PRICING AND HEDGING CDOs WITH LÉVY BASE CORRELATION

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We consider a collateralized debt obligation (CDO) with standard credit default swap (CDS) indices as the reference portfolio. Such a CDO is referred to as synthetic CDO, and it is designed to transfer the credit risk on a reference portfolio of assets between parties. CDOs have recently become very popular credit instruments.

A standard feature of a CDO structure is the tranching of credit risk, i.e., creating multiple tranches of securities which have varying degrees of seniority and risk exposure: the equity tranche is the first to be affected by losses in the event of one or more defaults in the portfolio. If losses exceed the value of this tranche, they are absorbed by the mezzanine tranche(s). Losses that have not been absorbed by the other tranches are sustained by the senior tranche and finally by the super-senior tranche. In such a way, each tranche protects the ones senior to it from the risk of loss on the underlying portfolio. The CDO investors take on exposure to a particular tranche, effectively selling credit protection to the CDO issuer, and in turn collecting premiums (spreads). Tranche spreads are daily quoted in the market.

To price a synthetic CDO, one needs a model that captures the dependency structure in the underlying portfolio and gives a good fit to the market prices of different tranches simultaneously. Moreover, the model is required to provide an arbitrage-free pricing of off-market tranches. The standard model for pricing CDOs established in the market is the Gaussian Copula model (see e.g. Vasicek [4]). It is basically a one-factor model with an underlying multivariate normal distribution. Actually, a very simple multivariate normal distribution is employed: all correlation between different components are taken equal. The one-factor Gaussian copula model is well-known not to provide an adequate solution for pricing simultaneously various tranches of a CDO, leading to the correlation smile. In order to deal with this problem, the base correlation concept was initiated (see e.g. O'Kane and Livasey [3]). Similarly to implied volatility in an equity setting, one uses a different base correlation for each tranche to be priced. Due to the construction, base correlation is quite adapted to interpolation for nonstandard tranches. One of the prime applications of base correlation is thus pricing bespoke tranches. The application of the Gaussian base correlation may, however, lead to arbitrage opportunities, providing higher prices for tranchlets with higher seniority. Another weakness of the Gaussian base correlation is that it significantly depends on the interpolation scheme.

A set of other one-factor models has recently been proposed in the literature. Albrecher, Ladoucette, and Schoutens [1] unified these approaches and proposed a generic one-factor Lévy model. Lévy models bring more flexibility into the dependence structure and allow tail dependence.

Several Lévy models that extend the classical Gaussian copula model were investigated and compared in Garcia, Goossens, Masol, and Schoutens [2]. In particular, shifted Gamma, shifted

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inverse Gaussian, and shifted CMY models were considered. The proposed models are very tractable and perform significantly better than the Gaussian copula model.

Furthermore, the concept of Lévy base correlation was introduced in [2]. The use of this Lévy base correlation is completely analogous as in the Gaussian case. We illustrate this by pricing tranchlets. By a historical study, we show that the Lévy base correlation curve is always much flatter than the Gaussian counterpart. This indicates that the Lévy models do fit the observed data much better. Additionally, flat base correlation curves are much more reliable for pricing of bespoke tranches because the curve flatness allows to reduce the interpolation error. Using Lévy base correlation, we price bespoke tranches of a synthetic CDO, and compare the prices obtained under Gaussian and Lévy models. Historical study shows that Lévy base correlation is much less dependent on the interpolation technique than the Gaussian counterpart and, even better, does not lead to arbitrage opportunities in the cases when Gaussian model does.

In this paper, we study and compare delta hedge parameters of the different models. We focus on two common approaches to delta-hedge a CDO tranche: first, hedging a tranche with the index, and, second, hedging the equity tranche with the mezzanine tranche. The dynamics of the deltas with respect to the index is similar under all models. The difference is only in scale: equity deltas under the Lévy models are approximately 25% higher than equity deltas under the Gaussian, while the mezzanine and senior deltas under the Lévy models are 20% and, respectively, 40% lower than the corresponding Gaussian deltas. As it concerns the dynamics of the hedge ratios over time, it is similar for all the models, hedge ratios under the Lévy models being approximately 30% higher than those under the Gaussian.

References

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