

Liquidity, Moral Hazard and Inter-Bank Market Collapse*

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Abstract

This paper proposes a framework to analyze the functioning of the inter-bank liquidity market and the occurrence of liquidity crises. The model relies on three key assumptions: (i) liquidity provisioning is not verifiable -it cannot be contracted upon-, (ii) banks face moral hazard when confronted with liquidity shocks -unobservable effort can help overcome the shock-, (iii) liquidity shocks are private information - they cannot be diversified away-. Under these assumptions, the equilibrium risk-adjusted return on liquidity provisioning is shown to increase with the aggregate equilibrium volume of ex ante liquidity provision. As a consequence, banks may provision too little liquidity compared with the social optimum. Within this framework we derive two main results. First inter-bank market collapse is an equilibrium. Second such an equilibrium is more likely when (i) the individual probability of the liquidity shock is lower, (ii) ex ante competition between banks on illiquid long term assets is larger.

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1 Introduction

The financial market turmoil that has been under way since the Summer of 2007 hit the core of the global financial system, the inter-bank market for liquidity. This has manifested itself through episodes of widening spreads on inter-bank interest rates (vs. policy rates), together with evidence of plummeting volumes in inter-bank lending transactions. As turmoil turned into a full blown crisis in the Fall of 2008, inter-bank transactions were widely reported as frozen, as bid-offer spreads widened dramatically, and interest rates peaked on term borrowing beyond overnight transactions. A significant part of this phenomenon has been ascribed to a reassessment of credit risk involved in dealing with bank counterparties. Yet a large share of the premium that has emerged on inter-bank rates has been attributed to “liquidity risk”. To be sure, liquidity needs on behalf of banks were to some extent related to concerns by financial institutions over their own balance sheets dynamics in the face of credit losses. More generally, banks certainly needed liquidity as they prepared for: (i) firms calls on contingent credit lines; (ii) re-intermediation of investments that had previously been funded off-balance sheet ; (iii) possible merger and acquisition opportunities.¹

This paper does not endeavour to account for all the features of the recent crisis, be it hard evidence or casual stories about the motivations of market players. However, it argues that a proper modelling of the collapse in the market for liquidity involves a close look at incentives to provision liquidity and moral hazard mechanisms in the inter-bank market. In addition, it makes sense to do so in a framework where banks can actually fail and default on their borrowing. These assumptions are both strongly vindicated by salient features of the recent crisis. Many observers have argued that securitization may have provided the wrong incentives regarding the monitoring of underlying asset quality, in a clear-cut case of moral hazard. In addition, recent developments have shown that bank failures scenarios are only too realistic.

We investigate the possible role of insufficient ex-ante liquidity provision, in paving the way to an inter-

¹The buzz among market participants suggested that strategic behaviors could have been at play in liquidity hoarding by banks. Some financial intermediaries may have been unwilling to provide funding to competitors that had cut into their market share. This would be hard to document. However, it sounds very likely that some banks may have held extra liquidity in order to be in a position to seize latter opportunities if competitors were forced to fire sales. Historical precedent is mentioned by Kindleberger (1996), in the context of financial crises: « Outsiders particularly suffered. The Bank of the United States was allowed to fail in New York in December 1930 by a syndicate of banks, not the Federal Reserve System, amid accusations that the Bank was being punished for its pushy ways » (p 158).

bank market collapse. We thus highlight the aggregate benefits of situations where banks set aside large amounts of liquid assets in order to better deal with shocks affecting their illiquid investments. By liquidity provisions we mean specifically holdings of assets that can be used to safely transfer wealth over a short period of time. This may be seen as a form of "balance sheet liquidity". In practice such liquid holdings could be remunerated reserves held at the central bank, or short-term Treasury securities.² Indeed, the secular decline in the share of liquid assets on banks' balance sheets is a striking stylized fact that has been underscored by Goodhart (2008) as a troubling feature of risk management. A situation where market and funding liquidity appeared to be high may thus have hidden vulnerabilities stemming from limited holdings of liquid assets.

Against such a background, this paper shows that across equilibria, the risk adjusted return on liquid assets can be increasing with the aggregate volume of such assets in the economy. In other words, there can be increasing returns at the aggregate level to provisioning liquidity. When a bank faces a liquidity shock, it needs to reinvest in its liquidity affected assets. When it has provisioned a large volume of liquidity ex ante, reinvestment is mostly financed through internal funds. Hence the distressed bank pays particular attention to improving the probability that reinvestment succeeds. Consequently the moral hazard problem is mitigated and the distressed bank benefits from a large capacity to borrow liquidity on the inter-bank market. This tends to raise the demand for liquidity and hence the price for liquidity which in turn raises incentives to provision liquid assets ex ante. As a result, both the risk adjusted return on liquidity provisioning and the total volume of liquidity in the economy are large.

By contrast, with low ex ante liquidity provision, the argument is reversed: the moral hazard problem is amplified through the aforementioned channel: reinvestment is mostly financed through external funds. Intact lending banks then impose a tight constraint on the volume of liquidity distressed banks can borrow on the inter-bank market as to restore their incentives to deliver effort. This however reduces the demand for liquidity and drives down the price of liquidity which in turn depresses banks incentives to provision liquidity ex ante. Consequently the risk adjusted return on liquidity provisioning and the total volume of

²We do not model a risk-free asset market as such however: we will simply assume that a technology providing a risk-free rate of return is available as an alternative to illiquid investments on the one hand, and to interbank lending on the other hand.

liquidity in the economy are low. The two polar cases of high and low liquidity provisions can therefore both be equilibria of the economy.

Turning to comparative statics, the credit rationing equilibrium happens to be more likely when the liquidity shock is less likely. We call this property *the curse of good times*, meaning that banks have more difficulties refinancing their illiquid investments when the probability of the liquidity shock is lower. Alternatively the equilibrium of large liquidity provision and large risk adjusted return on liquidity provisioning is more likely when the liquidity shock is more likely, a property we call *the virtue of bad times*: refinancing illiquid investment is easier and less costly when the probability of the liquidity shock is larger. When the probability of facing the liquidity shock is high, banks raise their liquidity holdings because they are more likely to need these provisions for reinvestment. This relaxes the moral hazard induced liquidity constraint, raises the demand for liquidity and thereby the price for liquidity on the inter-bank market which in turn raises incentives to provision liquidity ex ante. The equilibrium with large liquidity provision and high risk adjusted return on liquidity provisioning is therefore more likely when the liquidity shock is more likely, hence *the virtue of bad times* property. Conversely, under the same mechanisms, the credit rationing equilibrium is more likely when the liquidity shock probability is lower, hence *the curse of good times* property. When the probability of facing the liquidity shock is low, banks reduce their liquidity holdings because they are less likely to need these provisions for reinvestment. This tightens the moral hazard induced liquidity constraint, reduces the demand for liquidity and thereby the price for liquidity on the inter-bank market which in turn reduces incentives to provision liquidity ex ante. The equilibrium with low liquidity provision and low risk adjusted return on liquidity provisioning is therefore more likely when the liquidity shock is less likely, hence *the virtue of bad times* property.

Finally the paper investigates the role of banks' market power on the functioning of the inter-bank market. Credit rationing turns out to be more likely when the degree of competition between banks on long term illiquid investments is higher. With more intense competition ex ante, the return to illiquid investment is less sensitive to an individual bank volume of illiquid investment. As a result, banks tend to increase their long term illiquid investments and thereby reduce their ex ante liquidity provision. At the aggregate level,

the economy is more likely to fall into the credit rationing equilibrium.

The model in this paper builds on the standard literature on moral hazard and liquidity crisis. The moral hazard problem is modelled in a basic, standard fashion, similar to that of Holmström and Tirole (1998), whereby the effort choice by the agent (in our case the bank) has an impact on the project's probability of success. We however depart from their seminal paper in an important way, by assuming that idiosyncratic liquidity shocks cannot be diversified away: this opens the door to an inter-bank market where liquidity can be reallocated interim. Our paper is also connected to the literature on interbank markets, as a mechanism for managing, and potentially eliminating, risks stemming from idiosyncratic liquidity shocks. Bhattacharya and Gale (1987) in particular studied the case where neither banks investments in the illiquid technology, nor liquidity shocks are observable. Rochet and Tirole (1996) adapted the Holmström-Tirole framework to the interbank market in order to study systemic risk and "too-big to fail" policy. The existence of interbank market imperfections has been established empirically by Kashyap and Stein (2000), which showed the role of liquidity positions, the so-called "liquidity effect". Building on such evidence, Freixas and Jorge (2008) analyze the functioning of the interbank market in order to show the consequences of its imperfections for monetary policy. In particular, they establish the relevance of heterogeneity in banks' liquid asset holdings for policy transmission. Our work is also related to work on liquidity crises. In particular Caballero and Krishnamurthy (2008) provide a model of crises that features liquidity hoarding, and provides a motivation for lender of last resort intervention. However, their approach is primarily based on Knightian uncertainty that leads each agent to hedge against the worst-case scenario. Recent research on industry hedging behavior shows that hedging decisions are related to the degree of competition, with more heterogeneity in hedging in the more competitive industries (Adam, Dasgupta and Titman, 2007). Liquidity provisions on behalf of banks may be seen as a form of hedging; however we are not aware of any work studying the impact of bank competition on bank assets liquidity. Acharya, Gromb and Yorulmazer (2008) have studied the consequences of imperfect competition in the interbank market for liquidity. In a model where there are frictions in the money and asset markets, if banks that provide liquidity have market power, they may strategically underprovide liquidity, and thus precipitate fire sales. Their model does not however feature interbank liquidity

crises in the sense of a market breakdown. A common feature of this literature is that the public provision of liquidity, such as liquidity injections, can often improve on the allocation of liquidity resulting from the decentralized outcome.

In sum, this paper's contribution consists in combining standard features of the moral hazard literature in order to account for a collapse in inter-bank lending. To the best of our knowledge, it is original in providing an explanation for such a market failure without recourse to stronger assumptions such as adverse selection and non-measurable risk.

The paper is organized as follows. The following section lays down the main assumptions of the model. The first best allocation is derived in section 3. The behavior of intact and distressed banks in a second best environment is analyzed in section 4. Section 5 details the decentralized equilibrium highlighting the full reinvestment and the credit rationing equilibrium. Section 5 also discusses the nature of the externality at the source of the multiple equilibrium property of the model. Imperfect competition is analyzed in section 6. Conclusions are drawn in section 7.

2 Timing and technology assumptions

We consider an economy with a unit mass continuum of banks. Banks are risk neutral and maximize expected profits. The economy lasts for three dates; 0, 1 and 2. At date 0, each bank has a unit capital endowment and two investment possibilities. The first is to invest in a liquid asset: a unit of capital invested in the liquid technology at date t yields ρ units of capital at date $t + 1$. The volume of capital that a bank invests at date 0 in the liquid technology is denoted l .

Alternatively each bank can invest in an illiquid project.³ The volume of capital a bank invests in an illiquid project at date 0 is denoted k_0 . Each bank hence faces a date 0 resource constraint, $k_0 + l = 1$. The volume of capital invested in each technology is assumed to be observable but not verifiable. Contingent contracts on ex ante liquidity provisioning are thus precluded.⁴

Illiquid projects require investment at date 0. At date 1, they may face a liquidity shock. With a probability $1 - q$, the liquidity shock is avoided and the bank which has financed the project is said "intact". Then the illiquid project yields εk_0 at date 1 and Rk_0 at date 2 ($\varepsilon \ll R$).⁵ With a probability q , the liquidity shock occurs and the bank which has financed the project is said "distressed". Then the illiquid project yields no output at date 1. Moreover the project yields output at date 2 if and only if the bank that financed the project at date 0 makes a reinvestment at date 1. Banks whose illiquid investments face no liquidity shock cannot directly invest in projects facing a liquidity shock. These investments need to go through the inter-bank liquidity market.⁶ The total volume of capital a distressed bank reinvests at date 1 is denoted k_1 and is assumed to be limited by the size of the initial project: date 1 reinvestment cannot be larger than initial date 0 investment: $k_1 \leq k_0$.⁷ If a bank reinvests k_1 units of capital and delivers an effort e at date 1,

³Another interpretation is that banks invest in multiple firms with correlated pay-outs.

⁴The assumption where ex ante liquidity provisioning would be neither observable nor verifiable would be a sufficient condition (although not necessary), under which the results of the model would hold. In reality, the volume of liquid assets a bank holds at a given point in time may be observable. However the funding source for these assets -capital or short term deposits for instance- is much more difficult to assess for an outside agent in real time. Hence even observability can be an issue in practise.

⁵In what follows we will think of ε as being typically small. Note that ε can be made infinitely small without any consequence for the results of the paper.

⁶Note that the alternative arrangement under which banks would sign ex ante insurance contracts against liquidity shock is not possible here. If banks receive a payment when they declare to be distressed then "intact" banks would always report untruthfully their situation as "distressed" since (i) liquidity shocks are unobservable and (ii) banks can invest the payment from the insurance contract in the liquid technology from date 1 to date 2 and finally consume the output at date 2.

⁷This assumption ensures that the profits of a "distressed" bank will always be lower than those of an "intact" bank.

then reinvestment is successful with probability e and the project yields $R(e)k_1$ at date 2, $R(e)$ being the marginal return net of the non pecuniary cost of undertaking effort e . With probability $1 - e$, reinvestment is unsuccessful and the project yields no output. To simplify notations and further computations, we assume that effort e can either be low $e = e_l$ or high $e = e_h$ ($e_l < e_h$) such that $R(e_l) = R$ and $R(e_h) = \phi R$ with $0 < \phi < 1$. High effort e_h is assumed to be efficient while low effort reinvestment is dominated by the liquid technology: $e_l R < \rho < e_h \phi R$. Moreover we denote $\psi = \frac{\phi e_h - e_l}{e_h - e_l}$ and assume that $e_h \psi R < \rho$.⁸ Finally the effort e a bank delivers at date 1 is private information and hence a source of moral hazard. We assume that the illiquid project is on average more profitable than the liquid technology: $(1 - q)R > \rho^2$.

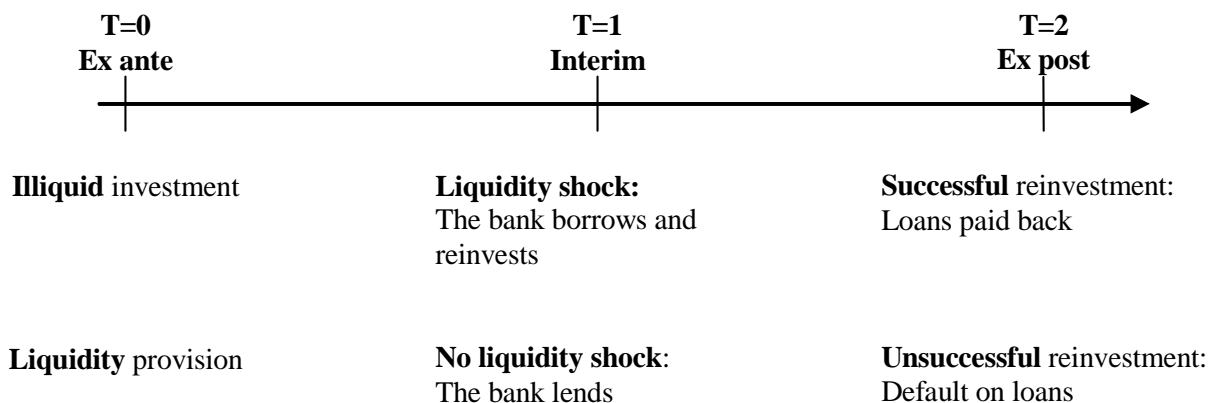


Figure 1: Timing of the model

To sum up, timing is as follows. At date 0, banks decide on capital allocation between liquid and illiquid assets. At date 1, a proportion q of banks face the liquidity shock. The inter-bank market then opens and intact banks can lend to distressed banks. Distressed banks reinvest their own liquidity plus borrowed funds in their illiquid project and deliver some effort. Finally at date 2, distressed banks learn if reinvestment has been successful or not. They pay their liabilities back if reinvestment is successful.

⁸This last assumption ensures that the moral hazard problem defines a non degenerate constraint for any possible inter-bank market interest rate.

3 The first best allocation

To derive the first best allocation, we remove two assumptions regarding market imperfections. First, date 0 allocation between liquid and illiquid assets is now verifiable. Second, the liquidity shock at date 1 and the effort e distressed banks deliver are both public information.

Let $c = (l; k_0; k_1; R_0; R_1(e))$ be a generic contract where l is date 0 investment in the liquid technology, k_0 is date 0 investment in the illiquid technology, k_1 is date 1 reinvestment in a project that faces a liquidity shock, R_0 is the date 2 payment to an intact bank and R_1 is the date 2 payment to a distressed bank. The first best allocation solves

$$\begin{aligned} & \max_e (1-q)R_0 + qR_1(e) \\ \text{s.t.} & \left\{ \begin{array}{l} k_0 + l \leq 1 \text{ and } R_0 \leq k_0R \\ qk_1 \leq (1-q)\varepsilon k_0 + \rho l \\ k_1 \leq k_0 \text{ and } R_1(e) \leq k_1eR(e) \end{array} \right. \end{aligned} \quad (1)$$

Each unit of capital endowment is divided between k_0 units of capital invested in the illiquid asset and l units of capital invested in the liquid asset. When the illiquid asset is intact, it returns k_0R at date 2. This happens with a probability $1-q$. On the contrary, the illiquid asset is distressed at date 1 with probability q . Given that there are $(1-q)\varepsilon k_0 + \rho l$ units of capital available at date 1 for reinvestment, total reinvestment qk_1 cannot be larger than total available capital $(1-q)\varepsilon k_0 + \rho l$. Moreover reinvestment cannot be larger than initial investment, $k_1 \leq k_0$. Finally each distressed project in which k_1 is reinvested returns an expected output $k_1eR(e)$. It is straightforward to note that the optimal contract is such that constraints on payments R_0 and R_1 are binding and that high effort is optimal. As a result, the first best allocation solves

$$\begin{aligned} & \max_l [(1-q)(1 + e_h\phi\varepsilon)(1-l) + e_h\phi\rho l]R \\ \text{s.t.} & \rho l \leq [q - (1-q)\varepsilon](1-l) \end{aligned} \quad (2)$$

We can then derive the following result

Proposition 1 *Denoting $[x]^+ = \max\{x; 0\}$ and assuming $(\rho - \varepsilon)e_h\phi > 1$, the first best capital allocation*

is such that each bank invests

$$l_{fb} = \frac{[q(1+\varepsilon) - \varepsilon]^+}{\rho + [q(1+\varepsilon) - \varepsilon]^+} \quad (3)$$

units of capital in the liquid technology at date 0.

Proof. The problem which solves the first best allocation writes as

$$\begin{aligned} \max_l & [(1-q)(1+e_h\phi\varepsilon)(1-l) + e_h\phi\rho l] R \\ \text{s.t. } & \rho l \leq [q - (1-q)\varepsilon](1-l) \end{aligned}$$

Assuming $(\rho - \varepsilon)e_h\phi > 1$, the objective function is always strictly increasing in l since $e_h\phi\rho > (1-q)(1+e_h\phi\varepsilon)$.

As a consequence, the first best allocation is such that the constraint $\rho l \leq [q - (1-q)\varepsilon](1-l)$ is binding.

As a result since liquidity provision cannot be negative, the first best optimal liquidity provision l_{fb} is imply

$$l_{fb} = \frac{[q(1+\varepsilon) - \varepsilon]^+}{\rho + [q(1+\varepsilon) - \varepsilon]^+}$$

and optimal investment in the illiquid technology k_0 is hence

$$k_0 = 1 - l_{fb} = \frac{\rho}{\rho + [q(1+\varepsilon) - \varepsilon]^+}$$

■

The first best optimal liquidity provision is $l_{fb} = \frac{[q(1+\varepsilon) - \varepsilon]^+}{\rho + [q(1+\varepsilon) - \varepsilon]^+}$ when $e_h\phi\left(\frac{\rho}{1-q} - \varepsilon\right) \geq 1$ and $l_{fb} = 0$ when $e_h\phi\left(\frac{\rho}{1-q} - \varepsilon\right) < 1$. Typically when the probability q of the liquidity shock is sufficiently low, i.e. $q < \frac{1 - (\rho - \varepsilon)e_h\phi}{1 + \varepsilon e_h\phi}$, then it is not worth provisioning liquidity because there will be very few illiquid projects hit by the liquidity shock and the expected return to illiquid investments $(1-q)R$ is very large. The social planner then prefers to maximize illiquid investments. In what follows, we will assume that the parameter restriction $(\rho - \varepsilon)e_h\phi > 1$ always holds so that first best liquidity provision is always given by (3).

4 Intact and distressed banks

We now return to the framework described in section 2. The model can be solved by backward induction. We first solve intact and distressed banks problem at date 1. Then we solve the date 0 problem of optimal liquidity provision.

4.1 Distressed banks' optimal demand for liquidity

Let us consider bank i which, at date 0, invested l_i units of capital in the liquid technology and $k_0 = 1 - l_i$ in an illiquid project. If bank i is distressed at date 1, it can either reinvest in its illiquid project or give up its illiquid project and lend its liquid assets on the capital market. In case a distressed bank reinvests in its illiquid project, d_i denotes the volume of capital it borrows at date 1 and e_i the effort it undertakes. Its date 2 expected profit (net of non pecuniary costs to deliver effort) then writes as

$$\pi_b = e_i [(\rho l_i + d_i) R(e_i) - r d_i] \quad (4)$$

where r is the borrowing rate on the inter-bank liquidity market. At date 1, a distressed bank uses the proceeds of its liquid investments ρl_i undertaken at date 0 and borrows d_i to reinvest in the illiquid project it financed at date 0. Hence reinvestment k_1 is equal to $\rho l_i + d_i$. Conditional on success, date 2 proceeds net of non pecuniary costs are $(\rho l_i + d_i) R(e_i)$, the face value of liabilities is $r d_i$, and e_i is the probability of successful reinvestment. Note that the interest rate r is independent of bank i decisions and in particular of its effort e_i , because effort is unobservable. The problem at date 1 of a distressed bank which reinvests in its illiquid project consists in choosing the effort level e_i and the volume of borrowing d_i which solve the problem

$$\begin{aligned} \max_{d_i; e_i} \pi_b &= e_i [(\rho l_i + d_i) R(e_i) - r d_i] \\ \text{s.t. } \rho l_i + d_i &\leq 1 - l_i \end{aligned} \quad (5)$$

The constraint that total reinvestment $(\rho l_i + d_i)$ cannot be larger than initial investment $(1 - l_i)$ imposes a constraint on the volume d_i that can be borrowed on the inter-bank market. This inequality can be written

as $d_i \leq \overline{d(l_i)}, \overline{d(l_i)}$ being the upper bound on the volume of liquidity a distressed bank needs to borrow. We can then derive the following proposition.

Proposition 2 *If the interest rate on the inter-bank market verifies $r \leq \phi R$, a distressed bank always borrows $d_i = \overline{d(l_i)}$ and delivers effort e_i such that*

$$e_i = \begin{cases} e_h & \text{if } r \geq \frac{\rho l_i + d_i}{d_i} \psi R \\ e_l & \text{if } r < \frac{\rho l_i + d_i}{d_i} \psi R \end{cases} \quad (6)$$

Proof. If bank i is distressed and reinvests in its illiquid project then optimal borrowing d_i^* writes as

$$d_i^* = \overline{d(l_i)} \mathbf{1} [R(e_i^*) \geq r] \quad (7)$$

Consequently as long as $r < \phi R$, $d_i^* = \overline{d(l_i)}$ and optimal effort e_i^* writes as

$$e_i^* = \begin{cases} e_h & \text{if } \psi R(\rho l_i + d_i^*) \geq r d_i^* \\ e_l & \text{if } \psi R(\rho l_i + d_i^*) < r d_i^* \end{cases} \quad (8)$$

■

A distressed bank is more likely to deliver low effort e_l when reinvestment is proportionally more financed through external funds, i.e. when borrowing d_i is larger and/or liquidity provisioning l_i is lower. This illustrates the trade-off an agent faces when it can borrow while facing moral hazard. Increased borrowing raises profits but reduces incentives to deliver effort. With large ex ante liquidity provision, borrowing is lower and incentives to deliver effort -which decrease with borrowing- are larger. Hence banks prefer to deliver high effort. On the contrary with low liquidity provisioning, borrowing is large $\overline{d(l_i)}$ is large- and incentives to deliver effort -which decrease with borrowing- are lower. Banks then prefer to borrow and deliver low effort. Note also that a distressed bank is more likely to deliver low effort e_l when the interest rate r is higher.

Having determined optimal borrowing and effort conditional on reinvestment, we can now examine whether distressed banks prefer to reinvest in their illiquid assets or to give up their illiquid project and lend their liquid holdings on the inter-bank market. The following lemma derives this choice.

Lemma 3 *If the interest rate on the inter-bank liquidity market verifies $r \leq \phi R$, then distressed banks always prefer to reinvest in their illiquid project than to lend their liquid assets on the inter-bank market.*

Proof. Denoting d_i^* the volume of capital a distressed bank borrows, when the interest rate on the inter-bank market verifies $r \leq \phi R$, its expected profits from reinvestment π_b then write as

$$\pi_b = e_i [R(e_i)(\rho l_i + d_i^*) - r d_i^*]$$

e_i being the distressed bank optimal effort. Expected profits π'_b from lending liquid assets on the inter-bank market are simply $\pi'_b = e_i r \rho l_i$ because the repayment probability of distressed banks is e_i . Given the assumption $R(e_i) \geq r$, d_i^* is always positive and profits from reinvestment π_b are always larger than profits from lending liquid assets on the inter-bank market. ■

4.2 Intact banks' optimal supply of liquidity

We now turn to the situation where bank j is intact at date 1. Recall that at date 0 it invested l_j units of capital in the liquid technology and $k_0 = 1 - l_j$ in an illiquid project. It reaps $(1 - l_j) R$ at date 2. Moreover it can lend its liquid assets to distressed banks at date 1. When the interest rate on the inter-bank market is r , and distressed banks deliver an effort e , intact bank j enjoys date 2 expected profits

$$\pi_g = (1 - l_j) R + [\varepsilon(1 - l_j) + \rho l_j] \max\{er; \rho\} \tag{9}$$

An intact bank can always invest its liquid assets $[\varepsilon(1 - l_j) + \rho l_j]$ at date 1 in the liquid technology. Hence intact banks supply their liquid holdings on the inter-bank market if and only if $er \geq \rho$. Moreover distressed

banks deliver high effort e_h when their liquidity provisioning l_i verifies

$$(\rho l_i + d_i) \psi R \geq r d_i \quad (10)$$

d_i being what distressed banks borrow from the inter-bank market. Given that distressed banks borrow at most $\overline{d(l_i)}$ on the inter-bank market, there can be two different situations:

(i) If (10) holds for $d_i = \overline{d(l_i)}$, then distressed bank i delivers high effort e_h and borrows up to the limit $\overline{d(l_i)}$. Intact banks supply their liquid holdings on the inter-bank market as long as the interest rate r verifies $e_h r \geq \rho$.

(ii) If on the contrary (10) does not hold for $d_i = \overline{d(l_i)}$, then the condition $e_r \geq \rho$ cannot hold since $e = e_l$, $r \leq R$ and $e_l r \leq R$. Intact banks then impose a borrowing constraint to distressed bank i to make sure that it delivers high effort. The volume of liquidity distressed banks can then borrow verifies the incentive constraint:

$$e_h ((\rho l_i + d) \phi R - d_i r) \geq e_l ((\rho l_i + d) R - d_i r)$$

This condition simplifies as $d \leq d(l_i)$ with

$$d(l_i) = \frac{\psi R}{[r - \psi R]^+} \rho l_i \quad (11)$$

In that case distressed bank i cannot achieve full reinvestment: $\rho l_i + d(l_i) < 1 - l_i$ and intact banks supply their liquid holdings on the inter-bank market as long as the interest rate r verifies $e_h r \geq \rho$. The next section is devoted to lay down the conditions under which each of these two situations can be an equilibrium.

5 The decentralized equilibrium

In the previous section, we derived the optimal date 1 decision rules for intact and distressed banks in terms of lending, borrowing, and effort. Based on these results, we now examine the problem of the optimal liquidity provision policy at date 0.

5.1 The full reinvestment equilibrium

5.1.1 Optimal ex ante liquidity provision with full reinvestment

Let us first consider the case where distressed banks choose to borrow from the inter-bank market, reinvest fully in their illiquid project and deliver the high effort e_h . The problem of a bank at date 0 then writes as

$$\begin{aligned} \max_{l_i} E\pi &= (1-q)[(1-l_i)R + (\varepsilon(1-l_i) + \rho l_i)e_h r] + qe_h[(\rho l_i + d_i)\phi R - rd_i] \\ \text{s.t. } \rho l_i + d_i &= 1 - l_i \text{ and } d_i \leq d(l_i) \end{aligned} \quad (12)$$

Proposition 4 Denoting $l(r) = \frac{r-\psi R}{r\rho+r-\psi R}$ and $r_{h,1} = \frac{R}{e_h} \frac{1-q+qe_h\phi}{\rho+q-(1-q)\varepsilon}$, optimal individual liquidity provision for a bank which reinvests fully in its illiquid project when distressed writes as

$$l_i^* = \begin{cases} l(r) & \text{if } r \leq r_{h,1} \\ [l(r); 1] & \text{if } r = r_{h,1} \\ 1 & \text{if } r \geq r_{h,1} \end{cases} \quad (13)$$

Proof. Program (12) is linear in ex ante liquidity provision l_i . Expected profits are decreasing in liquidity provision for $r \leq r_{h,1}$, since

$$\frac{\partial \pi}{\partial l_i} = e_h [\rho + q - (1-q)\varepsilon] \left[r - \frac{(1-q) + qe_h\phi}{\rho + q - (1-q)\varepsilon} \frac{R}{e_h} \right]$$

Banks then choose to provision as little liquidity as they can. Optimal liquidity provision $l(r)$ verifies $\rho l(r) + d(l(r)) = 1 - l(r)$. On the contrary expected profits are increasing in liquidity provision for $r \geq r_{h,1}$.

Banks then choose to provision as much liquidity as they can, i.e. $l^* = 1$. In between, i.e. for $r = r_{h,1}$ they are indifferent to liquidity provisioning and they choose any amount of liquidity l in $[l(r); 1]$ ■

When distressed banks are able to fully reinvest in their illiquid project, the interest rate $r_{h,1}$ beyond which they invest all their capital in liquid assets is decreasing in the individual probability q of facing a liquidity shock. A larger probability of the liquidity shock raises the expected profit of provisioning liquidity as opposed to investing in illiquid assets. Hence, banks accept to hold larger liquidity provision when the

liquidity shock is more likely even if the interest rate on liquid assets is lower. More formally, when a distressed bank reinvests fully in its illiquid project, the expected profits from liquidity provisioning increase with the interest rate r on the inter-bank market while profits from investments in illiquid assets decrease with the interest rate r :

$$E\pi = [(1 - q + qe_h\phi)R - (q - (1 - q)\varepsilon)e_hr](1 - l_i) + e_hr\rho l_i$$

Consequently, a decline in the probability q of liquidity shocks produces an increase in profits stemming from illiquid investments $(1 - l_i)$ relative to profits stemming from liquid investments l_i which is compensated by an increase in the inter-bank interest rate r to restore equilibrium. When distressed banks reinvest fully in their illiquid project, expected profits write as $\pi_h = \rho e_h r_{h,1}$.

5.1.2 Equilibrium with full reinvestment

The situation where full reinvestment is achieved is an equilibrium if and only if the following conditions are satisfied. First, aggregate supply of liquidity must balance aggregate demand for liquidity on the inter-bank market

$$\rho \int_{[0;1]} l_i^* di + (1 - q)\varepsilon \int_{[0;1]} (1 - l_i^*) di = q \int_{[0;1]} (1 - l_i^*) di \quad (14)$$

where l_i^* is given by (13). Second, distressed banks borrow from the inter-bank market and intact banks lend their liquid assets to distressed banks if and only if expected returns on the liquid technology, inter-bank lending and reinvestment verify

$$\rho \leq e_h r \leq e_h \phi R \quad (15)$$

Finally there should be no profitable deviation for banks in terms of ex ante liquidity provision. The next proposition wraps up the conditions under which the situation where distressed banks deliver high effort is an equilibrium.

Proposition 5 Denoting $r_{h,2} = \frac{\psi R}{(1-q)(1+\varepsilon)}$, the first best allocation is an equilibrium where distressed banks

provision the first best volume of liquidity $l = l_{fb}$ and are able to fully reinvest in their illiquid project if and only if the interest rate $r_h = \min\{r_{h,1}; r_{h,2}\}$ verifies

$$\rho \leq e_h r_h \leq e_h \phi R \quad (16)$$

Proof. cf. appendix ■

When the probability q to face the liquidity shock is larger, banks have more incentives to invest in liquid assets. As a result, the inter-bank market interest rate needs to decrease in order to reduce the expected return on liquidity lending to maintain the equilibrium on the inter-bank market. On the contrary a larger probability q to face the liquidity shock tends to raise the demand for liquidity. As a consequence, the interest rate on the inter-bank market needs to increase to maintain the equilibrium. If $r_h = r_{h,1}$, the first effect dominates and the equilibrium interest rate decreases when the liquidity shock is more likely. On the contrary if $r_h = r_{h,2}$, the second effect dominates and the equilibrium interest rate increases when the probability to face the liquidity shock is more likely.

The individual rationality constraints (16) are more likely to be verified when the individual probability q of the liquidity shock is larger. In other words the equilibrium with full reinvestment is more likely to hold in deteriorated environments. More precisely, when the equilibrium interest rate r is equal to $r_{h,1}$, the individual rationality constraint for intact banks, $e_h r_{h,1} \geq \rho$ is always verified since by assumption $e_h \phi R > \rho$ and $(1 - q) R > \rho^2$. Similarly, the individual rationality constraint for distressed banks, $r_{h,1} \leq \phi R$ always holds since by assumption we have $e_h \phi \rho > 1$. Alternatively when the equilibrium interest rate r is equal to $r_{h,2}$, the individual rationality constraint for distressed banks $r_{h,2} \leq \phi R$ is always verified since by assumption we consider the case where $r_{h,2} \leq r_{h,1}$ and we always have $r_{h,1} \leq \phi R$. Finally the individual rationality constraint for intact banks, $e_h r_{h,2} \geq \rho$ is more likely to be verified when the probability q to face the liquidity shock is relatively large since $r_{h,2}$ increases with the probability q .

When the probability to face the liquidity shock is low, the interest rate on the inter-bank market $r_{h,2}$ is relatively low. As a result, the individual rationality constraint for intact banks is more likely to be binding than the moral hazard problem for distressed banks. When the probability to face the liquidity shock increases, there are on the one hand more distressed banks but on the other hand, banks raise their liquidity holdings because they are more likely to need these provisions for reinvestment. At the aggregate level however, the former effect dominates and the demand for liquidity from distressed banks on the inter-bank market increases. As a consequence the inter-bank market interest rate $r_{h,2}$ increases and intact banks individual rationality constraint is more likely to be verified. The equilibrium with full reinvestment is therefore more likely when the liquidity shock is more likely, a property we refer to as *the virtue of bad times*. Note finally that the equilibrium where distressed banks achieve full reinvestment is efficient in the sense it replicates the first best capital allocation between liquid and illiquid assets.

5.2 The credit rationing equilibrium

In the equilibrium described in the previous section, distressed banks are able to carry out full reinvestment. When banks hold large liquidity provision ex ante, liquidity supply is large and liquidity demand is low on the inter-bank market. As a result, the interest rate on the inter-bank market is low and distressed banks can achieve full reinvestment while they deliver high effort. However what happens when the volume of liquidity that banks provision ex ante is not sufficiently large to ensure both full reinvestment and high effort? This section examines this case.

5.2.1 Optimal ex ante liquidity provision under credit rationing

When the constraint $d_i \leq d(l_i)$ on the volume of liquidity that can be borrowed from the inter-bank market is binding, each distressed bank borrows $d(l_i)$ from intact banks. Assuming the cost of borrowing liquidity is lower than the return on reinvestment, $r < \phi R$, the program of an individual bank i at date 0 therefore

consists in choosing the volume of ex ante liquidity provision l_i which solves

$$\begin{aligned} \max_{l_i} E\pi &= (1-q) [(1-l_i)R + e_h r (\rho l_i + \varepsilon (1-l_i))] + q e_h [(\rho l_i + d_i) \phi R - r d_i] \\ \text{s.t. } d_i &= d(l_i) \text{ and } \rho l_i + d_i \leq 1 - l_i \end{aligned} \quad (17)$$

Proposition 6 *Optimal individual liquidity provision for a bank which cannot fully reinvest in its illiquid project when distressed writes as*

$$l_i^* = \begin{cases} 0 & \text{if } \frac{\partial E\pi}{\partial l} \leq 0 \\ [0; l(r)] & \text{if } \frac{\partial E\pi}{\partial l} = 0 \\ l(r) & \text{if } \frac{\partial E\pi}{\partial l} \geq 0 \end{cases} \quad (18)$$

with

$$\frac{\partial E\pi}{\partial l} = \left[(1-q)(\rho - \varepsilon) + q \frac{(\phi - \psi)R}{r - \psi R} \rho \right] e_h r - (1-q)R$$

Proof. Program (17) is linear in ex ante liquidity provision l_i . When expected profits are decreasing in liquidity provision, then banks choose to provision as little liquidity as they can, i.e. $l_i^* = 0$. On the contrary when expected profits are increasing in liquidity provision, then banks choose to provision as much liquidity as they can. This level of liquidity provisioning $l(r)$ solves as previously $\rho l_i + d(l_i) = 1 - l_i$. ■

The function $\frac{\partial E\pi}{\partial l}$ is potentially non monotonic in the interest rate on the inter-bank market. On the one hand, a high cost of liquidity r raises the return to liquidity for intact banks. On the other hand however, it raises the cost of borrowing liquidity for distressed banks, and it reduces the volume of liquidity they can borrow on the inter-bank market. Banks therefore choose low liquidity provisioning when the interest rate on the inter-bank market is either very low or very large. Denoting π_c banks expected profits when the credit constraint induced by moral hazard binds, we have

$$\pi_c = (1-q)(R + e_h r \varepsilon)(1-l) + e_h r \left[1 - q + q \frac{(\phi - \psi)R}{r - \psi R} \right] \rho l$$

5.2.2 Equilibrium collapse of the inter-bank market

Given optimal date 0 liquidity provisioning (18), the aggregate demand for liquidity L_d at date 1 is

$$L_d = q \frac{\psi R}{r - \psi R} \rho \int_{[0;1]} l_i^* di$$

and the aggregate supply of liquidity L_s at date 1 is

$$L_s = (1 - q) \left[\varepsilon \int_{[0;1]} (1 - l_i) di + \rho \int_{[0;1]} l_i di \right]$$

We define a collapse of the inter-bank market as a situation where banks do not provision liquidity ex-ante, and intact banks do not lend to distressed banks. We can then derive the following proposition.

Proposition 7 *The collapse of the inter-bank market is an equilibrium if and only if*

$$\left[1 + q \frac{e_h \phi R - \rho}{\rho - e_h \psi R} \right] \rho^2 < (1 - q) (R + \rho \varepsilon) \quad (19)$$

In this equilibrium, the interest rate verifies $e_h r = \rho$. The inter-bank market collapse equilibrium is the unique credit rationing equilibrium when it exists.

Proof. To derive the proof of the proposition's first result, let us proceed in two steps. Assume first that the borrowing constraint $d_i \leq d(l_i)$, binds. Then distressed banks borrow $d_i = d(l_i)$ from the inter-bank market and the first order condition to the problem of an individual bank implies that zero ex ante liquidity provision is optimal if and only if $\frac{\partial E\pi}{\partial l} < 0$, i.e.

$$e_h r \left[(1 - q) (\rho - \varepsilon) + q \frac{(\phi - \psi) R}{r - \psi R} \rho \right] < (1 - q) R$$

When optimal liquidity provision is zero, $l_i^* = 0$ there is an excess supply of liquidity on the inter-bank market as aggregate liquidity supply is $S = (1 - q) \varepsilon > 0$ while liquidity demand is $D = 0$. As a consequence the individual rationality constraint for intact banks binds and the interest rate on the inter-bank liquidity

market verifies $e_h r = \rho$. Zero liquidity provision is therefore individually optimal if and only if

$$\left[1 + q \frac{e_h \phi R - \rho}{\rho - e_h \psi R}\right] \rho^2 < (1 - q)(R + \rho \varepsilon)$$

When this condition is verified, the situation where banks do not provision liquidity ex ante is an equilibrium if the initial assumption -that the borrowing constraint $d_i \leq d(l_i)$ is binding- holds. When $e_h r = \rho$, the condition $r < \phi R$ is always satisfied and reinvestment cannot be fully carried out when $l^* = 0$. If however banks decide to provision a larger volume of liquidity -so that the borrowing constraint $d_i \leq d(l_i)$ would not be binding if distressed- expected profits write as

$$E\pi_d = (1 - q) [(1 - l_i) R + \rho(l_i \rho + (1 - l_i) \varepsilon)] + q [(1 - l_i) (e_h \phi R - \rho) + \rho^2 l_i]$$

and the borrowing constraint $d_i \leq d(l_i)$ then does not bind if liquidity provision l_i verifies

$$l_i \geq l \left(\frac{\rho}{e_h} \right) = \frac{\rho - e_h \psi R}{\rho^2 + \rho - e_h \psi R}$$

Expected profits $E\pi_d$ are strictly decreasing in the volume l_i of ex ante liquidity provisioning since

$$\frac{(1 - q) R + q e_h \phi R}{\rho + q - (1 - q) \varepsilon} > \rho$$

As a consequence, the optimal liquidity provision of the bank verifies

$$l_i = l \left(\frac{\rho}{e_h} \right)$$

Optimal expected profits $E\pi_d$ can be written as

$$E\pi_d = (1 - q) [(1 - l_d) R + \rho(l_d \rho + (1 - l_d) \varepsilon)] + q e_h \frac{(\phi - \psi) R}{\rho - e_h \psi R} \rho^2 l_d$$

However when a bank does not provision liquidity, expected profits write as

$$E\pi(l^*) = (1 - q) [(1 - l^*)R + \rho(l^*\rho + (1 - l^*)\varepsilon)] + qe_h \frac{(\phi - \psi)R}{\rho - e_h\psi R} \rho^2 l^*$$

where $l^* = 0$. Hence expected profits when the liquidity constraint binds are larger than expected profits when the liquidity constraint does not bind if and only if $E\pi(l^*) > E\pi_d$ which simplifies as (19) and which by assumption is supposed to hold. As a consequence the liquidity constraint $d_i \leq d(l_i)$ always holds and the situation where banks do not provision liquidity is an equilibrium of and only if (19) holds.

The proof for the second result of the proposition is derived in appendix. ■

Condition (19) -under which the inter-bank market collapse equilibrium holds- is more likely to be satisfied when the probability q to face the liquidity shock is relatively low. When the liquidity shock is less likely, banks provision less liquidity and invest more in illiquid assets. Distressed banks are then more likely to deliver low effort when they reinvest in their illiquid project as reinvested funds will be mostly borrowed which intact lending banks solve by imposing credit rationing to ensure that distressed banks deliver high effort. However credit rationing depresses the return on liquidity provision for intact banks because it reduces the demand for liquidity and this in turn reduces ex ante incentives to provision liquidity especially when the probability to remain intact is large. The credit rationing equilibrium is therefore more likely when the liquidity shock is less likely, a property we refer to as *the curse of good times*: an environment with good fundamentals is conducive to credit rationing and inter-bank market collapse.

5.3 The general equilibrium externality

When ex ante liquidity provisioning is low, then liquidity supply on the inter-bank market is low but liquidity demand is also relatively low due to the presence of a moral hazard induced liquidity constraint for distressed banks. It turns out that with low liquidity provisioning, liquidity is in excess supply on the inter-bank market. Hence the inter-bank market interest rate is relatively low. This has two opposite consequences: on the one hand, a low interest rate on the inter-bank market reduces the return to liquidity provisioning for intact (lending) banks. On the other hand, it raises the return to liquidity provisioning for distressed (borrowing)

banks because (i) borrowing liquidity is not expensive and (ii) the volume of liquidity that can be borrowed on the inter-bank market increases with ex ante liquidity provisioning. When the probability q of facing the liquidity shock is relatively low, then the former effect dominates the latter and a positive feedback effect emerges: a low expected return to liquidity provisioning reduces bank incentives to provision liquidity and low liquidity provisioning generates an excess liquidity supply on the inter-bank market which depresses the expected return to liquidity provisioning. As a result, there is an equilibrium of low liquidity provisioning and low expected return on liquidity when the probability q to face the liquidity shock is relatively low.

Conversely, when ex ante liquidity provisioning is large, then liquidity supply is large but liquidity demand is also relatively large due to a diminished liquidity constraint for distressed banks. When the probability q of facing the liquidity shock is large, the interest rate on the inter-bank market is relatively high because the demand for liquidity is relatively large compared to the supply for liquidity. The expected return on liquidity provisioning is then large because intact (lending) banks enjoy a large return from lending liquidity on the inter-bank market while distressed (borrowing) banks enjoy a large shadow return on their liquidity assets. This gives rise to a positive feedback loop: on the one hand, a large expected return on liquidity provision raises bank incentives to provision liquidity while on the other hand, a large liquidity provision translates into a large expected return on liquid assets when the probability q of facing the liquidity shock is sufficiently large. As a result, there is an equilibrium of high liquidity provisioning and high expected return on liquidity when the probability q of facing the liquidity shock is relatively large. The existence of multiple equilibria -a full reinvestment equilibrium and a credit rationing equilibrium- is therefore entirely driven by the general equilibrium -feedback- effect of the aggregate liquidity provision on the inter-bank market interest rate. In partial equilibrium model with an exogenous cost of liquidity, the equilibrium would be unique. This property can be examined in a diagram representing the aggregate supply L_s and the aggregate demand for liquidity L_d as a function of aggregate ex ante liquidity provision l . Aggregate liquidity supply L_s is the sum of intact banks available liquid assets $(1 - q)l\rho$ and $(1 - q)(1 - l)\varepsilon$. Aggregate demand for liquidity L_d is the minimum of distressed banks' liquidity constraint $d(l) = \frac{\psi R}{r - \psi R}\rho l$ and the upper bound

on the volume of liquidity distressed banks need to borrow $\overline{d(l)} = 1 - l - \rho l$.

$$L_s = (1 - q)[l\rho + (1 - l)\varepsilon]$$

$$L_d = q \min \{d(l); \overline{d(l)}\}$$

Due to the existence of moral hazard, the aggregate demand for liquidity L_d is decreasing in the volume of aggregate liquidity provisioning l if and only if individual liquidity provisioning is sufficiently large. When provisioning is low, the moral hazard problem binds and the demand for liquidity increases with liquidity provisioning.

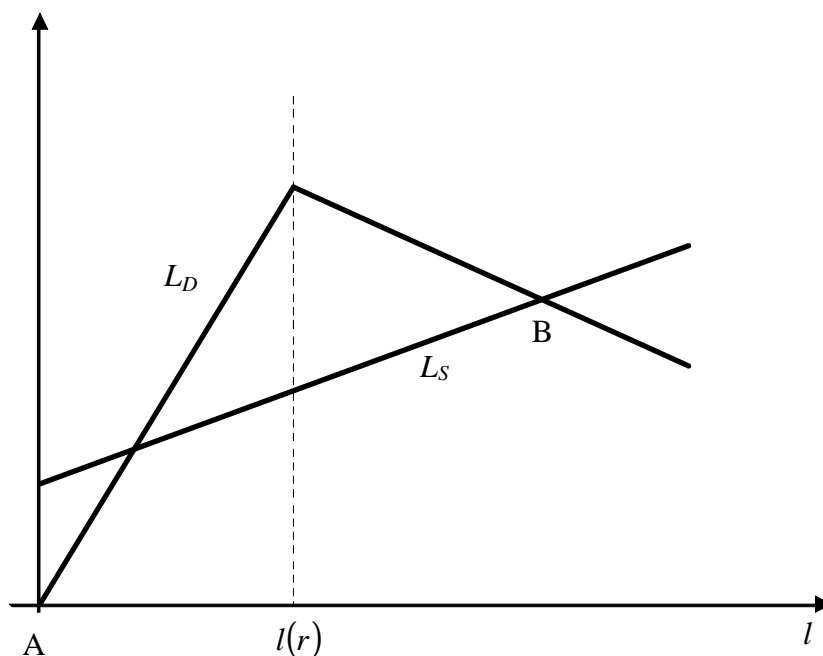


Figure 2: Aggregate supply and aggregate demand for liquidity

The supply for liquidity L_s is increasing in the volume of individual liquidity provisioning l . As a consequence, there are two equilibria. The credit rationing equilibrium is situated at point A where banks provision no liquidity. The moral hazard induced liquidity constraint then binds for distressed banks which cannot borrow liquidity and intact banks are compelled to store the interim output of their illiquid assets in the liquid technology. The equilibrium of full reinvestment is situated at point B . In this case the inter-bank

market clears and banks capital allocation between liquid and illiquid assets is identical to the first best allocation. In a partial equilibrium model, the liquidity supply L_s would be vertical and the equilibrium would always be unique.

Comparing the full reinvestment and the credit rationing equilibria shows that the risk adjusted return to liquidity provisioning is higher when liquidity provisioning is larger. Let us denote ρ_h (resp. ρ_c) the date 0 risk adjusted return on liquidity provisioning in the full reinvestment (resp. credit rationing) equilibrium. Given that the volume of liquidity provision in the full reinvestment (resp. credit rationing) equilibrium is l_{fb} (resp. 0), we have

$$\rho_h > \rho_c \text{ and } l_{fb} > 0$$

Across equilibria, the expected return on liquidity lending increases in the volume of liquidity that banks provision ex ante.

The credit rationing equilibrium could be eliminated if banks could ex ante contract on the volume of date 0 liquidity provisioning. Suppose banks agree at date 0 to contingent the cost of borrowing liquidity at date 1 on individual ex ante liquidity provisioning. For instance if the interest rate charged to distressed bank i writes as

$$r(l_i) = r_h + (R - r_h) \mathbf{1}[l_i < l_{fb}]$$

then all banks would ex ante provision $l_i \geq l_h$ and would be charged an interest rate r_h if distressed. Under this type of liquidity pricing the credit rationing equilibrium would be ruled out. Therefore, both moral hazard and non-verifiability of liquidity provisioning are required to obtain the credit rationing equilibrium.

6 The impact of imperfect competition

So far, we have assumed perfect competition between banks. We now introduce imperfect competition on investment in illiquid assets and focus on the impact on the likelihood of a credit rationing equilibrium. The framework we consider is essentially the same as the one considered so far. Imperfect competition is introduced assuming a continuum of local markets: each bank can finance a given illiquid project but the

return on that illiquid project depends on the total volume of illiquid investments in a given local market. The number of illiquid projects in a given local market is denoted n . When $n = 1$, each bank is local monopoly whereas a larger number n implies that each bank has a lower monopoly power. The date 2 return to an illiquid project which does not face the liquidity shock at date 1 is assumed to be linear and decreasing in the volume of capital invested in illiquid projects in the local market:

$$R_0 = R - \frac{\Delta}{n} \int_{j \in I_n} (1 - l_j) dj$$

where n is the number of illiquid projects in a given market at date 0, $(1 - l_j)$ is the bank j illiquid investment at date 0, R and Δ are positive scalars ($R > \Delta$), and I_n is the set of n banks in the local market.⁹ Denoting ϕR_1 the marginal return to reinvestment for distressed banks when they deliver high effort e_h , if distressed banks achieve full reinvestment, the optimal ex ante liquidity provision policy l_i of bank i solves the problem

$$\begin{aligned} \max_{l_i} (1 - q) [(1 - l_i) R_0 + ((1 - l_i) \varepsilon + \rho l_i) e_h r] + q e_h [(1 - l_i) (\phi R_1 - r) + r \rho l_i] \\ \text{s.t. } l_i \geq \frac{r - \psi R_1}{\rho r + r - \psi R_1} \end{aligned} \quad (20)$$

We can then derive the following result.

Proposition 8 *When distressed banks achieve full reinvestment, the equilibrium interest rate r_{ic} on the inter-bank market with imperfect competition writes as*

$$e_h r_{ic} = \frac{(1 - q) R + q e_h \phi R_1}{\rho + q - (1 - q) \varepsilon} - \frac{1 + n}{n} \frac{(1 - q) \Delta \rho}{[\rho + q - (1 - q) \varepsilon]^2} \quad (21)$$

Proof. The first order condition for problem (20) writes as

$$(1 - q) \left[- \left[R - \frac{\Delta}{n} \int_{j \in I_n} (1 - l_j) dj \right] + \frac{\Delta}{n} (1 - l_i) + (\rho - \varepsilon) r e_h \right] + q e_h [- (\phi R_1 - r) + r \rho] = 0$$

⁹To simplify the analysis, we assume that the marginal return ρ to an illiquid project which faces a positive liquidity shock at date 1 is constant. If it depended upon other banks capital allocation then the impact of imperfect competition on the likelihood of a credit rationing equilibrium would be amplified.

Summing this equality across all banks in the economy, we obtain

$$\int_{[0;1]} (1 - l_i) di = \frac{n}{1+n} \frac{(1-q)R + qe_h\phi R_1 - [\rho + q - (1-q)\varepsilon]re_h}{(1-q)\Delta}$$

The equilibrium between liquidity supply and demand for liquidity on the inter-bank market is such that

$$\rho \int_{[0;1]} l_i di + (1-q)\varepsilon \int_{[0;1]} (1 - l_i) di = q \int_{[0;1]} (1 - l_i) di \quad (22)$$

As a consequence the equilibrium interest rate r_{ic} on the inter-bank market verifies

$$e_h r_{ic} = \frac{(1-q)R + qe_h\phi R_1}{\rho + q - (1-q)\varepsilon} - \frac{1+n}{n} \frac{(1-q)\Delta\rho}{[\rho + q - (1-q)\varepsilon]^2}$$

■

Expression (21) shows that the equilibrium interest rate r_{ic} on the inter-bank market can be written for $R_1 = R$ as

$$r_{ic} = r_{h,1} - \frac{1+n}{n} \frac{\Delta}{e_h} \frac{(1-q)\rho}{[\rho + q - (1-q)\varepsilon]^2}$$

The equilibrium interest rate under imperfect competition is an increasing function of the number of competitors n . With more intense competition, banks are willing to invest more capital in illiquid projects because competition tends to dampen the negative effect of the individual illiquid investment decision on the market return to illiquid investment. Consequently with more capital invested in illiquid assets, less liquidity is provisioned ex ante and the inter-bank market can be in excess demand for liquidity. A larger interest rate r_{ic} on the inter-bank market is then needed to raise banks incentives to provision liquidity ex ante. The conditions under which the equilibrium with full reinvestment exists state that (i) equilibrium liquidity provision must be sufficiently large and (ii) intact (lending) banks as well as distressed (borrowing) banks are willing to participate to the inter-bank market. We can then derive the following proposition.

Proposition 9 *The full reinvestment equilibrium exists if and only if the number n of competitors on illiquid*

assets is intermediate.

Proof. The full reinvestment equilibrium exists if and only if two conditions are satisfied. First the reinvestment constraint -and not the moral hazard constraint- must be binding. Individual liquidity provision l_i must therefore verify $l_i \geq l(r)$. Since banks are ex ante symmetric, the equilibrium of the inter-bank market (22) implies that individual liquidity provision is $l_i = l_{fb}$. Given that the expression (21) for the equilibrium interest rate r_{ic} , these two conditions simplify as

$$r_{ic} \leq r_{h,2}$$

Given that the equilibrium interest rate r_{ic} is increasing in the number n of competitors, this inequality translates into an upper bound on n which we denote \bar{n} . Second participation constraints for intact (lending) banks and distressed (borrowing) banks must be verified. These conditions write as

$$\frac{\rho}{e_h} \leq r_{ic} \leq \phi R$$

Since the equilibrium interest rate r_{ic} is increasing in the number n of competitors, this condition can be written as $\underline{n} \leq n \leq \bar{n}$. A necessary and sufficient condition for the full reinvestment equilibrium to exist is therefore that the number n of competitors verifies $\underline{n} \leq n \leq \min \{\bar{n}; \bar{\bar{n}}\}$. ■

When competition between banks is either too low or too high, the full reinvestment equilibrium does not exist. Low ex ante competition means that the return on illiquid projects is highly sensitive to a change in a bank individual investment. This prompts banks to invest a relatively small amount of capital in illiquid projects but a relatively large amount of capital in liquid assets. At the interim date, the liquidity supply is therefore relatively large while the liquidity demand is relatively low. As a result, the equilibrium interest rate on the inter-bank market is low and possibly too low to meet intact (lending) banks participation constraint. Conversely, large ex ante competition implies that the return on illiquid projects is relatively insensitive to a change in a bank individual investment. Banks have therefore incentives to make large investments in illiquid projects. This contributes to raising the equilibrium inter-bank market interest rate.

However with a larger interest rate, a larger share of benefits from effort accrues to creditors and this reduces distressed bank incentives to deliver high effort. Lenders bypass this problem with a reduction in distressed banks' borrowing capacity. The moral hazard constraint becomes binding and distressed banks are unable to achieve full reinvestment. Consequently, beyond a certain level of competition, the interest rate on the inter-bank market is so high that distressed banks become unable to achieve full reinvestment.

7 Conclusions

The model we analyzed in this paper provides an useful framework for discussing policy responses to situations of inter-bank market collapses. To the extent that such a collapse may be explained by the ingredients we focus on (in particular moral hazard and liquidity provision non verifiability), this model may prove helpful to determine under what conditions a policy interest rate cut may be more effective than temporary liquidity provision by monetary authorities in restoring normalcy. In addition, this framework presumably lends itself well to the analysis of the role of outside liquidity and its impact on domestic liquidity provision in an open economy setting. These are possible research avenues for future work.

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8 Appendix

8.1 Proof of proposition 5: The full reinvestment equilibrium

When distressed banks achieve full reinvestment, the equilibrium interest rate cannot verify $r_h > r_{h,1}$ since banks would then invest their capital in liquid assets and the inter-bank market would be in excess supply at date 1. The equilibrium interest rate therefore always verifies $r_h \leq r_{h,1}$. When $r_h < r_{h,1}$ then each bank provisions $l = l(r)$ and the equilibrium interest rate is $r = r_{h,2}$ which yields an equilibrium liquidity provision $l = l(r_{h,2}) = l_{fb}$. Finally when $r_h = r_{h,1}$, then the equilibrium volume of liquidity each bank provisions ex ante is $l = l_{fb}$. When bank achieve full reinvestment, they always provision the first best volume of liquidity $l = l_{fb}$ and the equilibrium interest rate on the inter-bank market is $r_h = \min\{r_{h,1}; r_{h,2}\}$. To determine whether this case is an equilibrium, let us examine if there are profitable deviations. A bank can deviate by provisioning a lower level of liquidity. Assuming the interest rate on the inter-bank market verifies $r \leq \phi R$,

then the profit of a deviating bank is:

$$\pi_d = (1 - q) [R + e_h r \varepsilon] (1 - l_i) + e_h r \left(1 - q + q \frac{[\phi - \psi] R}{r - \psi R} \right) \rho l_i$$

Denoting $\frac{\partial E\pi}{\partial l} = e_h r \left((1 - q) (\rho - \varepsilon) + q \frac{(\phi - \psi) R}{r - \psi R} \rho \right) - (1 - q) R$, the optimal liquidity provision policy of the deviating bank l_d^* writes as:

$$l_d^* = \begin{cases} 0 & \text{if } \frac{\partial E\pi}{\partial l} \leq 0 \\ [0; l(r)] & \text{if } \frac{\partial E\pi}{\partial l} = 0 \\ l(r) & \text{if } \frac{\partial E\pi}{\partial l} \geq 0 \end{cases}$$

where r is the equilibrium interest rate when banks achieve full reinvestment; $r = r_h$. If the interest rate r_h is such that $\frac{\partial E\pi}{\partial l} \geq 0$, then the deviating bank provisions $l_d^* = l(r_h)$. In this case deviation is not strictly profitable since we have $\pi_d = \pi_h$. On the contrary if the interest rate on the inter-bank market r_h is such that $\frac{\partial E\pi}{\partial l} \leq 0$, then the deviating bank chooses to make no liquidity provision $l_d^* = 0$. Deviation is then profitable if and only if

$$(1 - q) [R + e_h \varepsilon r_h] > e_h r_{h,1} \rho$$

When $r_h = r_{h,2}$, this inequality simplifies as $e_h \phi \rho < (1 - q)$ for ε close to zero. By assumption this inequality never holds since we have $e_h \phi \left(\frac{\rho}{1 - q} - \varepsilon \right) \geq 1$. When the interest rate is $r = r_{h,1}$ deviation is profitable if and only if

$$1 - q < [1 - q + e_h \phi] \left(\frac{\rho}{1 - q} - \varepsilon \right)$$

However since by assumption we have $e_h \phi \left(\frac{\rho}{1 - q} - \varepsilon \right) \geq 1$, this condition cannot be satisfied. As a consequence there are no profitable deviations and the situation where banks achieve full reinvestment is an equilibrium.

8.2 Proof of Proposition 7: The impossibility of a credit rationing equilibrium with non zero liquidity provision

Apart from the situation where banks make no liquidity provision, there may be two other type of equilibria in the credit rationing regime.

First we examine the case where:

$$\left[(1 - q) (\rho - \varepsilon) + q \frac{(\phi - \psi) R}{r - \psi R} \rho \right] e_h r \geq (1 - q) R \quad (23)$$

and banks optimal liquidity provision is $l_i^* = l(r)$ can indeed be an equilibrium of the economy. When banks provision $l_i^* = l(r)$ the equilibrium inter-bank market interest rate is necessarily $r = r_{h,2}$. Otherwise the inter-bank market would not be balanced. Expected profits then write as $e_h \rho r_{h,1}$. Let us now show that the strategy consisting in provisioning a larger volume of liquidity is more profitable. When $r_{h,2} < r_{h,1}$ then a bank which wants to achieve full reinvestment chooses to provision the same volume of liquid assets $l_d = l_i^* = l(r)$ and expected profits are identical. On the contrary if $r_{h,2} > r_{h,1}$ then a bank which wants to achieve full reinvestment chooses to invest all its capital in liquid assets, $l_d = 1$ and its expected profit is $e_h \rho r_{h,2}$ which by definition is larger than $e_h \rho r_{h,1}$ since $r_{h,2} > r_{h,1}$. As a consequence the situation where banks provision a volume of liquidity $l_i^* = l(r)$ and (23) holds cannot be an equilibrium.

Second we turn to the case case where:

$$\left[(1 - q) (\rho - \varepsilon) + q \frac{(\phi - \psi) R}{r - \psi R} \rho \right] e_h r = (1 - q) R \quad (24)$$

and banks are indifferent to provisioning any volume l_i of liquid asset such that $0 \leq l_i \leq l(r)$. In this case banks expected profits write as $(1 - q) (R + \varepsilon e_h r)$. If the interest rate r which solves (24) is such that $r < r_{h,1}$ then a bank which wants to achieve full reinvestment would choose to provision a volume of liquidity $l(r)$. In this case expected profits are identical. On the contrary if the interest rate r which solves (24) is such that $r > r_{h,1}$ then a bank which wants to achieve full reinvestment would invest all its capital in liquid

assets $l = 1$ and its expected profit would be $e_h r \rho$. This situation is an equilibrium if and only if the interest rate r which solves (24) verifies the condition

$$\left(\frac{\rho}{1-q} - \varepsilon \right) e_h r < R \quad (25)$$

Given that we consider the case where $r > r_{h,1}$, a necessary condition for this situation to be an equilibrium is that (25) must hold for $r = r_{h,1}$. This necessary condition simplifies as

$$e_h \phi \left[\frac{\rho}{1-q} - \varepsilon \right] < 1$$

which by assumption does not hold. Consequently the situation where the inter-bank market interest rate verifies (24) and banks are indifferent to provisioning any amount l_i of liquid asset such that $0 \leq l_i \leq l(r)$ cannot be an equilibrium. The equilibrium with zero liquidity provision and inter-bank market collapse is therefore the only equilibrium, when it exists, in the credit rationing regime.