A new approach for measuring credit contagion

Jiwook Jang
Department of Actuarial Studies
Division of Economics and Financial Studies
Macquarie University, Sydney
Australia

Presentation to International Financial Research Forum in Paris

28 March 2008

Financial contagion

- Due to domestic and global business and financial links between lines of business (or companies), a primary event (or a shock) may initially only effect a line of business or a particular company.

- Primary events (or shocks) could be subprime mortgage meltdown, oil and commodity price movements, the governments' fiscal and monetary policies, the release of corporate financial reports, the political and social decisions, the romours of mergers and acquisitions among firms, collapse and bankruptcy of firms, September 11 WTC catastrophe and Hurricane Katrina etc.

Financial contagion (Cont.)

- In financial industry, a shock which initially affects a couple of institutions or a particular region of the economy spreads to the rest of the financial industry and then infect the larger economy. This is called 'financial contagion' (Allen and Gale, 2000, Bae et al, 2003).

- The stock price falling of *Countrywide Financial Corporation* and closing down of *New Century Financial Corporation* due to mismanagement of sub-prime mortgage in 2007 US are one of the particular and recent examples of financial contagion.

Financial contagion (Cont.)

- The prevalence of above financial contagion has led to bankruptcy and default of mortgage lenders in US announcing their significant losses.

- This crisis also has led to the collapse of stock prices in worldwide and it could shake global financial markets further due to new waves of default.

Credit contagion

- Main causes of defaults are due to domestic and global business and financial links or ties between firms.

- All firms financial stability is universally subject to macroeconomic factors such as the price of energy and minerals, interest rates, mortgage rate and exchange rate. Also the global economic system that allows free trades for goods & services and investment makes them highly dependent each other.

- As a result, a shock to a business sector or a region/country can create a series of default locally and globally. Hence we focus on credit market in financial contagion (simply we call it 'default or credit contagion').

A new approach

- In order to accommodate waves of default from a shock, we consider a new intensity process: **the multiple shot noise intensity**. This consists of n component processes, where each process acts as a jump intensity for the next one.

- The multivariate default intensity model we consider has the following structure:

Multiple shot noise intensity

$$d\lambda_t^{(n)} = -\delta^{(n)}\lambda_t^{(n)}dt + dC_t^{(n)}, \quad C_t^{(n)} = \sum_{j=1}^{M_t^{(n)}} Y_j^{(n)},$$

$$d\lambda_t^{(n-1)} = -\delta^{(n-1)}\lambda_t^{(n-1)}dt + dC_t^{(n-1)}, \quad C_t^{(n-1)} = \sum_{k=1}^{M_t^{(n-1)}} Y_k^{(n-1)},$$

$$\vdots$$

$$d\lambda_t^{(1)} = -\delta^{(1)}\lambda_t^{(1)}dt + dC_t^{(1)}, \quad C_t^{(1)} = \sum_{l=1}^{M_t^{(1)}} Y_l^{(1)},$$

where:

Multiple shot noise intensity (Cont.)

· $\left\{Y_j^{(i)}\right\}_{j=1,2,\cdots}$, $\left\{Y_k^{(i)}\right\}_{k=1,2,\cdots}$, $\cdots$, $\left\{Y_l^{(i)}\right\}_{l=1,2,\cdots}$ are sequences of independent but not identically distributed random variables with distribution function $G(y^{(i)})$ $\left(y^{(i)} > 0\right)$ and $i = n, n-1, \cdots, 1$.

· $M_t^{(n)}$ is the total number of events up to time t.

· $\delta^{(i)}$ is the rate of exponential decay for the firm $i = n$, $n-1$, $n-2$, $\cdots$, 1.

We also make the additional assumption that the point process $M_t^{(i)}$ and the sequences $\left\{Y^{(i)}\right\}$ are independent of each other.

$$d\lambda_t^{(n)} = -\delta^{(n)}\lambda_t^{(n)}dt + dC_t^{(n)}, \quad C_t^{(n)} = \sum_{j=1}^{M_t^{(n)}} Y_j^{(n)},$$

$$d\lambda_t^{(n-1)} = -\delta^{(n-1)}\lambda_t^{(n-1)}dt + dC_t^{(n-1)}, \quad C_t^{(n-1)} = \sum_{k=1}^{M_t^{(n-1)}} Y_k^{(n-1)},$$

$$\vdots$$

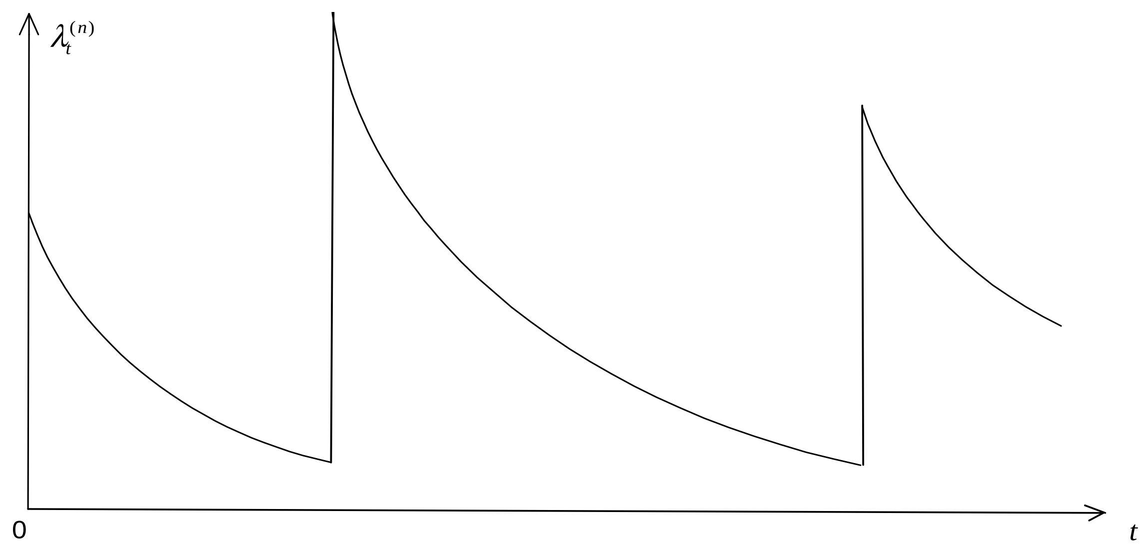
$$d\lambda_t^{(1)} = -\delta^{(1)}\lambda_t^{(1)}dt + dC_t^{(1)}, \quad C_t^{(1)} = \sum_{l=1}^{M_t^{(1)}} Y_l^{(1)},$$

$M_t^{(n)}$ follows a homogeneous Poisson process with frequency ρ and $M_t^{(i)}$ for $i = n-1,\ n-2,\ \cdots,\ 1$ follows the Cox process with frequency $\lambda_t^{(i+1)}$ respectively. So in this model, dependence between the intensities $\lambda_t^{(i)}$ comes from the structure that each process acts a jump intensity for the next one.

$$d\lambda_t^{(n)} = -\delta^{(n)}\lambda_t^{(n)}dt + dC_t^{(n)}, \quad C_t^{(n)} = \sum_{j=1}^{M_t^{(n)}} Y_j^{(n)}$$

The intensity $\lambda_t^{(n)}$ is triggered by a primary events (or a shock) such as subprime mortgage meltdown, oil and commodity price movements, the governments' fiscal and monetary policies, the release of corporate financial reports, the political and social decisions, the romours of mergers and acquisitions among firms, collapse and bankruptcy of firms, September 11 WTC catastrophe and Hurricane Katrina etc. that will result in a positive jump in intensity process. As time passes, the intensity process decreases, as the firm *n* in the market will do its best to avoid being in bankruptcy after the arrival of a primary event. This decrease continues until another event occurs which again will result in a positive jump in intensity process.

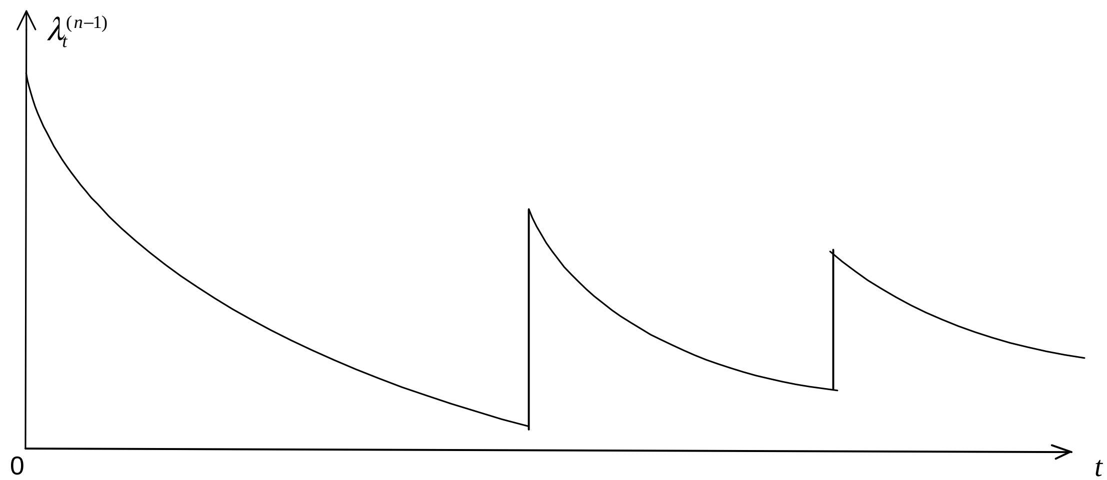
Illustration of shot noise process $\lambda_t^{(n)}$ over a period of time



where ρ is the primary event arrival rate in time period t. From now on, we assume that company *n*'s default intensity follows the shot noise process above.

The intensity $\lambda_t^{(n)}$ is the jump arrival rate for the $(n-1)^{th}$ firm's default intensity $\lambda_t^{(n-1)}$ and the intensity $\lambda_t^{(n-1)}$ is the jump arrival rate for the $(n-2)^{th}$ firm's default intensity $\lambda_t^{(n-2)}$ and so on. Hence the intensity $\lambda_t^{(n)}$ is the **prime trigger** in influencing all other relative local/global firms' default intensities. As time passes, the intensity processes for the firm $n-1$, $n-2$, $\cdots$ decrease, as these firms will also do their best to avoid being in bankruptcy from the influence by the prime company's default intensity $\lambda_t^{(n)}$ that is triggered by primary events (or shocks).

Illustration of shot noise intensity, $\lambda_t^{(n-1)}$ for the firm *n*-1



where $\lambda_t^{(n)}$ is the jump arrival rate for the $(n-1)^{th}$ firm's default intensity $\lambda_t^{(n-1)}$. From now on, we assume that company *n*-1's default intensity follows the shot noise process above.

Overview

- The multiple shot noise intensity to accommodate waves of default from a shock, i.e. credit contagion.

- The Cox process is used to model the multivariate default time and derive multivariate survival/default probabilities. In order to obtain these, the joint Laplace transform of the vector $\left(\Lambda_t^{(n)},\ \Lambda_t^{(n-1)}, \cdots, \Lambda_t^{(1)}\right)$ is derived, i.e.

$$\mathbb{E}\left(e^{-\nu_n \Lambda_t^{(n)}} e^{-\nu_{n-1}\Lambda_t^{(n-1)}} \cdots e^{-\nu_1 \Lambda_t^{(1)}} \mid \lambda_0^{(n)}, \lambda_0^{(n-1)}, \cdots, \lambda_0^{(1)}\right)$$

Overview (Cont.)

where $\nu_i \geq 0$ and $\Lambda_t^{(i)} = \int_0^t \lambda_s^{(i)} ds$ for $i = n, n-1, n-2, \cdots, 1$.

- An example using this model: measuring market credit default swaps (CDS) rates.

The generator of the process $\left(\Lambda^{(n)}, \Lambda^{(n-1)}, \cdots, \Lambda^{(1)}, \lambda^{(n)}, \lambda^{(n-1)}, \cdots, \lambda^{(1)}, t\right)$

Using the generator of the process $\left(\Lambda^{(n)}, \cdots, \Lambda^{(1)}, \lambda^{(n)}, \cdots, \lambda^{(1)}, t\right)$ acting on a function $f\left(\Lambda^{(n)}, \cdots, \Lambda^{(1)}, \lambda^{(n)}, \cdots, \lambda^{(1)}, t\right)$ belonging to its domain, we can find that

$$\prod_{i=1}^{n} e^{-\nu_i \Lambda_t^{(i)}} \prod_{i=1}^{n} e^{-A_i(t)\lambda_t^{(i)}} e^{D(t)}$$

is a martingale, where

$$A_1(t) = \frac{\nu_1}{\delta^{(1)}} + \left(k_1 - \frac{\nu_1}{\delta^{(1)}}\right) e^{\delta^{(1)} t},$$

$$
\begin{aligned}
A_i(t) &= k_i e^{\delta^{(i)} t} - \nu_i \left(\frac{e^{\delta^{(i)} t} - 1}{\delta^{(i)}} \right) \\
&\quad - e^{\delta^{(i)} t} \int_0^t e^{-\delta^{(i)} s} \left[1 - \hat{g}_{i-1} \left\{ A_{i-1}(s) \right\} \right] ds \\
\text{for } i &= n,\ n-1,\ \cdots,\ 2,
\end{aligned}
$$

$$
D(t) = \rho \int_0^t \left[1 - \hat{g}_n \left\{ A_n(s) \right\} \right] ds
$$

and $k_i \geq 0$ and $\hat{g}_i(\varphi) = \int\limits_0^{\infty} e^{-\varphi y^{(i)}} dG(y^{(i)})$.

Joint Laplace transform of the vector $\left(\Lambda_t^{(3)},\ \Lambda_t^{(2)}, \Lambda_t^{(1)}\right)$

- For simplicity, assuming that $n = 3$, we have the joint Laplace transform of the vector $\left(\Lambda_t^{(3)},\ \Lambda_t^{(2)}, \Lambda_t^{(1)}\right)$, i.e.

$$E\left\{e^{-\nu_3\left\{\Lambda_{t_2}^{(3)}-\Lambda_{t_1}^{(3)}\right\}}e^{-\nu_2\left\{\Lambda_{t_2}^{(2)}-\Lambda_{t_1}^{(2)}\right\}}e^{-\nu_1\left\{\Lambda_{t_2}^{(1)}-\Lambda_{t_1}^{(1)}\right\}}\right\},$$

where $\lambda_t^{(3)}, \lambda_t^{(2)}$ and $\lambda_t^{(2)}$ are assumed to be jointly stationary.

Survival probabilities

- If we set $\nu_1 = 0$, $\nu_2 = 0$ and $\nu_3 = 1$ then we have

$$\Pr(\tau_3 > t) = E\left\{e^{-\Lambda_t^{(3)}}\right\} =$$

$$e^{-\rho\times\int_0^\infty\left[1-\hat{g}_3\left\{\left(\frac{1-e^{-\delta^{(3)}(t_2-t_1)}}{\delta^{(3)}}\right)e^{-\delta^{(3)}s}\right\}\right]ds}\times e^{-\rho\int_0^t\left[1-\hat{g}_3\left\{\left(\frac{1-e^{-\delta^{(3)}s}}{\delta^{(3)}}\right)\right\}\right]ds},$$

which is **the survival probability for the prime company** that influences other relative local/global firms' default intensities.

- Using an exponential for $g(y^{(3)}) = \alpha e^{-\alpha y^{(3)}}$, it is given that

$$
\begin{aligned}
& E\left\{e^{-\Lambda_t^{(3)}}\right\} \\
= \; & \left\{\frac{\alpha e^{-\delta^{(3)}t}}{\alpha + \frac{1}{\delta^{(3)}}\left(1 - e^{-\delta^{(3)}t}\right)}\right\}^{\frac{\rho}{\delta^{(3)}}} \\
& \times \left\{\frac{\alpha + \frac{1}{\delta^{(3)}}\left(1 - e^{-\delta^{(3)}t}\right)}{\alpha e^{-\delta^{(3)}t}}\right\}^{\frac{\alpha\rho}{\delta^{(3)}\alpha+1}} .
\end{aligned}
$$

Survival probabilities (Cont.)

- Similarly, we can have the survival probability of the firm 2 infected from the firm 3, i.e.

$$\Pr\left(\tau_2 > t\right) = E\left\{e^{-\Lambda_t^{(2)}}\right\}$$

$$= e^{-\rho \int_0^t \left[1 - \hat{g}_3 \left\{ \begin{array}{c} e^{-\delta^{(3)} s} \\ \int_0^s e^{\delta^{(3)} u} \left[1 - \hat{g}_2 \left\{ \left(\frac{1 - e^{-\delta^{(2)} u}}{\delta^{(2)}} \right) \right\} \right] du \end{array} \right\} \right] ds}$$

$$\times e^{-\rho\times\int_0^\infty\left[1-\hat{g}_3\left\{\begin{array}{c}\left\{\int_0^t e^{-\delta^{(3)}(t_2-s)}\left[1-\hat{g}_2\left(\frac{1-e^{-\delta^{(2)}s}}{\delta^{(2)}}\right)\right]ds\right\}\\ \times e^{-\delta^{(3)}s}\\ +e^{-\delta^{(3)}s}\\ \times\int_0^s e^{\delta^{(3)}u}\left[1-\hat{g}_2\left\{\begin{array}{c}\left(\frac{1-e^{-\delta^{(2)}(t_2-t_1)}}{\delta^{(2)}}\right)\\ \times e^{-\delta^{(2)}u}\end{array}\right\}\right]du\end{array}\right\}\right]ds}$$

and

$$\Pr\left(\tau_1>t\right)=E\left\{e^{-\Lambda_t^{(1)}}\right\},$$

which is the survival probability of the firm 1 infected from the firm 3 and 2.

Joint probabilities

- For simplicity, it is assumed that $n = 3$ but it can be easily extended to the higher dimensions. From the martingale obtained, we derive the expression for the joint survival probability, i.e.

$$\Pr\left(\tau_3 > t,\ \tau_2 > t,\ \tau_1 > t \mid \lambda_0^{(3)}, \lambda_0^{(2)}, \lambda_0^{(1)}\right)$$

$$= \mathbb{E}\left\{e^{-\Lambda_t^{(3)}} e^{-\int_0^t \lambda_0^{(2)} e^{-\delta^{(2)} s} ds} e^{-\int_0^t \lambda_0^{(1)} e^{-\delta^{(1)} s} ds} \mid \lambda_0^{(3)}, \lambda_0^{(2)}, \lambda_0^{(1)}\right\}$$

$$= e^{-\lambda_0^{(3)}\left(\frac{1-e^{-\delta^{(3)}t}}{\delta^{(3)}}\right)} \times e^{-\rho\int_0^t\left[1-\hat{g}_3\left\{\left(\frac{1-e^{-\delta^{(3)}s}}{\delta^{(3)}}\right)\right\}\right]ds}$$

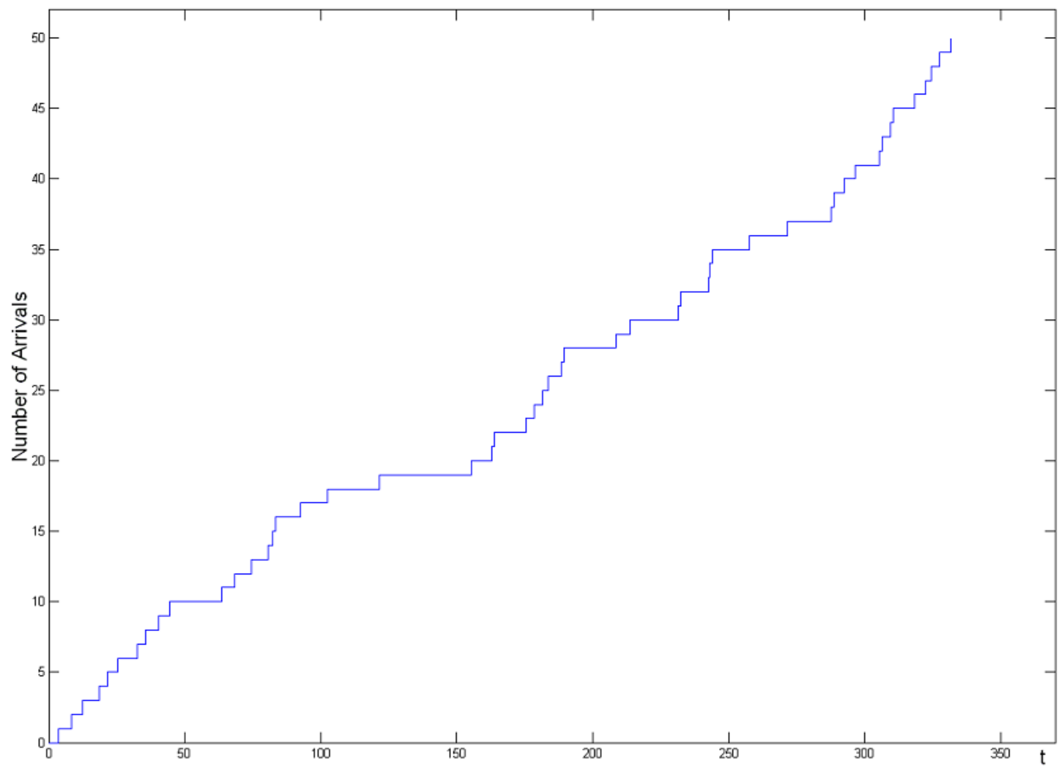
$$\times e^{-\lambda_0^{(2)}\left(\frac{1-e^{-\delta^{(2)}t}}{\delta^{(2)}}\right)} e^{-\lambda_0^{(1)}\left(\frac{1-e^{-\delta^{(1)}t}}{\delta^{(1)}}\right)},$$

where $\tau_i \equiv \inf\left\{t: \; N_t^{(i)} = 1 \mid N_0^{(i)} = 0\right\}$ is the default arrival time for the firm i that is equivalent to the first jump time of the Cox process $N_t^{(i)}$.

- We can also find the expression for the joint default probability, i.e.

$$\Pr\left(\tau_3 \leq t, \; \tau_2 \leq t, \; \tau_1 \leq t \mid \lambda_0^{(3)}, \lambda_0^{(2)}, \lambda_0^{(1)}\right)$$

A sample path of $N_t^{(3)}$ ~ **Poisson** $\left(\lambda_t^{(3)}\right)$



Joint probabilities

- If we consider for $i = 3, 2$ and use an exponential distribution of $G(y^{(3)})$ and $G(y^{(2)})$ for jump size respectively, i.e.

$$g(y^{(3)}) = \alpha e^{-\alpha y^{(3)}} \text{ and } g(y^{(2)}) = \beta e^{-\beta y^{(2)}} \text{ with } \alpha > 0, \ \beta > 0,$$

the corresponding joint survival probability, where $\lambda_t^{(3)}$and $\lambda_t^{(2)}$ are assumed to be jointly stationary, is given by

$$
\begin{aligned}
&\mathsf{Pr}\left(\tau_3 > t,\ \tau_2 > t\right) \\
= \ &\exp\left(-\rho\int_0^\infty \left(\frac{\left(\frac{1-e^{-\delta^{(3)}t}}{\delta^{(3)}}\right)e^{-\delta^{(3)}s} + e^{-\delta^{(3)}s}\int_0^s e^{\delta^{(3)}u}\left(\frac{\left(1-e^{-\delta^{(2)}t}\right)e^{-\delta^{(2)}u}}{\beta\delta^{(2)}+\left(1-e^{-\delta^{(2)}t}\right)e^{-\delta^{(2)}u}}\right)du}{\alpha+\left(\frac{1-e^{-\delta^{(3)}t}}{\delta^{(3)}}\right)e^{-\delta^{(3)}s} + e^{-\delta^{(3)}s}\int_0^s e^{\delta^{(3)}u}\left(\frac{\left(1-e^{-\delta^{(2)}t}\right)e^{-\delta^{(2)}u}}{\beta\delta^{(2)}+\left(1-e^{-\delta^{(2)}t}\right)e^{-\delta^{(2)}u}}\right)du}\right)ds\right) \\
&\times\exp\left(-\rho\int_0^t\left(\frac{1-e^{-\delta^{(3)}s}}{\alpha\delta^{(3)}+\left(1-e^{-\delta^{(3)}s}\right)}\right)ds\right).
\end{aligned}
$$

Conditional default probabilities

- Using Bayes' rule, the conditional default probabilities between firm 3 and 2, denoted by $p_{3|2}$ and $p_{2|3}$ are given by

$$p_{3|2} = \frac{p_{32}}{p_2} = \frac{\Pr(\tau_3 \le t,\ \tau_2 \le t)}{\Pr(\tau_2 \le t)} \quad \text{and} \quad p_{2|3} = \frac{p_{32}}{p_3} = \frac{\Pr(\tau_3 \le t,\ \tau_2 \le t)}{\Pr(\tau_3 \le t)}$$

where $p_{3|2}$ denotes the probability that the firm 3 defaults before t, given that the firm 2 has defaulted before t and $p_{2|3}$ denotes the probability that the firm 2 defaults before t, given that the firm 3 has defaulted before t.

Linear correlation coefficient

- The linear correlation coefficient between indicator random variables for two firms, denoted by $\rho\left(1_{(\tau_3 \leq t)}, 1_{(\tau_2 \leq t)}\right)$, is given by

$$\rho\left(1_{(\tau_3 \leq t)}, 1_{(\tau_2 \leq t)}\right) = \frac{p_{32} - p_3 p_2}{\sqrt{p_3(1-p_3)}\sqrt{p_2(1-p_2)}}.$$

Example: Joint survival/default probabilities

- We assume that the magnitude of the contribution to the default intensity of the firm 3 from the primary events is **higher** than that of the firm 2. We also assume that the decay rate for the firm 3, that measures how quick the firm gets out of the influence of primary events lowering their default intensity rate, is **lower** than that for the firm 2. So the parameter values used to calculate the joint probabilities are

$$\alpha = 5,\ \beta = 10,\ \delta^{(3)} = 0.3,\ \delta^{(2)} = 0.5 \text{ and } \rho = 4.$$

- Setting $t_1 = 0$ and $t_2 = 1$, the calculations of the joint survival/default probability, relevant joint probabilities, the conditional default probability and the linear correlation coefficient are given by

$$
\begin{aligned}
\Pr(\tau_3 > 1,\ \tau_2 > 1) &= 0.060059 \\
\Pr(\tau_3 \le 1,\ \tau_2 > 1) &= 0.54294, \\
\Pr(\tau_3 > 1,\ \tau_2 \le 1) &= 0.026231, \\
\Pr(\tau_3 \le 1,\ \tau_2 \le 1) &= 0.37077.
\end{aligned}
$$

and

$$
p_{3|2} = 0.93393, \quad p_{2|3} = 0.40579 \text{ and } \rho\left\{1_{(\tau_3 \le t)}, 1_{(\tau_2 \le t)}\right\} = 0.058428,
$$

where

$$
\Pr(\tau_3 > 1) = 0.08629 \quad \text{and} \quad \Pr(\tau_2 > 1) = 0.603,
$$

$$
\Pr(\tau_3 \le 1) = 0.91371 \quad \text{and} \quad \Pr(\tau_2 \le 1) = 0.397.
$$

Example: Credit default swap on Daimler Chrysler

- **The trade**: At time $t = 0$, A and B enter a credit default swap on Daimler Chrysler, A as protection buyer and B as protection seller. They have agreed on:

(i) The reference credit : Daimler Chrysler AG.

(ii) The term of the credit default swap: 5 years.

(iii) The notional of the credit default swap: 20m USD.

(iv) The credit default swap rate (or fee): $\overline{s} = 116bp = 1.16\% = 0.0116$.

- **The fee payment**: The credit default swap fee $\overline{s} = 116bp$ is quoted per annum as a fraction of the notional. A pays the fee in regular interval, semi-annally. To make our life easier, we simplify the day count fractions to $\frac{1}{2}$ such that A pays to B:

$$116bp \times \frac{20\text{m}}{2} = 116,000 \text{ USD at } T_1 = 0.5,\ T_2 = 1,\ \cdots,\ T_{10} = 5.$$

These payments are stopped and the CDS is unwound as soon as a default of Daimler Chrysler occurs.

- **The default payment**: Let us assume that at date $t = \tau$, Daimler Chrysler has failed to pay a coupon that was due at this date on one of the bonds listed above. Then the proctection buyer A will notify the proctection seller B of the occurence of a credit event and the credit default swap contract is unwound. Assuming that the price of the defaulted bonds is determined immediately, e.g. 430 USD for a bond of 1000 USD notional. Now the proctection seller pays the difference between this price and the par value for a notional of 20m USD, i.e.

$$\frac{1000-430}{1000} \times 20\text{m USD} = 11.4\text{m USD}.$$

Mathematical expression for CDS rate $\overline{s}$

$$\overline{s} = (1-\pi)\,\frac{\sum_{k=1}^{k_N} e^{rc,s}(0,t_{k-1},t_k)}{\sum_{n=1}^{N}(t_{k_{n+1}}-t_{k_n})\overline{B}^b(0,t_{k_n})},$$

i.e.

$$\overline{s} \times \sum_{n=1}^{N}(t_{k_{n+1}}-t_{k_n})\overline{B}^b(0,t_{k_n}) = (1-\pi)\sum_{k=1}^{k_N} e^{rc,s}(0,t_{k-1},t_k),$$

where we need to consider between **r**eference **c**redit denoted by 'rc' and protection **s**eller denoted by 's'. 'b' denotes protection **b**uyer. The detail expression for $\overline{B}^b(0,t_{k_n})$ and $e^{rc,s}(0,t_{k-1},t_k)$ will be shown later.

A couple of scenarios

- The worst case for the protection buyer: When reference credit and protection seller default.

- The worst case for the protection seller: When reference credit and protection buyer default.

Protection buyer's point of view, need to assess the defaultability of protection seller and vice versa and those defaultabilities should be considered in the credit default swap rate, $\overline{s}$.

$$\frac{\overline{B}^b(0, t_{k_n})}{}$$

- Firstly, let us look at

$$\$1 \times \overline{B}^b(0, t_{k_n}),$$

which can be treated as a zero-coupon **defaultable** bond price. If the protection buyer 'b' defaults, \$1 cannot be paid to protection seller.

- The price of zero-coupon defaultable bond paying $1_{(\tau_b > t)}$ at time t is given by

$$
\begin{aligned}
\overline{B}^{b}(0,t) &= \mathbb{E}\left\{\exp\left(-\int_0^t r_s ds\right) 1_{(\tau_b>t)} \mid r_0,\ \lambda_0^{(b)}\right\} \\
&= \mathbb{E}\left[\exp\left\{-\int_0^t \left(r_s+\lambda_s^{(b)}\right) ds\right\} \mid r_0,\ \lambda_0^{(b)}\right] \\
&= e^{-rt}\times\mathbb{E}\left(e^{-\int_0^t \lambda_s^{(b)} ds} \mid \lambda_0^{(b)}\right),
\end{aligned}
$$

where we assume that deterministic instantaneous rate of interest r for a zero-coupon default-free bond.

The deterministic recovery rate, π

- In reality, the lenders (i.e. the buyers of defaultable bonds) can receive the part (or whole) of coupon payments and principle after the liquidation of borrowers' assets. We can consider the recovery of par model introduced by Duffie (1998). For fractional recovery, we refer you Duffie and Singleton (2003).

- For simplicity, we can assume a deterministic recovery rate π.

$e^{rc,s}(0, t_{k-1}, t_k)$: the value of a deterministic payoff

- The value of a deterministic payoff, 1 that is paid at t_k if and only if the **r**eference **c**redit **defaults** in $]t_{k-1}, t_k]$ and the protection seller **survives** up to t_k, denoted by $e^{rc,s}(0, t_{k-1}, t_k)$, is given by

$$e^{rc,s}(0, t_{k-1}, t_k) =$$

$$\mathbb{E}\left[\exp\left(-\int_0^{t_k} r_s ds\right)\left(1_{\left\{N^{rc}_{t_{k-1}}=0\right\}} - 1_{\left\{N^{rc}_{t_k}=0\right\}}\right)\left(1_{\left\{N^{s}_{t_k}=0\right\}}\right) \mid r_0,\ \lambda_0^{rc}, \lambda_0^{s}\right]$$

$$= e^{-rt_k}$$

$$\times \left[\begin{array}{c} \mathbb{E}\left(e^{-\int_0^{t_{k-1}} \lambda_s^{(rc)} ds} \times e^{-\int_0^{t_{k-1}} \lambda_s^{(s)} ds} \times e^{-\int_{t_{k-1}}^{t_k} \lambda_s^{(s)} ds} \mid \lambda_0^{(rc)}, \lambda_0^{(s)}\right) \\ -\mathbb{E}\left(e^{-\int_0^{t_k} \lambda_s^{(rc)} ds} \times e^{-\int_0^{t_k} \lambda_s^{(s)} ds} \mid \lambda_0^{(rc)}, \lambda_0^{(s)}\right) \end{array}\right],$$

where $0 = t_0 < t_{k-1} < t_k$.

Expressions we need

- $\mathbb{E}\left(e^{-\int_0^t \lambda_s^{(b)} ds} \mid \lambda_0^{(b)}\right)$,

$$\mathbb{E}\left(e^{-\int_0^{t_{k-1}} \lambda_s^{(rc)} ds} \times e^{-\int_0^{t_{k-1}} \lambda_s^{(s)} ds} \times e^{-\int_{t_{k-1}}^{t_k} \lambda_s^{(s)} ds} \mid \lambda_0^{(rc)}, \lambda_0^{(s)}\right) \text{ and}$$

$$\mathbb{E}\left(e^{-\int_0^{t_k} \lambda_s^{(rc)} ds} \times e^{-\int_0^{t_k} \lambda_s^{(s)} ds} \mid \lambda_0^{(rc)}, \lambda_0^{(s)}\right).$$

- The parameter values used to calculate CDS rate are $r = 0.05$, $\pi = 50\%$, $N = 2$, $t_{k_0} = 0$, $t_{k_1} = 0.5$ and $t_{k_2} = 1$. We assume that the default intensity process of reference credit follows $\lambda_t^{(3)}$, i.e.

$$\lambda_t^{(3)} = \lambda_t^{(rc)}$$

and that the default intensity processes of CDS buyer and seller follows $\lambda_t^{(2)}$, i.e.

$$\lambda_t^{(2)} = \lambda_t^{(b)} = \lambda_t^{(s)}.$$

- Using the same parameter values used in Example 1, the calculations of market credit default swaps (CDS) rates caused by changes in the value of ρ, α and $\delta^{(rc)}$ for the **r**eference **c**redit are shown in Table 1-3, respectively.

Table 1.

	$\overline{s}$
$\rho = 50$	427.46bp
$\rho = 30$	1329.3bp
$\rho = 10$	3439.4bp
$\rho = 4$	3910.8bp
$\rho = 1$	2020.9bp

Table 2.

	$\overline{s}$
$\alpha = 0.1$	14.620bp
$\alpha = 1$	2354.2bp
$\alpha = 5$	3910.8bp
$\alpha = 10$	3249.6bp
$\alpha = 20$	2152.6bp

Table 3.

	$\overline{s}$
$\delta^{(rc)} = 0.01$	6.4243bp
$\delta^{(rc)} = 0.1$	3182.5bp
$\delta^{(rc)} = 0.3$	3910.8bp
$\delta^{(rc)} = 0.5$	3441.8bp
$\delta^{(rc)} = 1$	2361.6bp

- Using the same parameter values used in Example 1, the calculations of market credit default swaps (CDS) rates caused by changes in the value of β and $\delta^{(s)}$ for the CDS **s**eller are shown in Table 4-5, respectively.

Table 4.

	$\overline{s}$
$\beta = 0.1$	507.31bp
$\beta = 1$	1612.5bp
$\beta = 5$	3430.6bp
$\beta = 10$	3910.8bp
$\beta = 20$	4187.1bp

Table 5.

	$\overline{s}$
$\delta^{(s)} = 0.01$	0.060789bp
$\delta^{(s)} = 0.1$	2039.0bp
$\delta^{(s)} = 0.3$	3552.7bp
$\delta^{(s)} = 0.5$	3910.8bp
$\delta^{(s)} = 1$	4182.6bp