications four different pure Nash equilibria are possible, denoted as $H/H$, $H/L$, $L/H$, and $L/L$, where the first position indicates the strategy used by bank 1 ($H$ corresponds to the rating system with high accuracy, $L$ to the rating system with low accuracy). If there exists a Nash equilibrium in mixed strategies, it is denoted as $mixed$. Apart from showing the Nash equilibria for varying parameters, we also present the corresponding profits for both banks, the credit spreads and credit volumes if the banks play the strategy corresponding to the Nash equilibrium.

We start with a representative base case where the following parameters are chosen: $\alpha_1 = \alpha_2 = 0.5, \beta_1 = \beta_2 = 5, \gamma_1 = \gamma_2 = 2, \delta = 0.5, \pi = 3\%$. Thus, the distribution of the true PD is replaced by a constant value. This facilitates the calculation of the expected profits done by numerical integration. However, we assume that banks cannot take advantage of this simplification but they still assume that the true PDs are drawn from a distribution. The two banks can choose between two rating technologies characterized by $\sigma_H = 0$ and $\sigma_L = 0.7$. Rating technology $H$ therefore always provides the correct rating, i.e. the true PD of the obligor, whereas when using rating technology $L$, a normally distributed error (with mean $0$ and standard deviation $\sigma_L = 0.7$) is added to the score corresponding to the true PD. We assume that rating technology $L$ involves no fixed costs, i.e. $c_L = 0$, while we vary the fixed costs of rating technology $H$, $c_H$, to investigate their influence on the Nash equilibria.

It is the main purpose of our numerical analysis to examine the trade-off between additional rating accuracy and the respective additional costs of this accuracy. Thus, for the choice of the parameters representing the accuracy and the costs of the rating technologies only their *relative* magnitude is important. The parameters characterizing the demand functions are calibrated in such a way that the pricing game leads to empirically reasonable outcomes. However, extensive numerical calculations not presented in the paper showed that the main results are virtually identical across different parameter constellations. Only the absolute level of fixed costs where changes in equilibria are observed varies, the relative pattern remains unchanged.

The fixed costs of rating technologies have a great influence on the decision to invest into the rating system. Fig. 1 shows the Nash equilibria of the matrix for varying fixed costs of the high accuracy rating technology $H$. Note that given the banks technology decisions, varying the fixed costs does not alter the quantity and credit spread the banks offer, but only changes their expected profits. Due to this fact, the Nash equilibria of the matrix $M$ may change.

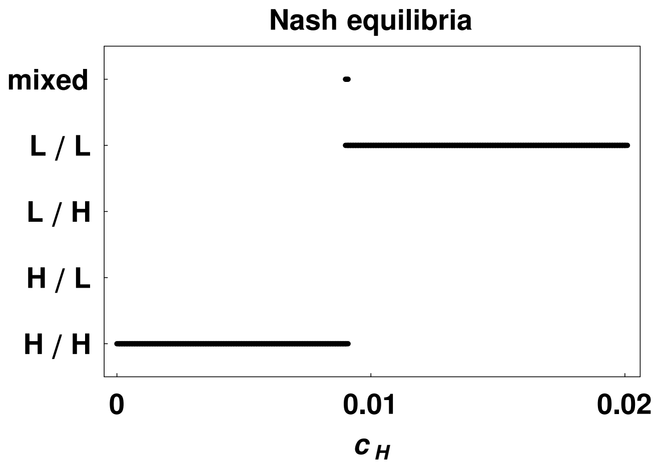


Figure 1: Nash equilibria of the matrix game with expected profits $M$ for the base case parameters. The four possible Nash equilibria in pure strategies are denoted as $H/H$, $H/L$, $L/H$, and $L/L$, where the first position indicates the strategy used by bank 1 ($H$ corresponds to the rating system with high accuracy, $L$ to the rating system with low accuracy). We vary fixed costs of the high accuracy rating technology $H$, $c_H$, and set $c_L = 0$. For high $c_H$, the Nash equilibrium strategy is to choose rating technology $L$, whereas for sufficiently small $c_H$, the Nash equilibrium strategy is to choose rating system $H$, for both banks. Parameters: $\alpha_1 = \alpha_2 = 0.5, \beta_1 = \beta_2 = 5, \gamma_1 = \gamma_2 = 2, \delta = 0.5, \pi = 3\%, c_L = 0, \sigma_H = 0$ and $\sigma_L = 0.7$.

For very high cost of the rating technology both banks decide not to invest. When the costs fall below a certain threshold it is optimal for both banks to simultaneously change to the accurate rating technology. The expected profits when both banks play the Nash equilibrium strategy are shown in Fig. 2.

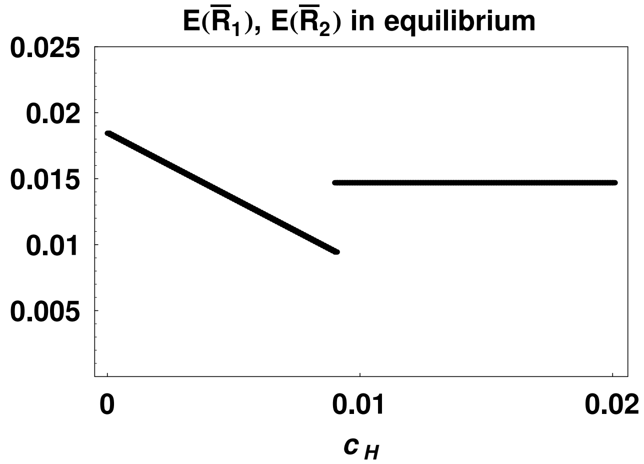


Figure 2: Expected profits for both banks when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$ using the base case setup. As the parameters and the resulting Nash equilibria in pure strategies are symmetric, $E(\bar{R}_1)$ is identical to $E(\bar{R}_2)$. Parameters as in Fig. 1.

As long as the banks decide not to invest (i.e. as long as the accurate rating technology is too "expensive") but to use the worse rating technology, the expected profits stay at a certain constant level, as we set $c_L = 0$. When the fixed costs $c_H$ fall below a certain threshold (the 'investment threshold'), the Nash equilibrium changes from $L/L$ to $H/H$. Both banks invest but the profits drop to a lower level with both banks being worse off than before. Whereas Nash equilibrium $L/L$ is Pareto-optimal, the Nash equilibrium $H/H$ no longer is, at least for fixed costs close to the investment threshold. For very small fixed costs, the profits become even higher than for the scenario where both banks do not invest, and the Nash equilibrium $H/H$ becomes Pareto-optimal.

Thus, the availability of a cheaper rating technology is not always in the interest of banks. If the costs are just below the investment threshold, both banks are worse of than before. Only when the cost of the rating technology $H$ is substantially lower than the investment threshold, banks are better off.

Now we analyze the consequences of the rating technology decision for the credit market itself. Fig. 3 and Fig. 4 present the credit volumes and the credit spreads for the base case.

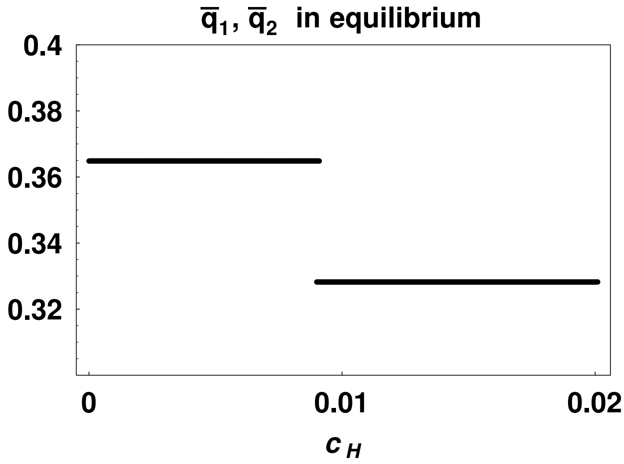


Figure 3: Credit volumes for both banks when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$ using the base case setup. As the parameters and the resulting Nash equilibria in pure strategies are symmetric, $\bar{q}_1$ is identical to $\bar{q}_2$. Parameters as in Fig. 1.

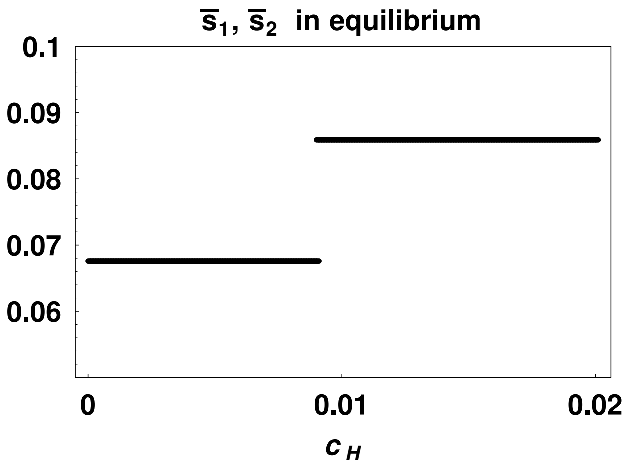


Figure 4: Credit spreads offered by both banks when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$ using the base case setup. As the parameters and the resulting Nash equilibria in pure strategies are symmetric, $\bar{s}_1$ is identical to $\bar{s}_2$. Parameters as in Fig. 1.

Basically the situation for obligors is improved when banks invest into their rating technology. When both banks invest into the accurate rating system, the credit volume increases and the credit spreads decrease, i.e. more obligors have access to loans and the loans are cheaper for all obligors. Thus for the obligors in the credit market it is of clear advantage when the accurate rating technology is available at a price below the investment threshold.

So far we have focused on banks with symmetric parameter configuration and cost structures. In the following we assume that banks differ in their autonomous demand parameter $\alpha_i$. (we set $\alpha_1 = 0.75$ and $\alpha_2 = 0.25$). Fig. 5 shows the resulting Nash equilibria.

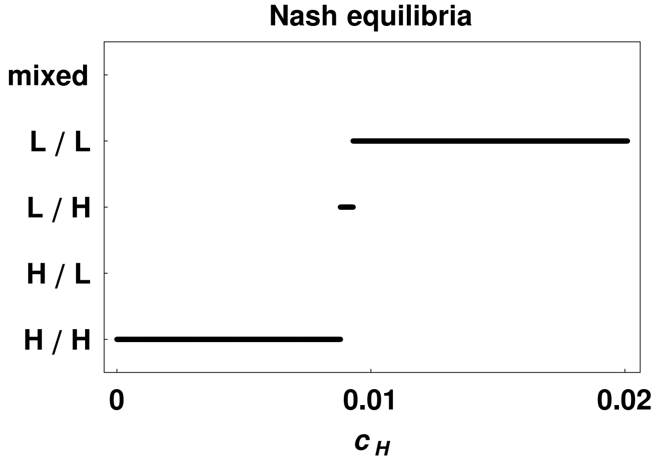


Figure 5: Nash equilibria of the matrix game for expected profits $M$ when banks differ in their autonomous demand parameters $\alpha_i$. The four possible Nash equilibria in pure strategies are denoted as $H/H$, $H/L$, $L/H$, and $L/L$, where the first position indicates the strategy used by bank 1 ($H$ corresponds to the rating system with high accuracy, $L$ to the rating system with low accuracy). We vary fixed costs of the high accuracy rating technology $H$, $c_H$, and set $c_L = 0$. For high $c_H$, the Nash equilibrium strategy is for both banks to choose rating technology $L$, whereas for intermediate $c_H$, bank 2 with lower autonomous demand changes to rating technology $H$ while bank 1 remains with rating technology $L$. For small $c_H$ the Nash equilibrium strategy is for both banks to choose rating technology $H$. Parameters: $\alpha_1 = 0.75, \alpha_2 = 0.25, \beta_1 = \beta_2 = 5, \gamma_1 = \gamma_2 = 2, \delta = 0.5, \pi = 3\%, \sigma_H = 0$ and $\sigma_L = 0.7$.

When the rating technology is expensive both banks do not invest as in the base case. Interestingly, as the costs for the high precision rating technology $H$ get cheaper there exists a threshold where only the bank with the lower autonomous demand invests into the high precision rating system. As the rating technology costs get particularly low, both banks invest into the rating system $H$.

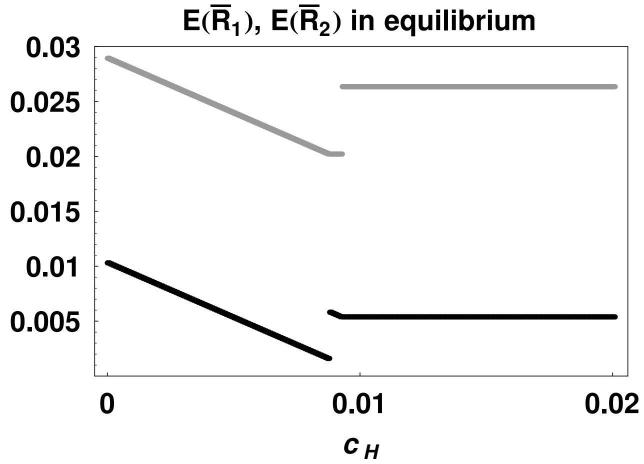


Figure 6: Expected profits $E(\bar{R}_1)$ (light shading) and $E(\bar{R}_2)$ (dark shading) for bank 1 and bank 2 when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$. In contrast to the base case banks differ in their autonomous demand parameter in this setup. Parameters as in Fig. 5.

Concerning the expected profits, the situation is similar to the base case, see Fig. 6. Both banks earn certain levels of profits when they do not invest. These levels are different representing the different market power of the banks. When the investment thresholds for both banks are reached the profits jump to a lower level from which it increases as the costs get lower, and again the profits rise above the non-investment levels as the costs get particularly low. An important fact to note is that for bank 2 with lower autonomous demand, the profits are close to zero at the investment threshold. Thus the bank is dangerously close to the point where it would leave the market or where the probability for bankruptcy is very high for the bank.

The effects for the credit market are basically the same as in the base case. Overall the loans get cheaper and more credit volume is available when banks adopt high accuracy rating systems, see Fig. 7 and Fig. 8. It is important to note that the two banks offer different levels of credit spreads and credit volumes because banks are heterogeneous.

In the analysis presented above, we assume that banks make their technology decision by taking into account the adverse selection effect, i.e. banks anticipate the errors of their rating technology. In further numerical calculations (not shown in details) we analyze the situation for banks not aware of (or not being able to quantify) the adverse selection effect, i.e. banks

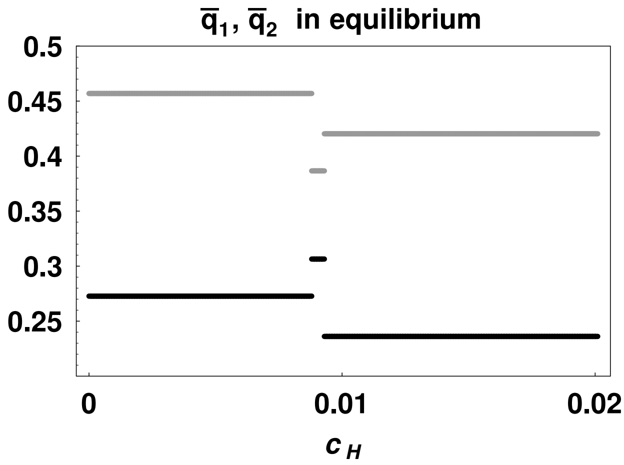


Figure 7: Credit volumes $\bar{q}_1$ (light shading) and $\bar{q}_2$ (dark shading) for bank 1 and bank 2 when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$. In contrast to the base case banks differ in their autonomous demand parameter in this setup. Parameters as in Fig. 5.

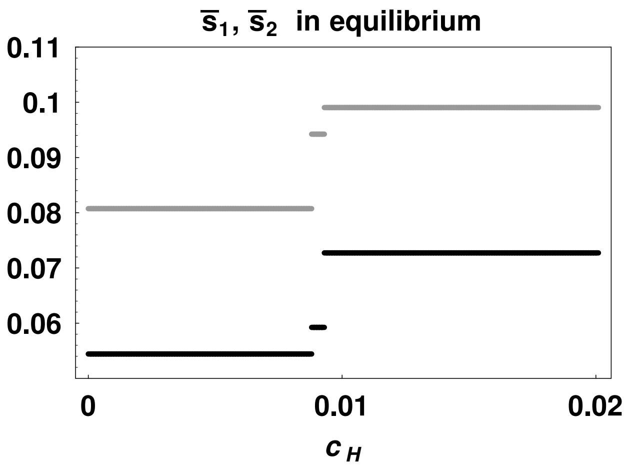


Figure 8: Credit spreads $\bar{s}_1$ (light shading) and $\bar{s}_2$ (dark shading) offered by bank 1 and bank 2 when they play the Nash equilibrium strategy for varying fixed costs $c_H$ of the high accuracy rating technology $H$. In contrast to the base case banks differ in their autonomous demand parameter in this setup. Parameters as in Fig. 5.

estimate the expected profits assuming that the estimated PDs for the obligors correspond to the true PDs. The main result is that banks will choose to invest into the high accuracy rating system far too late (i.e. when cost are much lower compared to the investment threshold of the previous analysis) with negative consequences on their profits. Even for particular low costs for the high accuracy rating system the decision to invest into the rating system appears not to be Pareto-optimal for the banks. Furthermore, ignoring adverse selection effects is critical for small banks which are assumed to have lower autonomous demand parameters $\alpha_i$. Here the suboptimal technology choice will result in realized profits which are potentially negative. The effect on the credit markets is basically the same as in the base case (reduction in credit spreads and the increase in credit volume) for the cost ranges where the banks invest into the high accuracy rating system.

Our findings have two important implications for banking regulation. First, it seems clear from our results that changes in the costs of the better rating technology have a potential impact on the Nash equilibrium strategy of banks. The new standards for minimum capital requirements set forth by Basel II are intended to create incentives to adopt a more accurate rating technology. In particular, banks are deemed to expect a certain reduction of their costs of equity when improving their rating technology. Depending on the size of the cost reduction we can distinguish three different scenarios in our model, when the equilibrium without cost reduction is $L/L$. (i) The cost reduction is very small such that the new equilibrium remains at $L/L$, i.e. both banks do not change their strategy. In this scenario the incentives set by the regulator is simply too small to have an impact on bank behavior. (ii) The cost reduction is 'medium' such that the new equilibrium is $H/H$ but with lower profits for the banks. In this scenario the regulators will only partially achieve their goals: Banks will improve their rating technology and - as desired - competition will decrease the loan rates. On the other hand competition will deteriorate the profitability of banks with ambiguous impact on financial stability. (iii) The cost reduction is sufficiently large such that banks will adopt the $H/H$ strategy by ensuring lower loan rates *and* higher profitability of banks at the same time. In this scenario regulators will additionally foster financial stability.

Second, in those scenarios where the cost reduction leads to decreasing profits for banks, banks with higher autonomous demand (usually larger banks with strong customer relations) are better off. As larger banks are likely to have access to a better rating technology at lower costs than smaller banks, we conclude that smaller banks are particularly exposed to disadvantageous effects of banking regulation as described above.

Summarizing, is seems of particular importance for the judgement of the effects of regulatory activities whether the implied cost reduction is large enough to ensure a new equilibrium which is Pareto-optimal with advantages for both obligors and banks.

# 4 Discussion

The management of credit risk exposures is an important challenge for the whole banking sector. Accurate rating systems providing proper estimates of the risk parameters are of key importance for banks. In our model rating systems estimate the default probabilities for the individual borrowers and thus provide important input for the loan pricing. While under perfect competition all banks will invest in the most accurate available rating technology by equating marginal costs and profits, the situation under imperfect competition is more complex. In this paper we analyze the choice of the rating technology in an oligopolistic market. We model the technology choice and the pricing as a two-stage game. In the first stage banks choose the rating technology and in the second stage banks choose their pricing policy given the imperfect (oligopolistic) market.

This choice is derived under the assumption that each bank is sequentially rational. This implies that banks are aware of the impact of the rating technology choice on the pricing strategy. Applying a backward induction we solve the pricing game first and define a matrix game for the technology choice. The pricing game is modeled in a risk-based lending environment which is the logic choice since banks have estimates of the individual default probabilities of its borrowers available. Since in such a framework the credit spread offered to one obligor might be different among banks, it seems natural to model the loan market as a Bertrand-type oligopoly. In this environment adverse selection leads to an advantage for banks with a more accurate rating system.

Our presented probabilistic framework and the modeling of the technology choice is novel in the banking literature and can provide important insight. In a comparative static analysis we study the implications for a market with two banks, which can employ two different rating systems (low or high accuracy). An equilibrium were both banks use the low accuracy rating system is only left when the cost for the high accuracy rating system is sufficiently low. Interestingly, at this threshold banks are worse off than in the previous equilibrium whereas the borrowers are better off (higher credit volume and lower credit spread). In particular we show that this decrease in profits can be severe for banks with low market power or with a suboptimal technology choice. Moreover, we provide evidence that for particularly small costs of the accurate rating system the profits of the banks increase. In such a case the adoption of a new rating technology would lead to lower interest rates *and* higher profits for the banks at the same time.

This has important implications for banking regulation which aims to provide incentives to use high accuracy rating system (e.g. Basel II regulation). If the incentives results in low enough costs for high accuracy rating systems, banks and borrowers will be better off. However a small reduction of the costs can lead to unwanted disadvantages for certain banks.

Dynamic frameworks of strategy adaptations for the rating technology choice are left for future research.

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