



# **Choice of Rating Technology and Price Formation in Imperfect Credit Markets**

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# Motivation



Introduction Model Second stage First stage Equilibrium Results Conclusions

- How to structure credit portfolios is a key factor of success for banks.
- The choice of rating technologies and efficient credit risk management practices plays a crucial role in this process.
- Banks employ rating technologies to
  - ◆ Prices loans and manage their credit risk.
  - ◆ Determine their regulatory capital (Basel II).
  - ◆ Mitigate possible adverse selection effects.
- Substantial resources are currently used by banks to set up appropriate rating technologies.



# This paper



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- The choice of a rating technology has an important impact on estimates of default probabilities and hence on loan prices for any individual customer.
- Rating technologies generate private signals about the PD of a customer. These signals are used in the pricing game.
- We explore the interaction of rating technology choice and loan pricing in an oligopolistic banking sector in a two stage game.
  - ◆ In the first stage banks choose among two alternative rating technologies: **high** and **low accuracy**.
  - ◆ In the second stage banks set their loan spreads under the assumption that the loan market is a **differentiated product market** and banks play a loan pricing game.



# This paper



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- We use this model to shed light on the following questions:
  - ◆ Why do different banks simultaneously employ rating technologies with different levels of accuracy (standard approach versus IRB-approach)?
  - ◆ Can a regulatory agent set incentives for banks to switch to more accurate rating technologies?
  - ◆ What are the consequences for banks if regulatory agents force banks to use more accurate rating technologies?
  - ◆ Do banks and borrowers both benefit from more accurate rating technologies?



# Related Literature



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- Bank competition and credit standards:  
Ruckes (2004), Hauswald and Marquez (2003), Thakor (1996), Bernanke and Gertler (1989, 1990), and Almazan (2002).
- Bank competition and risk taking of banks: **Franchise value paradigm** and its challenge.  
Papers include Repullo (2004).
- Rating technology choice:  
Jankowitsch et al. (2007).
- Technology investments in oligopolistic industries:  
Brander and Spencer (1983), and Tirole (1989).



# The Model



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- We consider a banking industry with 2 banks.
- Banks face two types of decisions:
  - ◆ First stage: Choice of rating technology
  - ◆ Second stage: Setting of credit spreads
- Agents are sequentially rational and anticipate the interaction of technology choice and loan pricing when making their decisions.
- This game is therefore solved via backward induction.

# The Model

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- Defaults occur with prior probability  $\pi$  for the whole population of customers.
- Customers can either default  $I = 1$  or not default  $I = 0$ .
- For a single borrower, bank  $i$  receives a signal  $\tau_i \in \{0, 1\}$  from its rating technology whether or not default will occur ( $I = 0$ ...non-default,  $I = 1$ ...default).
- Signal  $\tau_i$  takes value 1 with probability  $1 - \epsilon_i$  if default occurs, and with probability  $\epsilon_i$  in case of no-default.

# Rating technologies

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- Banks chose between two rating systems  $H$  and  $L$ , characterized by parameters  $\epsilon_H < \epsilon_L$ .
- Signals produced by the low accuracy rating technology  $L$  are less informative than those produced by  $H$ .
- According to the signals, banks update the probabilities of default for each customer which in case of two banks results in:

$$P(I = 1 | \tau_i = 0) = \frac{\epsilon_i \pi}{(1 - \epsilon_i)(1 - \pi) + \epsilon_i \pi},$$

$$P(I = 1 | \tau_i = 1) = \frac{(1 - \epsilon_i) \pi}{\epsilon_i(1 - \pi) + (1 - \epsilon_i) \pi}.$$

- Rating technologies involve a certain level of fixed costs  $c_H > c_L$ .

# Credit spreads and volumes

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- Banks operate in a differentiated product oligopoly and compete by setting their credit spreads.
- Banks charge credit spreads  $S_i(\tau_i) : \{0, 1\} \rightarrow \mathbb{R}_+$  for a given customer.
- Given two credit spreads  $s_i$  and  $s_j$  offered by bank  $i$  and  $j$  loan demand at bank  $i$  is characterized by linear demand functions

$$Q_i(s_i, s_j) = \alpha_i + \beta_i(s_j - s_i) - \gamma_i s_i.$$

- ◆  $\alpha_i > 0$  autonomous loan demand of bank  $i$ .
- ◆  $\beta_i > 0$  substitutability between a loan offered by bank  $i$  and one offered by bank  $j$ .
- ◆  $\gamma_i > 0$  absolute effect of credit spread on the loan demand.

# Expected profits

- Given a strategy  $S_j$  for bank  $j \neq i$ , bank  $i$  will choose to maximize expected profits based on the signal  $\tau_i$  by setting the optimal spread  $s_i$ :

$$R_{i,\tau_i}(s_i, S_j) = \begin{cases} s_i Q_i(s_i, S_j) & \text{in case of no-default} \\ -\delta Q_i(s_i, S_j) & \text{in case of default} \end{cases}$$

$$= P(I = 0 | \tau_i) s_i [(1 - \epsilon_j) Q_i(s_i, S_j(0)) + \epsilon_j Q_i(s_i, S_j(1))] - \\ - P(I = 1 | \tau_i) \delta [(1 - \epsilon_j) Q_i(s_i, S_j(0)) + \epsilon_j Q_i(s_i, S_j(1))]$$

- ◆  $S_j(\tau_j)$ ...spreads chosen by bank  $j$  (random variable)
- ◆  $\delta$ ... loss rate in default
- ◆  $Q_i(s_i, S_j)$ ... quantity of loan at bank  $i$

# Equilibrium Loan Spreads

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- The (unique) equilibrium loan spreads can be calculated explicitly solving the f.o.c.:

$$\bar{S}_i(\tau) = \frac{2\alpha_i(\beta_j + \gamma_j) + \alpha_j\beta_i + \left[\frac{\beta_i\beta_j}{2} + \beta_i(\beta_j + \gamma_j)\right]\frac{\pi\delta}{1-\pi}}{4(\beta_1 + \gamma_1)(\beta_2 + \gamma_2) - \beta_1\beta_2} + \frac{1}{2}\left[(1 - \tau)\frac{\epsilon_i}{1 - \epsilon_i} + \tau\frac{1 - \epsilon_i}{\epsilon_i}\right]\frac{\pi\delta}{1 - \pi}.$$

- Note that as the signal becomes almost perfect ( $\epsilon_i \rightarrow 0$ ) a bank receiving a "default" signal ( $\tau_i = 1$ ) will set an excessively high spread ( $S_i(1) \rightarrow \infty$ ).

# Technology Choice

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- Technology choice must be based on expected equilibrium profits:

$$\bar{R}_i = P(\tau_i = 0)R_{i,0}(\bar{S}_i, \bar{S}_j) + P(\tau_i = 1)R_{i,1}(\bar{S}_i, \bar{S}_j).$$

with  $P(\tau_i = 0)$  and  $P(\tau_i = 1)$  being the unconditional probabilities for bank  $i$  to receive signal "no-default" and "default" given by

$$P(\tau_i = 0) = (1 - \epsilon_i)(1 - \pi) + \epsilon_i\pi, \quad \text{and}$$

$$P(\tau_i = 1) = \epsilon_i(1 - \pi) + (1 - \epsilon_i)\pi$$

# Technology Choice (cont'd)

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- Technology choice is binary (H or L) and based on the matrix game

$$\begin{pmatrix} \bar{R}_i^{H,H} - c_H & \bar{R}_i^{H,L} - c_H \\ \bar{R}_i^{L,H} - c_L & \bar{R}_i^{L,L} - c_L \end{pmatrix}.$$

- At this stage the fixed costs of the rating technologies have to be taken into account.
- The equilibrium of this matrix game is the solution of the two-stage game.



# Summary of analytical results



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- Spreads for "default" signals are always higher than for "non-default" signals.
- More accurate rating technologies result in lower (higher) loan spreads in case of a "non-default" ("default") signal.
- Mean spreads set by banks increase and hence loan volumes decrease with the level of rating accuracy. Mean loan volumes increase with the rating accuracy of the rival bank.
- Expected bank profits increase with an increase in the accuracy of its own rating technology and decrease with an increase in that of the rival.

# Results (cont'd)

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- In case of high (low) investment costs choosing the low (high) accuracy technology is a dominant strategy: Strategy  $L_i$  is a dominant strategy iff

$$\begin{aligned} c_i^H &> \bar{R}_i(\epsilon_i^H, \epsilon_{3-i}) - \bar{R}_i(\epsilon_i^L, \epsilon_{3-i}) = \\ &= \frac{(\epsilon_i^L - \epsilon_i^H)(1 - \epsilon_i^L - \epsilon_i^H)\delta^2\pi^2(\beta_i + \gamma_i)}{4(1 - \epsilon_i^H)(1 - \epsilon_i^L)\epsilon_i^H\epsilon_i^L(1 - \pi)}. \end{aligned}$$

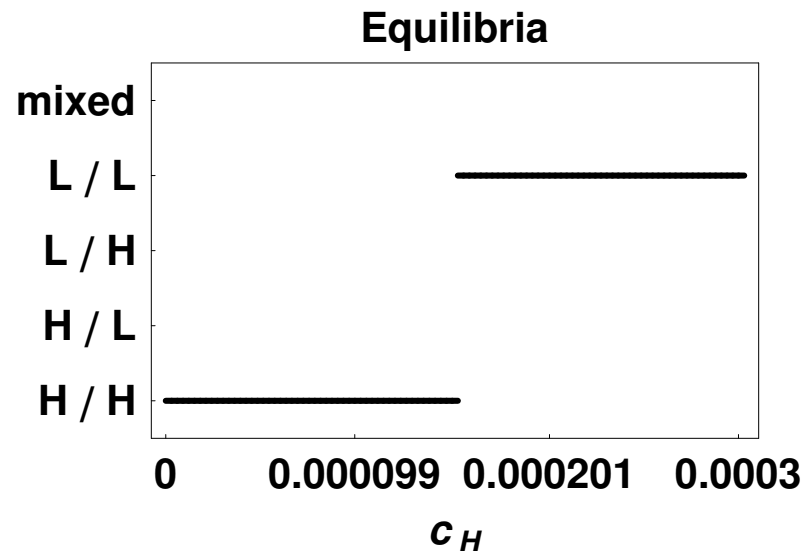
For smaller costs,  $H_i$  is the dominant strategy.

- The above results hold similarly for the case of  $N > 2$  banks.

# Base case: Equilibria

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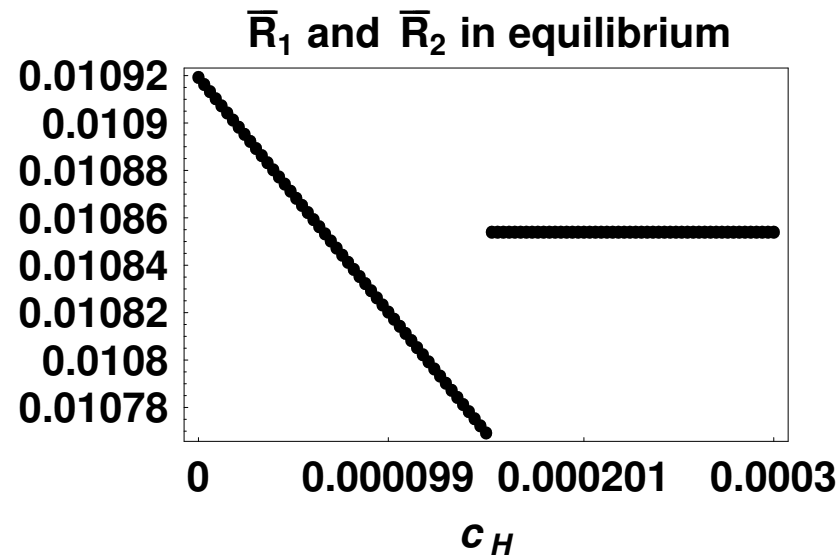
- Rating technologies characterized by  $\epsilon_H = 0.15$  and  $\epsilon_L = 0.35$ .
- No fixed costs for rating system L, we only vary  $c_H$ .
- Parameters:  
 $\alpha_1 = \alpha_2 = 0.5, \beta_1 = \beta_2 = 2, \gamma_1 = \gamma_2 = 5, \delta = 0.5, \pi = 1\%$ .



# Base case: Expected profits

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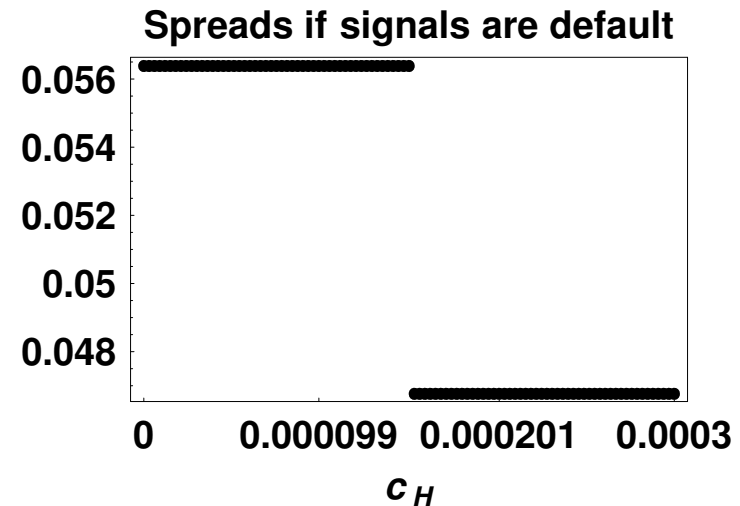
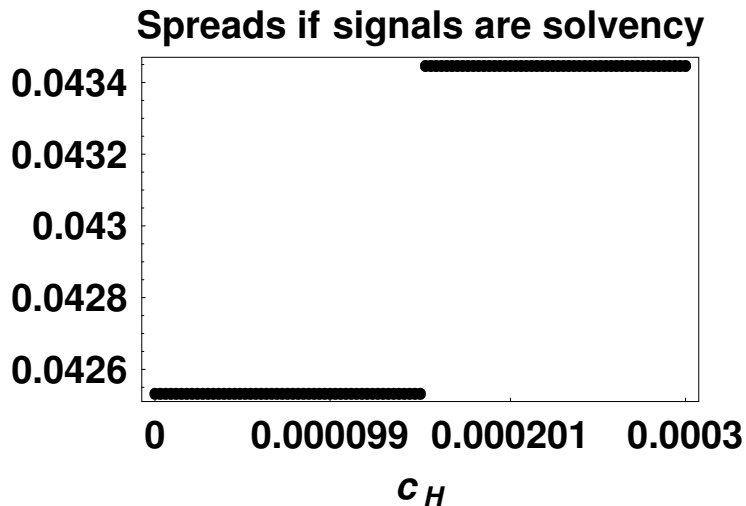
- Expected profits may fall for both banks when they switch to high accuracy rating system.



# Base case: Credit spreads

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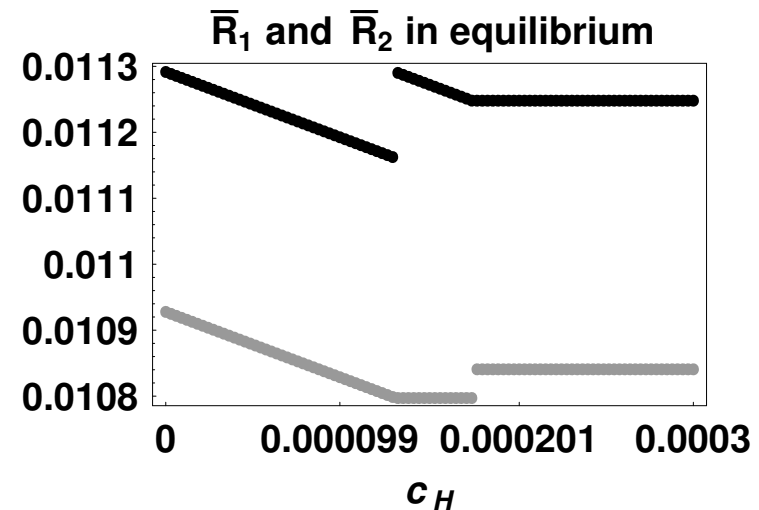
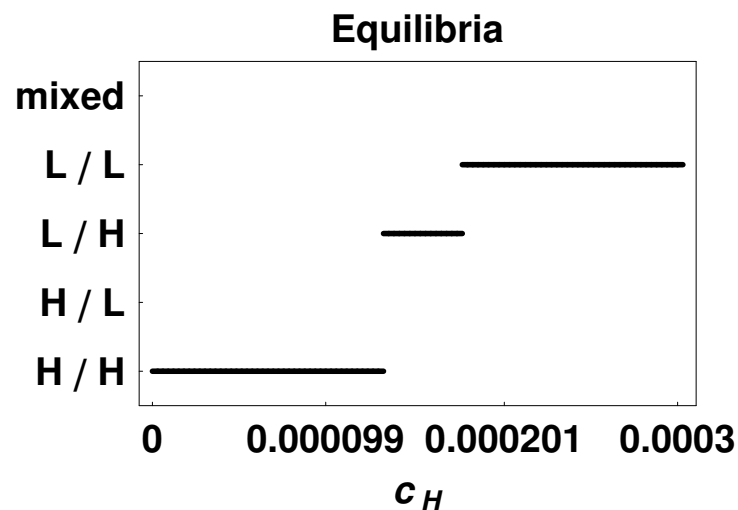
- If the signal is "solvency", credit spreads decrease when banks use the high accuracy rating system



# Asymmetric case

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- Banks now differ in their substitutability parameter ( $\beta_1 = 1, \beta_2 = 3$ ).
- For intermediate costs only bank 2 (black) with higher substitutability parameter invests into high accuracy rating system, bank 1 (gray) stays with L.



# Conclusions

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We observe three scenarios of regulator's incentives:

- Small cost reduction / too small incentives: Equilibrium stays at L/L.
- Medium cost reduction: Banks switch to high accuracy system H, but profitability decreases. Small banks may have difficulties to survive.
- High cost reduction: Banks adopt H, higher profitability for banks.

Oligopolistic banking competition can generate incentives to invest into accurate rating technology (depending on the level of investment costs).

Only in case of sufficiently high incentives, adoption of the high accuracy rating technology results in a Pareto improvement for banks.