

Dynamic Pricing of Synthetic CDOs

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Discussion

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Pricing of synthetic CDOs using extension of industry-standard Vasicek (1991) model:

- Build dynamic model of period losses based on AR representation of common risk factor
- Bootstrap the unconditional default probabilities from quotes of index spreads (single-tranche CDOs)
- Fit consistently prices of synthetic CDO tranches of different tenors at a point in time using simple algorithm

Advantages of the model

- Flexibility (big improvement in fit)
- Consistency (no correlation smile)

Limits

- Statistical dynamic model (arbitrage opportunities):
fully dynamic?
- Implied correlation depends on choice of model and estimated parameters

Dynamic model

- Standard conditional default probability

$$p^X = \Phi \left(\frac{c - \sqrt{\rho}X}{\sqrt{1-\rho}} \right)$$

- Replace cumulated losses with period t losses θ_t
- Apply LLN so $\theta_t \rightarrow p_t^X$
- Assume X_t is $AR(p)$ process
- Normal idiosyncratic risk Φ replaced with general H .

- Then

$$\begin{cases} H^{-1}(\theta_t) = \frac{c_t}{\sqrt{1-\rho}} - \sqrt{\frac{\rho}{1-\rho}}X_t \\ X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sigma \eta_t \end{cases}$$

Comments

- Nice paper, difficult to find fault with
- Improvement in fit:
 - (i) Index spreads are precisely fit in dynamic, hazard rate is constant in static
 - (ii) Idiosyncratic shock not normal (Student, G mixture)
 - (iii) Common factor is dynamic AR(p)
 - (iv) Time horizon split in discrete periods

$$1 - L_t = \prod_{i=1}^t (1 - \theta_i)$$

- Since best fit appears to be mixture model with one lag, comparison should be carried out with static model based on
 - (i) Gaussian mixture
 - (ii) Perfect fit of unconditional probabilities
 - (ii) Same discretization of time horizon

- Some concerns about "high stability" of ρ
 - (i) Possibly, statistical artifact coming from the short length of the period (robustness w.r.t. number of periods chosen?)
 - (ii) Comes at the cost of a high variability in the autoregressive parameter (variable persistence or misspecification?)