Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk*

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PRELIMINARY VERSION

Abstract

We study the functioning and possible breakdown of the interbank market due to asymmetric information about counterparty risk. We allow for privately observed shocks to the distribution of asset risk across banks after the initial portfolio of liquid and illiquid investments in chosen. Our model generates several interbank market regimes: 1) low interest rate spread and full participation; 2) elevated spread and adverse selection; and 3) liquidity hoarding leading to a market breakdown. The regimes are in line with observed developments in the interbank market prior to and during the financial crisis of 2007/2008. We use the model to examine various policy responses.

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## 1 Introduction

The 2007/2008 turmoil in financial markets severely affected the functioning of short-term money markets and led central banks around the world to intervene by adjusting their liquidity supply. More recently, as liquidity in money markets has more and more dried up, governments and central banks have introduced depositor guarantees and other measures aimed at re-establishing confidence in banks. The market for short-term funds is usually closely controlled by central banks, who are monopolist suppliers. Central banks have a particular interest in the well-functioning of this market, as they regard the implementation of their key interest rate in this market as the first step in the transmission of monetary policy.

Under normal market conditions, the short-term market for liquidity works rather smoothly, especially at the very short end where credit concerns play hardly any role. As long as the aggregate supply of liquidity is appropriately chosen by the central bank, interest rates are strongly linked to central bank policy rates. Markets tend to be very liquid, with a high turnover and a large number of participants. Since August 2007, however, the functioning of money market has become impaired in several countries, notably in the US and in the euro area. Banks tend to keep liquidity on their accounts instead of lending excess funds in the interbank market.

This paper provides a model of the interbank market that can generate such a dry-up of liquidity in money markets, and can be used to illustrate the effects of various policy interventions. We use a model in the spirit of Diamond and Dybvig (1983), in which consumers deposit their endowments into a bank and face uncertainty in which of two periods they need to consume. In order to model an interbank market, we assume that there is an infinite number of banks, each hit differently by the liquidity shock in the interim period. Moreover, banks face an uncertain return on their long-run investment, facing the probability of becoming
insolvent. Banks can be of different types, either having a risky or a less risky technology, with differing liquidation values. Bank type is private information.

We determine the equilibrium prices and investment on the interbank market – first, in the benchmark setting where banks are of the same type, and then in a richer setup where banks’ assets differ in risk. We show how, in the presence of different types and with asymmetric information, the interbank market may break down if the population of risky banks, or the extent of risk involved, increases. In this environment, we study banks’ incentives for holding appropriate liquidity buffers. Finally, we study possible policy implications and ask whether a central bank may by means of appropriate liquidity policy or other tools be able to avoid a breakdown of the interbank market or possibly alleviate its consequences.

Next, we present some interbank market facts. Our model aims at understanding potential mechanisms underlying interbank market functioning both in normal times and during the current financial turmoil.

**Some interbank market facts**

One of the standard ways to measure tensions in the unsecured interbank markets is to look at the spread between three-month bank borrowing costs and the overnight index swap in three months’ time. It shows the premium banks have to pay for short-term funds compared with borrowing free of credit risk. Figure 1 plots the spread for the euro area from July 2006 to November 2008 (red line). The blue bars show recourses to the deposit facility of the ECB (daily average per week in billions of euros). Banks may use the deposit facility to make overnight deposits with the Eurosystem. For completeness, the green bars depict liquidity-absorbing fine tuning operations (daily average per week in billions of euros). The Eurosystem carries those out relatively infrequently to manage the liquidity situation and steer interest rates in the money markets.

From Figure 1, it seems that the interbank market experienced three qualitatively different phases in the time period depicted. The initial period is character-
Figure 1: Interbank spread, recourses to the ECB deposit facility, and liquidity absorbing fine tuning operations, 07/2006 - 11/2008

This is consistent with the fact that in normal times, banks try to avoid using the ECB’s permanent overnight lending facility because the deposit rate is punitive compared with the rates usually available on interbank markets. The second phase is characterized by an increased spread but still very low amounts deposited with the ECB overnight (except the 2007 year-end effect). The spread in the interbank market starts rising following the beginning of the financial turmoil on August 9, 2007.

The third phase, which is depicted using daily data in Figure 2, can be distinguished by a dramatic increase in the usage of the deposit facility by banks, in addition to a continuing rise in the spread. The amounts deposited with the ECB rise from a daily average of 0.09 billion euros in the week starting September 1 to a daily average of 169.41 billion in the week of September 29. Some of the major developments of the financial turmoil are also indicated in Figure 2. The amounts deposited with the ECB start rising after the collapse of Washington Mu-
tual when the crisis spreads outside the investment banking realm. Importantly, this rise precedes the ECB announcement of a change in its tender procedure and in the standing facilities corridor on October 8. The ECB reduced the corridor of standing facilities from 200 basis points to 100 basis points around the interest rate on the main refinancing operation as of October 9, thus making depositing at the deposit facility relatively more attractive.¹

A similar pattern of the three-month interbank market spread can be observed in the United States in the aforementioned time period, as documented in Figure 3. It plots the interbank market spread for the US (blue line) as compared to the euro area (red line) from July 2006 to November 2008.

Our model generates interbank market regimes similar to those just described.

¹The rate of the marginal lending facility was reduced from 100 to 50 basis points above the interest rate on the main refinancing operation, i.e. currently to 4.25%, and the rate of the deposit facility was increased from 100 to 50 basis points below the interest rate on the main refinancing operation, i.e. currently to 3.25%. Moreover, as from the operation settled on October 15, the weekly main refinancing operations is carried out through a fixed rate tender procedure with full allotment at the interest rate on the main refinancing operation, i.e. currently 3.75%.
In the first regime, there is full participation of borrowers and lenders in the interbank market. In the second regime, the interbank market is characterized by adverse selection. There is still borrowing and lending in the market. However, safe borrowers drop out of the market and the interest rate rises to reflect that only riskier borrowers stay. In the third regime, the market breaks down because lenders prefer to hoard liquidity instead of lending it out, despite the fact that the interest rate in the market remains high. There is an additional, fourth, regime in the model in which the interbank market can break down since even risky borrowers find the interest rate too high and choose to drop out.

*Related literature*

Both repo and unsecured interbank lending allow banks to cope with liquidity shocks. We focus on the unsecured markets, which are based on peer monitoring. They introduce market discipline in a way akin to unsecured deposits in the environment in which depositors receive information (Calomiris and Kahn, 1991).

The role of the interbank market to cope with bank specific liquidity shocks
and avoid unnecessary liquidation of long term investments was first acknowledged in Bhattacharya and Gale (1987). Later contributions built upon this role while introducing either moral hazard (Rochet and Tirole, 1996), aggregate liquidity risk (Allen and Gale, 2000) or else by introducing credit risk (Freixas, Parigi, and Rochet 2000). Furthermore, Bhattacharya and Fulghieri (1994) analyze the efficiency of an interbank market in a framework where banks face uncertain timing of liquidity returns, and Holmström and Tirole (1998) discuss the role of liquidity provision by the public sector.

Our model economy borrows two essential features from the seminal paper by Diamond and Dybvig (1983). First, consumers are uncertain about the timing of their consumption needs. This generates liquidity shocks on an individual bank level. Second, the investment technologies available to banks exhibit a trade-off between liquidity and return: the long-term asset is more productive than the short-term asset over the long run but its early liquidation entails a loss.

The focus of our paper is studying the effects of information asymmetry about the credit risk on functioning of the interbank market. We are particularly concerned about the possibility of a break down in the interbank market. From that perspective, our work builds on Stiglitz and Weiss (1981) and is related to Broecker (1990) and to Flannery (1996) who also consider models of asymmetric information and credit risk. A recent paper by Freixas and Jorge (2007) examines how financial imperfections in the interbank market affect the monetary policy transmission mechanism. They show that interbank market imperfections induce an equilibrium with rationing in the credit market.

The remainder of the paper is organized as follows. In section 2, the model setup is described. Section 3 studies the benchmark case in which the riskiness of the illiquid asset is the same for all banks. We solve for the equilibrium interest rate and determine banks’ ex-ante investment in liquidity. In section 4, the full model where banks’ illiquid asset can be of different types is laid out. Sections 5
and 6 discuss policy responses, and section 7 concludes.

2 The model

The baseline model is a version of Freixas and Holthausen (2004). There are three time periods, \( t = 0, 1, \) and 2, and a single homogeneous good. Consumption and asset returns are measured in terms of the good. There is no aggregate uncertainty about fundamentals of the economy.

Consumers. There is a \([0, 1]\) continuum of consumers, each lives for three periods. Every consumer has an endowment of 1 unit of the good in period 0. Consumers are risk averse with twice differentiable concave utility functions. Ex ante (as of period 0), consumers are identical. In period 1, consumers receive an idiosyncratic liquidity shock as in Diamond and Dybvig (1983): some of them become “impatient” consumers (they only value period-one consumption, \( u = u(c_1) \)) and some “patient” consumers (who only value period-two consumption, \( u = u(c_2) \)). A consumer’s realized type is private information.

In period 0, consumers deposit their endowments with the bank in exchange for a deposit contract which promises them a consumption \( c_1 \) if they withdraw in period 1 or \( c_2 \) if they withdraw in period 2. Deposits are fully insured by a deposit insurance so there are no bank runs.

Liquidity shocks. In period 1, banks are uncertain about the liquidity demand they face. For a fraction \( \pi_h \) of banks, a high fraction of consumers, denoted by \( \lambda_h \), is impatient and wishes to withdraw in period 1. A fraction \( \pi_l \) of banks faces a low liquidity demand \( \lambda_l \), \( \lambda_l < \lambda_h \). We assume that \( \pi_h + \pi_l = 1 \) and that aggregate demand for liquidity is known. The remaining depositors withdraw in period 2.

In our setup, an interbank market can develop at \( t = 1 \) in which banks with excess liquidity lend it to those with a liquidity shortage. We let \( L_l \) denote an amount a bank lends in the interbank market and \( L_h \) an amount a bank borrows
in the interbank market. Note that banks are risk-neutral and that we solve for banks’ optimal behavior.

**Assets.** There is a \([0,1]\) continuum of risk neutral, profit maximizing banks. We assume that banking industry is perfectly competitive. Thus, in equilibrium, banks make zero profits.

Banks can invest the consumers’ endowments in two types of real assets, a long-term illiquid asset or reserves (costless storage). Let \(\alpha^I\) denote the fraction invested in the illiquid asset at \(t = 0\). Each unit invested in reserves offers a return equal to 1 after one period. In the benchmark model, we assume that each unit invested in the illiquid asset yields an uncertain payoff \(\hat{R}\) after two periods, where \(\hat{R} = R\) with probability \(\bar{p}\) (success), and \(\hat{R} = 0\) with probability \(1 - \bar{p}\) (failure). We assume that investment in the illiquid asset is ex ante efficient: \(\bar{p}R > 1\).

Whether an illiquid project will yield a positive return becomes known to a bank only at time 2. Any fraction \(\alpha^L_k\) of the investment into the illiquid asset can be liquidated after one period at \(t = 1\) by a bank that experience a liquidity shock of type \(k = h, l\) yielding a per unit return of \(\bar{l} = \gamma\bar{p}R < 1\), where \(\gamma\) captures that early liquidation entails a loss. Finally, \(\alpha^R_k\) is the fraction of the reserves \((1 - \alpha^I)\) reinvested for an additional period at \(t = 1\).\(^2\)

Note that there is a trade-off between liquidity and return: the illiquid asset is more productive over the long run but its early liquidation entails a loss.

The sequence of events is summarized in figure 4 below.

**Safe and risky illiquid investments.** In Section 4, we suppose that a shock occurs to risk of the illiquid investment at the same time as, but uncorrelated with, the liquidity shock. It thus occurs after the allocation into liquid and illiquid assets took place but before the interbank market opens. With probability \(q\) the illiquid asset will be safer than expected and with probability \((1 - q)\) it will be riskier than

\(^2\)Note that when the return from liquidation is zero, banks runs cannot occur even in the absence of deposit insurance. For \(0 < l < 1\), banks runs considerations are eliminated by the assumption of deposit insurance. We assume that it is financed at unit cost and that banks have to bear the expected cost of failure.
Banks offer deposit contracts \((c_1, c_2)\).

Banks invest fraction \(\alpha\) into a risky illiquid long-term asset.

\[\text{Idiosyncratic liquidity shocks and shocks to illiquid asset's risk realized.}\]

Banks borrow and lend on an inter-bank market at an interest rate \(r\).

Additionally, they can liquidate a fraction \(\alpha^L\) of the illiquid long-term asset and reinvest a fraction \(\alpha^R\) into second period reserves.

Impatient consumers withdraw deposits and consume \(c_1\).

The return of the illiquid long-term asset realizes.

Interbank loans are repaid.

Patient consumers withdraw their deposits and consume \(c_2\).

Figure 4: The timing of events

expected, \(\bar{p} = qp_s + (1-q)p_r\) and \(p_s > p_r\). Even though the investment is ex-ante profitable, \(\bar{p}R > 1\), we allow that if the asset turns out to be riskier than expected, it may no longer be profitable, \(p_r R < 1\). Whether the illiquid investment turns out to be riskier or safer than expected, i.e. the type of the illiquid asset \(\theta = s, r\), is private information to each bank.

Since the liquidation value of the illiquid asset is a fraction of its expected gross value, \(l_\theta = \gamma_p R\), the shock also means that a safer illiquid asset is easier to liquidate, \(l_s > \bar{l} > l_r\).\(^3\)

Given that the type of illiquid investment is not known, this assumption will allow us to study an interbank market with adverse selection.

**Time 1 liquidity management.** In period 1, banks can manage their liquidity by 1) borrowing/lending in the interbank market, 2) by liquidating the long-term asset, and 3) by storing reserves until period 2. We assume that the interbank market is anonymous and competitive.

A bank of type \(k\) chooses an amount to borrow \(L_h\) or lend \(L_l\) in the interbank market, a fraction \(\alpha^L_k\) of the long-term asset to be liquidated, and a fraction \(\alpha^R_k\)

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\(^3\)This is isomorphic to assuming that \(R_s < \bar{R} < R_r\) instead. All that is needed is that \(\frac{R}{\bar{R}}\), i.e. the opportunity cost of liquidation, is higher for a riskier bank. However, type-specific liquidation values considerably improve tractability in our setup.
of reserves to be reinvested for an additional period. Assume that if state $\tilde{R} = 0$ is realized, then the bank is insolvent and fails. Let $r$ denote interest rate on the interbank loan.

We assume that a type-$h$ bank, a borrower in the interbank market, always repays its interbank loan if it succeeds (even if its own lender defaults, the loan is repaid - the proceeds are used by the deposit insurance scheme). However, a type-$l$ bank, a lender in the interbank market, only receives the repayment if the borrower was successful, too.

We solve the model backwards. We start with the decisions taken by the banks in period 1 and characterize equilibrium in the interbank market. We then solve for the optimal portfolio choice in period 0.

3 Benchmark: No shock to the quality of the illiquid asset

Assume for the moment that there is no shock to the quality of the illiquid asset. Hence, all banks face the same probability of having a positive return on their illiquid investment, $\tilde{p}$, and the same liquidation value, $\tilde{l}$.

In this simplified setup, we will develop the equilibrium in the interbank market and the banks’ investment decision. This will facilitate the derivation of results later on.

At $t = 1$, banks face liquidity shocks and realize that they either have too many or too little reserves for paying out to impatient consumers. In order to meet their time-1 budget constraint, they have the possibility to liquidate, to borrow or lend in the interbank market, and, if they have excess liquidity, to store this again in reserves until the next period. At time 1, thus they maximize their profits subject to their time-1 budget constraint and non-negativity constraints.
Type-\(l\) banks (liquid banks) solve the following problem:

\[
\max_{\alpha^L_l, \alpha^R_l, L_l} \tilde{p}[R(1 - \alpha^L_l)\alpha^I + \alpha^R_l((1 - \alpha^I) + \alpha^L_l \alpha^I \bar{l}) + \tilde{p}(1 + \bar{r})L_l - (1 - \lambda_l)c_2] \quad (1)
\]

subject to

\[
\lambda_l c_1 + \alpha^R_l((1 - \alpha^I) + \alpha^L_l \alpha^I \bar{l}) + L_l \leq (1 - \alpha^I) + \alpha^L_l \alpha^I \bar{l} \quad [\mu^l_1] \\
L_l \geq 0 \quad [\mu^l_2] \\
0 \leq \alpha^L_l \leq 1 \quad [\mu^l_3, \mu^l_4] \\
0 \leq \alpha^R_l \leq 1 \quad [\mu^l_5, \mu^l_6]
\]

Similarly, type-\(h\) banks (illiquid banks) solve:

\[
\max_{\alpha^L_h, \alpha^R_h, L_h} \tilde{p}[R(1 - \alpha^L_h)\alpha^I + \alpha^R_h((1 - \alpha^I) + \alpha^L_h \alpha^I \bar{l}) - (1 + \bar{r})L_h - (1 - \lambda_h)c_2] \quad (2)
\]

subject to

\[
\lambda_h c_1 + \alpha^R_h((1 - \alpha^I) + \alpha^L_h \alpha^I \bar{l}) \leq (1 - \alpha^I) + \alpha^L_h \alpha^I \bar{l} + L_h \quad [\mu^h_1] \\
L_h \geq 0 \quad [\mu^h_2] \\
0 \leq \alpha^L_h \leq 1 \quad [\mu^h_3, \mu^h_4] \\
0 \leq \alpha^R_h \leq 1 \quad [\mu^h_5, \mu^h_6]
\]

We now derive optimality conditions. The first-order condition for a type-\(l\) w.r.t. \(L_l\) is

\[
\tilde{p}^2(1 + \bar{r}) - \mu^l_1 + \mu^l_2 = 0 \quad (3)
\]

while the first-order condition for a type-\(h\) bank w.r.t. to \(L_h\) is

\[
-\tilde{p}(1 + \bar{r}) + \mu^h_1 + \mu^h_2 = 0 \quad (4)
\]

The first-order conditions for a type-\(k\) bank w.r.t. to \(\alpha^L_k\) and \(\alpha^R_k\) are:

\[
-\tilde{p} R \alpha^I + \alpha^L_k \alpha^I \bar{l}(\alpha^R_k + \mu^k_1(1 - \alpha^R_k)) + \mu^k_3 - \mu^k_4 = 0 \quad (5)
\]

\[
((1 - \alpha^I) + \alpha^L_k \alpha^I \bar{l})(\tilde{p} - \mu^k_1) + \mu^k_5 - \mu^k_6 = 0 \quad (6)
\]
Equation (3) implies that \( \mu_l^1 > 0 \). Thus, the resource constraint of the lender always binds.

Let us consider the case in which the type-\( l \) bank decides to provide liquidity in the interbank market, \( L_l > 0 \) and consequently \( L_h > 0 \), so that \( \mu_l^2 = \mu_h^2 = 0 \). This means that \( \mu_l^1 = \bar{p}^2(1 + \bar{r}) \) and \( \mu_h^1 = \bar{p}(1 + \bar{r}) \). The marginal value of being in the interbank market is lower for a lender since he may not be repaid at \( t = 2 \).

Substituting \( \mu_h^1 = \bar{p}(1 + \bar{r}) \) into (6) yields

\[
((1 - \alpha^I) + \alpha_h^L \alpha_h^I)(-\bar{p}\bar{r}) + \mu_h^5 - \mu_h^6 = 0
\]

We can rule out that \( \alpha_h^R = 1 \) (which implies \( \mu_h^5 = 0 \)) since \( \mu_h^6 \) must be positive. Moreover, \( \alpha_h^R \in (0, 1) \) would requires that \( \alpha^I = 1 \) (and \( \alpha_h^L = 0 \)). But investing everything into into the illiquid asset at \( t = 0 \) must be suboptimal if the interbank market is active. If it turns out to be a type-\( l \) bank that lends in the interbank market, it would have to liquidate some of the long-term asset.\(^4\) But the return on liquidation is less than one, which is the return on investing in the short-term asset at \( t = 0 \). A borrower in the inter-bank market does not use second-period storage (reserves) \( \alpha_h^R = 0 \).

If the interbank market exists, then the lender must not use any second-period storage (reserves), \( \alpha_l^R = 0 \). The decision between lending in the interbank market or reserves is an either-or decision. The expected return on lending any amount in the interbank market is \( \bar{p}(1 + \bar{r}) \) while the rate on any amount stored as reserves is 1. If the bank prefers reserves then nothing is lent out at \( t = 1 \). A necessary condition for an active interbank market is:\(^5\)

\[
\bar{p}(1 + \bar{r}) \geq 1
\]  

\(^4\)Moreover, it can never be the case that the lender liquidates while the borrower does not (see below).

\(^5\)We abstract from the borderline case and assume that a bank prefers to stay in the interbank market when it offers the same return as second-period storage.
Turning to the liquidation decision of a lender, substituting \( \mu'_1 = \tilde{\mu}^2 (1 + \tilde{r}) \) and \( \alpha^R_l = 0 \) into (5) yields

\[
\alpha^L_l \tilde{p} (\tilde{p} (1 + \tilde{r}) \tilde{l} - R) + \mu'_3 + \mu'_4 = 0
\]

If a lender does not liquidate \((\alpha^L_l = 0, \mu'_3 > 0, \mu'_4 = 0)\) then it must be that\(^6\)

\[
\tilde{p} (1 + \tilde{r}) \tilde{l} \leq R \quad (8)
\]

The left hand side is the expected return at \( t = 2 \) from liquidation (per unit return \( \tilde{l} \) reinvested at \( 1 + \tilde{r} \) paid back with probability \( \tilde{p} \)).

Similarly, a borrower does not liquidate if

\[
(1 + \tilde{r}) \tilde{l} \leq R \quad (9)
\]

By liquidating, a type-\( h \) bank saves \( 1 + \tilde{r} \) per unit return \( \tilde{l} \) by not having to borrow in the interbank market. A borrower liquidates earlier than a lender since the expected benefit of not borrowing is larger than the expected benefit of lending.

If a borrower liquidates his long-term asset, then the interbank market cannot be active. The borrower could not repay the interbank loan since he has no cash-flow at \( t = 2 \) anymore. Hence, the lender would not lend. For the inter-bank market to exist, the interest rate must neither be too high or too low,

\[
\frac{1}{\tilde{p}} \leq 1 + \tilde{r} \leq \frac{R}{\tilde{l}} \quad (10)
\]

At \( t = 0 \), a bank decides on the allocation between liquid and illiquid assets by maximizing its expected profit, not knowing whether it will face high or low

\(^6\)Again, we rule out the borderline case by assuming that a bank does not liquidate if it is indifferent.
liquidity demand at time 1:

\[
\max_{\alpha^l} \pi_l \bar{p}[R \alpha^I + \bar{p}(1 + E(\bar{r}))L_l - (1 - \lambda_l)c_2] \\
+ \pi_h \bar{p}[R \alpha^I - (1 + E(\bar{r}))L_h - (1 - \lambda_h)c_2]
\]  

(11)

subject to

\[L_l = (1 - \alpha^I) - \lambda_l c_1\]
\[L_h = \lambda_h c_1 - (1 - \alpha^I)\]

where we have used \(\alpha^R_k = \alpha^L_k = 0\) and the fact that the resource constraints at \(t = 1\) are binding.\(^7\)

The first-order condition for an interior solution gives\(^8\)

\[R - (1 + E(\bar{r}))(\bar{p} \pi_l + \pi_h) = 0\]  

(12)

Essentially, this means that whenever the return on the illiquid asset is high (low) compared to the expected return from participating on the interbank market, it is better to invest only the illiquid asset (reserves). Since all banks are ex-ante equal, we can only have a meaningful equilibrium (with investment in both types of assets) whenever (12) holds with equality, and \(E(\bar{r}) = \bar{r}\). Of course, the interest rate will be ultimately determined by the time-0 investment, as will be shown further on.

The interest rate in the interbank market is given by a no-arbitrage between liquid and illiquid assets. The return on liquidity, \((1 + \bar{r})(\bar{p} \pi_l + \pi_h)\), comes from being able to lend and borrow in the inter-bank market. The interbank market in turn allows to smooth out any future frictions due to the uneven liquidity shock \(\lambda_k\). Note the return on liquidity is the interest rate in the interbank market times a discount factor \(\delta = \bar{p} \pi_l + \pi_h \in (\bar{p}, 1)\). The discount is less than one since the

\(^7\)Note that each bank is a price taker in the competitive interbank market so that \(\bar{r}\) does not depend on \(\alpha^I\) in the optimization program.

\(^8\)Investing everything either in the liquid or the illiquid asset cannot be optimal.
lending but not borrowing in the interbank market is risky. The discount will be larger if i) it is more likely to be a lender, or ii) it is less likely to be repaid.

Note that condition (7) is satisfied and the lender does not use reserves. The return on liquidity is larger than 1, which is the return on first (and second) period storage. For the borrower not to liquidate (condition (9)), it must be that \( \bar{l} < \delta \).

The interbank market fails to exist if the cost of liquidation is larger than the discount factor \( \delta \) (which is small if there is a lot of risk in the interbank market or if there are mainly lenders who worry about not being repaid).

The following proposition summarizes the pricing in the interbank market when there is no shock to the quality of the illiquid investment.

**Proposition 1** A necessary and sufficient condition for the interbank market to exist is \( \bar{l} < \delta \).\(^9\) If the inter-bank market is active \( (L_l > 0 \text{ and } \alpha^I \in (0, 1)) \), then second period reserves are not used, liquidation never takes place, and \( \delta(1+\bar{r}) = R \).

### 3.1 Investment decision

The allocation between the liquid and illiquid constant returns to scale investments, i.e. the amounts invested, is determined by i) market clearing in the interbank market (given by \( \pi_h L_h = \pi_l L_l \)) and ii) the condition that competition forces a full pay-out of cash-flows at \( t = 2 \) to depositors.

Market clearing yields

\[
\tilde{\lambda} c_1 = 1 - \alpha^I
\]

where \( \tilde{\lambda} = \pi \lambda_l + (1-\pi)\lambda_h \) is the average fraction of early consumers. The interbank market fully solves the problem of uneven demand for liquidity across banks.

Since competition forces banks to pay-out everything to depositors at \( t = 2 \) we

\(^9\)To see sufficiency suppose that \( \bar{l} < \delta \) and the interbank market is not used. This means autarky. However, higher welfare for depositors can be achieved if banks use the interbank market to smooth out liquidity shocks.
have for a type-\(l\) bank:

\[
(1 - \lambda_l) c_2 = \bar{p}(1 + \bar{r})((1 - \alpha^I) - \lambda_l c_1) + R\alpha^I
\]

and for a type-\(h\) bank

\[
(1 - \lambda_h) c_2 = (1 + \bar{r})((1 - \alpha^I) - \lambda_h c_1) + R\alpha^I
\]

so that after eliminating \(c_2\) and substituting for \(c_1\) using (13) we have

\[
\frac{\alpha^I}{1 - \alpha^I} \frac{R}{1 + \bar{r}} = \frac{1 - \bar{\lambda} - \pi_h(1 - \bar{p})(1 - \lambda_h)}{\lambda} \tag{14}
\]

The condition says that the ratio of the expected total return on investment in the illiquid asset \((\alpha^I R)\) to the expected total return on the liquid asset \(((1 - \alpha^I)(1 + \bar{r}))\) should equal the ratio of average deposit withdrawals at \(t = 2\) to \(t = 1\). Here, average deposit withdrawals at \(t = 1\) are simply \(\bar{\lambda}\); the term for \(t = 2\) withdrawals is more complex, because the risk of bank failure reduces aggregate late withdrawals.\(^{10}\) On average a fraction \(\pi_h\) of banks have high liquidity needs at \(t = 1\) and borrow in the interbank market, with probability \((1 - \bar{p})\) their illiquid investment then fails so that their late withdrawals \((1 - \lambda_h)\) are no served.\(^ {11}\)

Equation (14) also determines the interest rate in the interbank market \(1 + \bar{r}\) as a function of investment in the illiquid asset. It is easy to check that, as expected, the interest rate is higher when investment in the illiquid asset is high (so that the level of reserves is low), and vice versa.

Substituting for \((1 + \bar{r})\) using the pricing equation (12) and solving for \(\alpha^I\) we obtain

\[
\alpha^I = \frac{1 - \bar{\lambda} - \Delta}{1 - (1 - \delta)\bar{\lambda} - \Delta} \tag{15}
\]

\(^{10}\)The \((1 - \bar{p})\pi_h(1 - \lambda_h)\) patient depositors of failed banks will be reimbursed by a deposit insurance company (which for simplicity we do not model explicitly).

\(^{11}\)These late consumers are reimbursed by the deposit insurance scheme.
where $\Delta = (1 - \tilde{p})\pi_h(1 - \lambda_h)$ is the expected mass of impatient depositors of failed banks. Note that the amount invested $\alpha^I$ does not depend on the return $R$.

### 3.2 First best: The illiquid asset is safe

Up to now, there are three frictions in the model: i) individual liquidity shocks at $t = 1$ (solved by the deposit contract), ii) uneven liquidity need across banks and inability to make deposit contract contingent on it (solved by the interbank market) and iii) risk in the interbank market that may lead to inefficient liquidation and the failure of the interbank market to exist. In order to see what is the first best benchmark, let $\tilde{p} = 1$ so that the return on the long-term asset is risk-free, and consequently, there is no risk in the interbank market.

**Proposition 2 (First best)** Suppose that the return on the long-term asset is risk-free, $\tilde{p} = 1$. Then, the inter-bank market always exists (since $\tilde{l} < 1$), the interest rate in the inter-bank market is $1 + \tilde{r} = R$ (the expected returns from investing in the liquid and illiquid asset are equal) and the amount invested in the illiquid asset is equal to the expected amount of late withdrawals:

$$\alpha^I_{FB} = 1 - \bar{\lambda}$$

where $\bar{\lambda} = \pi \lambda_l + (1 - \pi)\lambda_h$. The payout to depositors is $c_1^{FB} = 1$, $c_2^{FB} = R$.

Note that the amount invested in the illiquid technology in the first best case does not depend on its return $R$ (constant returns to scale).

### 3.3 Comparative statics of second best investment

It is easy to verify that the amount invested in the illiquid long-term technology $\alpha^I$ in (15) is decreasing in the expected amount of early withdrawals $\bar{\lambda}$. More early withdrawals for a lender in the interbank market also reduces the investment into the illiquid asset since $\lambda_l$ enters (15) only via $\bar{\lambda}$. More early withdrawals for a
borrower affect long-term investment via both average early withdrawals $\lambda$ and the expected reduction in late withdrawals $\Delta$. Both effects work in the same direction. For example, more early withdrawals for a borrower lower the expected reduction of late withdrawals $\Delta$, which in turn means that less illiquid investment is needed.

The comparative statics with respect to the probability of being a lender $\pi_l = (1 - \pi_h)$ and the risk of the illiquid asset $\tilde{p}$ are ambiguous. There are two channels, which is best see in equation (14). A price channel working through the interest rate in the interbank market and a quantity channel working through the relative amount of early and late withdrawals. These two channels work into different directions for $\pi_l$ and $\tilde{p}$.

The risk of the illiquid asset increases the interbank interest rate. An increase of interest rate in the interbank market leads to a larger allocation in the illiquid asset ceteris paribus. Since the liquid asset becomes more valuable relative to the illiquid one, more can be invested in the latter in order to meet the relative demand for early and late withdrawals. But a higher risk increases the expected reduction $\Delta$ in the late payout. Expecting to payout less at $t = 2$ means that less needs to be invested in the illiquid asset.

The following proposition states that the price channel dominates with respect to the effect of risk on the amount of illiquid investment if a bank is (weakly) more likely to be a lender in the interbank market. This is intuitive since the price channel relates to being a lender. He is the one who suffers the risk of not being repaid in the interbank market. The expected reduction in late withdrawals that is responsible for the quantity channel however relates to borrowers since it is them who may default.

**Proposition 3** A riskier illiquid investment leads to more investment in it if a bank is (weakly) more likely to be a lender, $\pi_l \geq \frac{1}{2}$.

**Proof.** It can be shown that the derivative of (15) with respect to $\tilde{p}$ is negative if
and only if

\[ 1 - 2\pi_l + \pi_l^2\lambda_l - \pi_h^2\lambda_h < 0 \]

This can be rewritten as

\[ (1 - 2\pi_l)(1 - \lambda_l) < \pi_h^2(\lambda_h - \lambda_l) \]

which holds if \( \pi_l \geq 1/2 \) since \( \lambda_h > \lambda_l \). ■

4 Privately observable shocks to the risk of the illiquid asset

In this Section, we consider the case when there is a privately observed shock to the riskiness of the illiquid asset. We show that two cases need to be considered. First, the case without adverse selection, i.e. all banks stay in the interbank market. This situation will be indistinguishable from the previous Section. Second, there can be adverse selection in the interbank market leading to two possible types of market breakdowns.

4.1 Case 1: Both types of banks with high liquidity needs borrow in the interbank market

Let \( (1 + \bar{r}) \) denote the interest rate when borrowers with both safer and higher than expected illiquid assets stay in the interbank market.\(^{12}\)

At \( t = 1 \), type-\((l, \theta)\) banks solve the following problem:

\[
\max_{\alpha_{l,\theta}, \alpha_{h,\theta}, L_{l,\theta}} p_{\theta}[R(1 - \alpha_{l,\theta}^L)\alpha^I + \alpha_{l,\theta}^R((1 - \alpha^I) + \alpha_{l,\theta}^L\alpha^I I_{l,\theta}) + \bar{p}(1 + \bar{r})L_{l,\theta} - (1 - \lambda_l)c_0] \quad (17)
\]

subject to

\(^{12}\)No screening of type is possible since there is no contractual variable other than the interest rate. Other contractual variables to screen borrowers could be collateral (but we consider the unsecured market) or the size of the loan (but we have constant returns to scale technologies).
\[
\lambda c_1 + \alpha^R_{l,\theta}((1 - \alpha^l) + \alpha^L_{l,\theta}\alpha^l l_\theta) + L_{l,\theta} \leq (1 - \alpha^l) + \alpha^L_{l,\theta}\alpha^l l_\theta \\
L_{l,\theta} \geq 0 \\
0 \leq \alpha^L_{l,\theta} \leq 1 \\
0 \leq \alpha^R_{l,\theta} \leq 1
\]

Similarly, type-\((h, \theta)\) bank solves:

\[
\max_{\alpha^R_{h,\theta}, \alpha^L_{h,\theta}, L_{h,\theta}} \quad p_\theta[R(1 - \alpha^R_{h,\theta})\alpha^l + \alpha^R_{h,\theta}((1 - \alpha^l) + \alpha^L_{h,\theta}\alpha^l l_\theta) - (1 + \tilde{r})L_{h,\theta} - (1 - \lambda_h)c_2] \quad (18)
\]

subject to

\[
\lambda h c_1 + \alpha^R_{h,\theta}((1 - \alpha^l) + \alpha^L_{h,\theta}\alpha^l l_\theta) \leq (1 - \alpha^l) + \alpha^L_{h,\theta}\alpha^l l_\theta + L_{h,\theta} \\
L_{h,\theta} \geq 0 \\
0 \leq \alpha^L_{h,\theta} \leq 1 \\
0 \leq \alpha^R_{h,\theta} \leq 1
\]

The first-order condition for a type-\((l, \theta)\) w.r.t. \(L_{l,\theta}\) is

\[
\begin{align*}
p_\theta(1 + \tilde{r}) - \mu^l_{1,\theta} + \mu^l_{2,\theta} &= 0 \quad (19)
\end{align*}
\]

while the first-order condition for a type-\((h, \theta)\) bank w.r.t. \(L_{h,\theta}\) is

\[
\begin{align*}
-p_\theta(1 + \tilde{r}) + \mu^h_{1,\theta} + \mu^h_{2,\theta} &= 0 \quad (20)
\end{align*}
\]

The first-order conditions for a type-\((k, \theta)\) bank w.r.t. \(\alpha^L_{k,\theta}\) and \(\alpha^R_{k,\theta}\) are:

\[
\begin{align*}
-p_\theta R\alpha^l + \alpha^l l_\theta(\alpha^R_{k,\theta} + \mu^k_{1,\theta}(1 - \alpha^R_{k,\theta})) + \mu^k_{3,\theta} - \mu^k_{4,\theta} &= 0 \quad (21) \\
((1 - \alpha^l) + \alpha^L_{k,\theta}\alpha^l I_\theta)(p_\theta - \mu^k_{1,\theta}) + \mu^k_{5,\theta} - \mu^k_{6,\theta} &= 0 \quad (22)
\end{align*}
\]

We proceed as in the case without the shock to the risk of the illiquid asset.

Equation (19) implies that \(\mu^l_{1,\theta} > 0\) and the resource constraint for a lender always
binds. Since we assume that the interbank market is active and all banks borrow and lend, we have $L_{k,\theta} > 0$ so that

$$p_{\theta}\bar{p}(1 + \bar{r}) = \mu_{1,\theta}$$  \hspace{1cm} (23)

$$p_{\theta}(1 + \bar{r}) = \mu_{h,\theta}$$  \hspace{1cm} (24)

As in the case without the shock, the marginal value of being in the interbank market is lower for the lender. The difference is that while $a$ knows her own type (and thus her success probability $p_{\theta}$), a lender does not know the borrower’s type and thus expects to be repaid with probability $\bar{p}$.

The shock to the illiquid asset does not change a borrower’s decision to never use second period storage (reserves), $\alpha_{h,\theta}^{R} = 0$. Similarly, a lender must not use reserves if the interbank market exists, $\alpha_{l,\theta}^{R} = 0$. With the shock the necessary condition (7) becomes

$$\bar{p}(1 + \bar{r}) > 1$$  \hspace{1cm} (25)

since a lender does not know what type of borrower he is facing. Note that this lower bound on the interest rate does not depend on the lender’s type $\theta$.

The shock will however affect the liquidation decision since a bank will know whether the illiquid asset has turned out to be safer or riskier than thought. Thus, a bank also knows whether is illiquid asset is more easily liquidated or not.

Following the argument leading to equations (8) and (9), a lender does not liquidate if

$$\bar{p}(1 + \bar{r})l_{\theta} > R$$  \hspace{1cm} (26)

and a borrower does not liquidate if

$$(1 + \bar{r})l_{\theta} > R$$  \hspace{1cm} (27)

As before a type-$\theta$ borrower liquidates earlier than a type-$\theta$ lender. Moreover, a
safer bank liquidates earlier than a riskier bank since the former’s illiquid asset has higher expected value and has therefore also a higher liquidation value. Since we assume that both safe and risky borrowers stay in the interbank market and the interest rate is \((1 + \bar{r})\), it must be that

\[
(1 + \bar{r})l_s < R \quad (28)
\]

A safe borrower is the first to liquidate his illiquid asset and to drop out of the interbank market.\(^{13}\)

To sum up, when there are unobservable shocks to the risk of the illiquid asset but all banks still use the interbank market, the interest rate must satisfy

\[
\frac{1}{\bar{p}} \leq (1 + \bar{r}) \leq \frac{R}{I_s} \quad (29)
\]

The shock lowers the upper bound since some borrowers, the safe ones, know that the shock improves their outside opportunity of liquidation.

As before, the interest rate in the interbank market is given by the optimal allocation across liquid and illiquid assets at \(t = 0\). A bank solves

\[
\max_{\alpha_l^t} \pi_l \bar{p}[R\alpha_l^t + \bar{p}(1 + \bar{r})L_l - (1 - \lambda_l)c_2] + \pi_h \bar{p}[R\alpha_h^t - (1 + \bar{r})L_h - (1 - \lambda_h)c_2] \quad (30)
\]

subject to

\[
L_l = (1 - \alpha_l^t) - \lambda_l c_1 \\
L_h = \lambda_h c_1 - (1 - \alpha_h^t)
\]

where we have used \(\alpha_{R,\theta}^t = \alpha_{L,\theta}^t = 0\) and the fact that the resource constraints at \(t = 1\) are binding and do not depend on the shock to the risk of the illiquid asset.

\(^{13}\)We implicitly assume that a safe borrower cannot liquidate and use the interbank market, i.e. we assume that liquidation is observable.
An optimal allocation across the liquid and illiquid asset requires that

$$(1 + \tilde{r})(\tilde{p}_l \pi_l + \pi_h) = R$$

(31)

The interest rate in the interbank market is not affected by the shock as long as all banks continue to use the interbank market.

Again, the lender does not use reserves (condition (25) is satisfied when (31) holds). The upper bound on the interest rate now is $l_s < \delta$ which is a stricter condition than in the case without the shock. It is this condition and the possibility that safe borrowers drop out of the interbank market which opens up the possibility of an adverse selection in, and even a breakdown of, the interbank market.

Since all types of banks access the interbank market and the demand and supply of loans does not depend on the shock, we can just follow the derivation in the previous section to obtain the asset allocation $\alpha^I, (1 - \alpha^I)$:

$$\frac{\alpha^I}{1 - \alpha^I} \frac{R}{1 + \tilde{r}} = \frac{1 - \bar{\lambda} - \pi_h (1 - \tilde{p})(1 - \lambda_h)}{\bar{\lambda}}$$

(32)

and after substituting for the interest rate

$$\alpha^I = \frac{1 - \bar{\lambda} - \Delta}{1 - (1 - \delta) \lambda - \Delta}$$

(33)

where $\delta$ and $\Delta$ are the same as in the case without the shock.

The following proposition summarizes the case when all banks use the interbank market even though they are privately informed about the shock to the risk of their illiquid investment.

**Proposition 4** Under private information about the shock to the risk of the illiquid asset all banks use the interbank market iff $l_s < \delta$. If all banks use the interbank market, the interest rate and asset allocation are identical to the situation without the shock.
4.2 Case 2: Adverse selection in the interbank market

We now examine the situation when \( l_s > \delta \), i.e. the interest rate in the interbank market with full participation of all types of banks is too high for safe borrowers to stay in the market. The interest rate is too high because either the average risk is too high, because a bank is likely to be a lender or because the positive shock increases the liquidation value a lot.

Let \((1 + r_r)\) denote the interest rate when only risky borrowers stay in the interbank market. Since lenders know that only risky banks borrow, their marginal value of lending is now reduced while the (risky) borrower’s stays the same (when compared to (23) and (24)) ceteris paribus.

\[
\begin{align*}
    p_r(1 + r_r) & = \mu^{l,g}_1 \\
    p_r(1 + r_r) & = \mu^{h,r}_1
\end{align*}
\]

The condition for a type-\( l \) bank to indeed lend and not use second period storage (equation (25)) becomes

\[
p_r(1 + r_r) \geq 1
\]

Both types of lenders are assumed to not liquidate their long-term asset so it must be that

\[
p_r(1 + r_r)l_s \leq R
\]

since a safe lender starts to liquidate earlier than a risky one. Finally, a risky borrower must not liquidate

\[
(1 + r_r)l_r \leq R
\]

Since we assume that \( l_\theta = \gamma p_\theta R \), equation (38) actually implies (37). A risky borrower liquidates earlier than a safe lender. In the case of an adverse selection
of risky borrowers in the interbank market, the interest rate must therefore satisfy

\[
\frac{1}{p_r} \leq (1 + r_r) \leq \frac{R}{l_r} = \frac{1}{\gamma p_r} \quad (39)
\]

Again, the allocation at \( t = 0 \) between illiquid and illiquid assets determines the interest rate in the interbank market (the price of liquidity). A bank now solves

\[
\max_{\alpha^I} \quad \pi_I \bar{p}[R\alpha^I + p_r(1 + r_r)L_I - (1 - \lambda_I)c_2] \\
+ \pi_h(1 - q)p_r[R\alpha^I - (1 + r_r)L_h - (1 - \lambda_h)c_2] \quad (40)
\]

subject to

\[
L_I = (1 - \alpha^I) - \lambda_I c_1 \\
L_h = \lambda_h c_1 - (1 - \alpha^I)
\]

Ex-ante, a bank expects to drop out of the interbank market if it is a safe borrower, which occurs with probability \( \pi_h q \). The safe borrower liquidates his long-term asset to maximize the possible pay-out at \( t = 1 \) (which will be less than one since \( (1 - \alpha^I) + \alpha^I l_s < 1 \)).\(^{14}\) Having liquidated, a safe borrower will not have any funds at \( t = 2 \) and will thus not be able to repay \( c_2 \). Similar for a bank that fails, a safe borrower goes bankrupt at \( t = 1 \).

Taking the derivative with respect to \( \alpha^I \) and rearranging, we obtain

\[
(\pi_I \bar{p}p_r + \pi_h(1 - q)p_r)(1 + r_r) = (\pi_I \bar{p} + \pi_h(1 - q)p_r)R \quad (42)
\]

The adverse selection of risky borrowers means that a lender expects to repaid only with probability \( p_r \) instead of \( \bar{p} \). What complicates the expression relative to (31) is that adverse selection also lowers the expected return of the illiquid investment, as well as the expected cost of being in the interbank market, for borrowers from

\(^{14}\)If anything is left after banks pay out \( c_1 \), the remaining funds are absorbed by the deposit insurance company.
We can rewrite (42) as

\[(1 + r_r)\delta_r = R \] (43)

where

\[\delta_r = \frac{\pi_l p_r + \pi_h \zeta}{\pi_l + \pi_h \zeta} \] (44)

and

\[\zeta = \frac{1}{1 + \frac{q}{1 - q} p_r} \] (45)

is the adverse selection effect on the probability of being a borrower. An decrease in $\zeta$ will decrease $\delta_r$, i.e. both adverse selection effects raise the interest rate.

The next result verifies that the interest rate with adverse selection is indeed higher than the one when all borrowers stay in the interbank market.

**Proposition 5** The interest rate under adverse selection $(1 + r_r)$ is always larger than the interest rate $(1 + \bar{r})$ when no borrower drops out of the interbank market, i.e. $\delta_r < \delta$

**Proof.** Since $\bar{p} = q p_s + (1 - q) p_r$, we can write

\[q = \frac{\bar{p} - p_r}{\Delta p} \]

where we used $p_s = p_r + \Delta p$. We need to show that $\delta_r < \bar{\delta}$. Substitution for $p_s$ and $q$ in (44) and (45), and using $\pi_h = 1 - \pi_l$ yields

\[p_r(- (1 - \pi_l)(p_r + \Delta p) + \bar{p}(1 - \pi_l - \pi_l \Delta p)) \]

\[= -(1 - \pi_l)p_r(p_r + \Delta p) + \bar{p}(p_r(1 - \pi_l) - \pi_l \Delta p) \]

\[< \pi_l \bar{p} + (1 - \pi_l) \]

This holds as long as

\[(1 - \bar{p})(1 - \pi_l)p_r + \Delta p(1 - (1 - \bar{p})\pi_l) > 0 \]
which simplifies to
\[ \pi_l < \frac{p_r(1 - \tilde{p}) + \Delta p}{p_s - p_r \tilde{p}} \]

This always holds since the right-most expression is larger than 1. ■

It can further be shown that adverse selection in the interbank market leads to ex-ante overinvestment in the illiquid asset.

**Proposition 6** The amount invested in the illiquid asset under adverse selection is larger than in the case when no borrower drops out of the interbank market, i.e. \( \alpha^I_r > \alpha^I \).

When the interest rate under adverse selection is outside the bounds imposed by condition (39) then there will be a breakdown of the interbank market. More precisely, the market can either break down because even risky banks find it too expensive to borrow or because lenders stop providing liquidity to an adverse selection of borrowers.

*Market break-down: no borrowers*

Borrowers drop out of the interbank market and liquidate their long-term asset if the interest rate is too high. Using (42), condition (38) can be written as
\[ \delta_r < l_r \]

*Market break-down: liquidity hoarding by lenders*

Lenders no longer lend to an adverse selection of risky borrowers if condition (36) does not hold, i.e. if they obtain more by using second period storage (reserves) than by lending in the interbank market
\[ p_r(1 + r_r) < 1 \]
Using (42), the market breaks down due to lenders hoarding liquidity if

\[ p_r R < \delta_r < 1 \]

A necessary condition for such a market breakdown there is that an illiquid asset that is riskier than expected, also turns out to be unprofitable, \( p_r R < 1 \). Alternatively, when the illiquid asset turns out to be riskier than expected but is still profitable, then the market cannot break down due to liquidity hoarding.

**Transition between regimes**

Four regimes that can emerge in the interbank market in our economy are summarized in Figure 4. The plot captures the effects of changes in the counterparty risk in the economy, \( \bar{p} \), and changes in the “dispersion” of risk, \( \Delta p \) where \( \Delta p \equiv p_s - p_r \). A quadrangle whose borders are given by the x-axis and the three lines in bold represents feasible \((\bar{p}, \Delta p)\) pairs.

![Figure 5: Comparative statics: Transition between regimes](image)

The interbank market is in the Regime I (full participation of borrowers and lenders) whenever the average level of counterparty risk is low (\( \bar{p} \) is high). In
this case, changes in the dispersion of risk that keep $\bar{p}$ unchanged cannot move the interbank market to a different regime. In other words, for low levels of counterparty risk, the dispersion of risk is irrelevant for the determination of the interbank market equilibrium.

Once the interbank market is in the Regime II (adverse selection), which occurs as the level of counterparty risk rises, changes in both the average riskiness and the dispersion of risk can cause a move to the Regime III (market breakdown due to the liquidity hoarding by lenders). For high levels of counterparty risk, the interbank market breaks down because all borrowers choose to drop out of the market (Regime IV).

In the next Section, we turn to a discussion of policy interventions that may be used to ensure that the interbank market operates in Regime I and inefficient liquidation of illiquid investments is prevented.

5 Policy interventions

So far, we have analyzed the pure market equilibrium of the interbank market and banks’ optimal investment decision. As we have seen, depending on parameters, the market doesn’t always achieve the constrained efficient outcome, notably because adverse selection may lead to a withdrawal from the market and inefficient liquidation of safe banks. This section analyses how the central bank - or a governmental agency - may be able to influence the allocation by appropriate policy intervention.

It is worth noting that in our model, the equilibrium outcome is known to agents already at time 0: whether or not an equilibrium with full participation in the interbank market will exist depends on parameters, namely the relationship between the illiquid asset’s liquidation value and lenders’ and borrowers’ characteristics. Therefore, if parameters are such that the market is characterized by adverse selection, a central bank could try to intervene in order to achieve a dif-
ferent allocation than the one of adverse selection. In the following, we consider two main regulatory interventions, the enhancement of market transparency, and the introduction of liquidity ratios by banks.

5.1 Transparency

In the model, we take as given that information about the riskiness of the illiquid asset is private information to the bank. It is precisely the fact that the information is only private, that leads safe banks to drop out of the interbank market. One possible regulatory measure is thus to improve transparency.

Suppose that the government was able to devise a regulation implying that all banks need to credibly reveal their type. This information would become known at time 1, which is the point in time when the type is realized. Then, all banks would be able to distinguish whether they are facing a borrower with a risky or a safe technology. The effect would be the emergence of two markets: one for risky banks with an interest rate \( 1 + r_r \), and one for safe banks with an interest rate \( 1 + r_s \), with \( r_s < r < r_r \).

This has implications for two types of banks (in the case where parameters would otherwise imply adverse selection): safe illiquid banks are now able to finance their liquidity needs via the interbank market and do not have to inefficiently liquidate. Lenders will be able to lend out more of their excess reserves at time 1, as also safe borrowers will be on the market. Moreover, this regulation will affect the case where parameters would - in the absence of regulation - have led to an equilibrium with full participation: also this equilibrium will now be replaced by an equilibrium where both types of borrowers operate in a different market.

Anticipation of this type of regulation would impact on banks’ time-0 investment decision. Knowing that borrowers of all types will, at different interest rates, obtain interbank market loans, influences banks’ expected payoffs from investing in the liquid or the illiquid asset. Note that this regulation would not imply that
no banks with a risky illiquid investment would ever exist. This is because we are not considering a moral hazard problem in which the bank knowingly invests in the risky asset. Ex-ante, at the time the investment decision is made, all banks remain equal.

5.2 Liquidity Ratios

The second type of regulatory intervention considered here is one of imposing certain liquidity requirements. In the adverse selection case, safe illiquid banks liquidate instead of borrowing. In order to avoid this inefficiency, a regulator could impose certain liquidity levels that banks need to fulfill.

For simplicity, let us assume that the regulation demands all banks to hold $\lambda_h$ reserves, that is reserves are high enough so that banks can satisfy impatient depositors’ liquidity demand even if this turns out to be high. In this case, there is no need for any bank to borrow on the interbank market, or to liquidate at time 1. Instead, banks with a low liquidity shock will have excess reserves which they are not able to lend because of lack of demand.

Of course, the downside of this regulation is that imposing a higher than otherwise chosen level of reserves implies to forego higher expected returns from investment in the illiquid asset. This shortcoming has to be weighed against the benefits for safe borrowers from not having to liquidate. The benefit increases with the proportion of safe borrowers, $\pi_h q$ and with the gain that is implied from not liquidating, $1 - l_s$. The shortcoming on the other hand depends positively on the expected foregone return $\overline{p}R$. Therefore, for low $l_s$ or $\overline{p}R$, a liquidity ratio could improve overall welfare.

\footnote{Alternatively, one could assume a regulation putting a cap on the illiquid investment which still left room for an interbank market to develop. In this case, the imposed higher level of reserves would lead to a reduction in interbank interest rates at time 1, from equation (14). If the cap on $\alpha^f$ was chosen appropriately, it could lead to an interest rate just low enough for safe illiquid banks to participate in the interbank market.}
6 Crisis resolution

The ex-ante regulatory and supervisory interventions just described may help in preventing a breakdown of the interbank market, as happened during the current 2007/2008 financial crisis. A different topic are interventions in a situation in which a market breakdown occurs unexpectedly. In this section, we consider appropriate governmental intervention in the case that an unexpected shock leads to a (partial) breakdown of the interbank market. We focus on the situation in which the market is characterized by adverse selection, that is, only banks with a risky investment appear as borrowers in the interbank market.

Notice that we assume that the regulatory intervention as well as the shock are unanticipated. We therefore abstract, for the moment, from their impact on banks’ ex-ante investment decision. The moral hazard that could occur as a result of such interventions will be dealt with at a later stage.

6.1 Liquidity provision by the central bank

As a way of avoiding inefficient liquidation by banks, the central bank could offer to provide liquidity directly to those institutions in need. Because it has no information other than the one publicly available, such a liquidity provision would need to be offered to all banks in need, including risky illiquid banks. The provision of liquidity would be designed in such a way as to prevent inefficient liquidation by safe illiquid banks. Thus, the interest on loans from the central bank, $1 + r_{CB}$, would need to fulfill $1 + r_{CB} \leq \frac{R}{L}$.

On such operations, the central bank would make a loss, since the interest rate it obtains would necessarily be lower than the one reflecting the risk of the pool, i.e. $1 + \tau > 1 + r_{CB}$. This loss needs to be weighed against the benefit from the operation, i.e. avoiding inefficient liquidation of the safe banks.

If the central bank provides temporary liquidity to some banks at time 1, this implies that some banks - lenders - are left with excess liquidity they are unable
(or, in some cases, unwilling) to lend out on the market. In order to have a fair intervention in markets (or with the aim of achieving a certain market interest rate), the central bank can consider borrowing these funds from the market. In this case, it would take on an intermediation function: the central bank would be counterparty to any liquidity transaction taking place at time 1, and these transactions would replace the interbank market.\textsuperscript{16}

6.2 Interbank loan guarantees

A different way of obtaining an active interbank market with safe illiquid banks’ participation is the provision of guarantees for interbank loans.\textsuperscript{17} Depending on their scope, guarantees reduce or even eliminate the risk for lenders of not being repaid. This in turn lowers the interest rate in the interbank market since lenders demand lower compensation for risk.

In the case of full interbank loan guarantees, the probability of the interbank loan being repaid becomes 1 thus ensuring that all lenders and borrowers participate in the interbank market. Again, such intervention eliminates the need for the safe banks to liquidate their assets. Moreover, the extreme case of the complete market breakdown, which occurs in our model when $p_r (1 + r_r) < 1$, is avoided.

The costs of this intervention are similar to the case above (liquidity provision): ultimately, the central bank takes on the credit risk prevailing in the interbank market, and thus bears an expected loss. This cost needs to be weighed against the benefits from no liquidation for safe illiquid banks, $\pi_h g (1 - L)$.\textsuperscript{16}

\textsuperscript{16}Indeed, many central banks have during the 2007/08 crisis taken on intermediary functions. For instance, this became apparent when the European Central Bank started to provide unlimited amounts of liquidity in its weekly Main Refinancing Operations, but at the same time, took in significant amounts of deposits (remunerated, albeit at a penalty rate) from banks.

\textsuperscript{17}Many governments have, as a reaction to the 2007/2008 financial crisis, clarified and enhanced depositor guarantees. In our model, we assume that deposits are fully insured which eliminates the possibility of runs on banks.
6.3 Asset purchases

We assume that early liquidation is costly with \( l_{s,r} < 1 \). In a crisis, this reflects the fact that illiquid assets often have to be liquidated at “fire-sale” prices (see, e.g., Allen and Gale (2004)). Note that a central bank or a government authority does not face liquidity problems. It can offer to buy illiquid assets from banks at a specified price \( P > l_s \) which can be higher than the fire-sale price since the liquidity risk does not need to be priced in. The price \( P \) can only reflect credit risk faced by the central bank. Moreover, by setting the price appropriately, the central bank can attract both safe and risk illiquid borrowers and take advantage of knowing the average quality of the asset pool. This way of obtaining liquidity will be preferred by the safe banks to liquidation at \( l_s \).

7 Conclusion

In this paper, we have modelled an interbank market which can be characterized by adverse selection. We have shown that depending on parameter constellations, capturing in particular the distribution of risk of among banks and banks’ asset liquidation values, an equilibrium in which all banks with a negative shock (illiquid banks) borrow, and all those with excess liquidity lend, may break down. The market can either break down because even risky banks find it too expensive to borrow or because lenders stop providing liquidity to an adverse selection of borrowers.

As in the model any liquidation is equivalent to a welfare loss, there is a benefit of maintaining an interbank market open to all banks, or to provide alternative financing to banks. Thus, we have studied different possible interventions by a regulator or central bank that could improve efficiency. An overall increase in transparency is one method of helping the interbank market to reemerge. A direct intervention by the central bank, be it by providing temporary liquidity, acting as
an intermediary in the market, by purchasing assets or by providing guarantees on interbank loans, would also help. However, all these measures entail costs which need to be carefully weighed against their benefits, even in the case of possible resolution of a crisis.
8 Selected references


