

A Copula-Based
Model
of Term Structure
of CDO tranches

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Discussion
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What the paper does

- Use copulas to represent temporal loss dependence
- Start with cumulated losses at beginning of period $F_X(x)$
- Use given copula to represent dependence with period losses $C_{X,Y}(w, \lambda)$
- Recover cumulated losses at end of period $F_Z(z)$ AND copula $C_{X,Z}(u, v)$ specifying dependence between X and $Z = X + Y$

Further results (or promises)

- Model consistent with Markov specification of dynamic losses (Section 2)
- Stationary increments: the cumulated loss process is characterized by marginal distribution at some date and some given copula
- Special cases: independent or co-monotonic increments
- Construct recursively the joint distributions of losses (Section 5)

Overall assessment

- Very preliminary
- Overlap with companion paper (Copula-based martingale processes and financial dynamics)
- Elegant, flexible approach
- All the more useful as link with Markov specification of losses can be reinforced

Comments

- Dependent increments: in the co-monotonic case

$$\hat{C}(u, v) = u \wedge v \quad (7)$$

- Stationary increments: Same copula is used irrespective of the length of the period

$$C_{s,t}(u, v) = \int_0^u \frac{\partial C}{\partial w}(w, \dots) dw \quad (9)$$

- OK if period length $\epsilon = 1/n$ is fixed
- Approaching real by rational numbers requires changing period length

- Reverse engineering: Given two marginals F_s and F_t ($s < t$), what are the period losses and copulas C that are consistent with them (i.e. F_t can be recovered from F_s using (9) and some period loss distribution)