Constant Proportion Portfolio Insurance Strategies for Credit Portfolios

Meng-Lan Yueh *

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*Meng-Lan Yueh, Department of Finance, National Chengchi University. Email: mlyueh@nccu.edu.tw, Tel: +886 2 29393091#88074

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Abstract

To analyze the capacity of a credit CPPI strategy as a capital protection method, this paper investigates how the features of a CPPI strategy along with the characteristics of credit risk exposures affect the performance of a credit CPPI investment. The empirical results show that the performance of a credit CPPI portfolio is highly dependent on the aggressiveness of the leverage strategy employed. The larger the multiplier assigned to a CPPI strategy, the greater the net asset value of a CPPI portfolio, but also the higher the incidence of lower net asset value when the market performs badly. Moreover, risky investments on the tranche with a larger implied leverage will offer a more variable risk return profile than those on tranches with smaller degree of leverages. The additional leverages that might be introduced into a credit portfolio via investments on CDO tranches is the distinguished feature that differs a credit CPPI portfolio from CPPI portfolios of other asset classes. In general, the credit CPPI strategy performs poorly in a highly volatile market. When the value of risky investments falls so sharply before the portfolio can be readjusted, the CPPI mechanism may even fail to provide the principal protection.

1 Introduction

Constant proportion portfolio insurance (CPPI) is an investment strategy that has widely been employed to create structured credit products with principal-protected features. However, most of these portfolio insurance-related investments have failed to provide the specified percentage of principal redemption during the recent financial market turmoil. To what extent can the implementation of a CPPI mechanism reach the goal of principal protection? In this paper, I investigate how the characteristics of credit risk exposures along with the features of a CPPI strategy affect the performance of a credit CPPI investment.

CPPI is a rule-based portfolio insurance strategy first introduced by Black and Jones (1987). Its rationale is to maintain a highly leveraged exposure in risky assets while assuring principal protection at any point in time. Due to financial innovation in the credit derivatives market, credit CPPI products which offer investors a return linked to the performance of a portfolio of CDSs or CDOs have become more prevalent than ever before. Because synthetic iTraxx CDOs have the major advantage of lower transaction cost, pricing transparency and enhanced liquidity, they are often taken to be the underlying risky investments in a credit CPPI strategy.

The DJ iTraxx Europe index includes 125 equally weighted investment-grade rated European corporate issuers that are actively traded in the credit default swap market. Based on the iTraxx index, standardized CDO tranches are created for investors to gain braoder exposures on the credit derivatives market. In this paper, I will use different iTraxx index tranches: (i) iTraxx Master index; (ii) iTraxx mezzanine tranche ; (iii) iTraxx equity tranche; and (iv) their iTraxx HiVol index counterparts as the eligible risky assets within a credit CPPI structure to explore how a CPPI's features affect the net asset value of a credit CPPI portfolio.

Most studies investigating the performance of a portfolio insurance strategy are conducted under hypothetical market scenarios. For example, Perold and Sharpe (1995) examine how a portfolio consisting of stocks and bills performs in bull, bear and flat markets. Cesari and Cremonini (2003) provide a comparison of nine dynamic strategies for managing a portfolio based on simulation methods. Based on different scenarios, Montenary et al. (2005) study a CPPI strategy in the context of credit portfolios. The very few studies that use empirical data to assess the effectiveness of portfolio insurance like CPPI are still focusing on portfolio composed of stocks and risk-free assets. For example, Do and Faff (2004) use the Australian

All Ordinaries Index as the risky investment to empirically examine the performance of a CPPI strategy. To our knowledge, there is no empirical work examining the CPPI strategy in the context of credit portfolios. In this paper, I present the empirical results regarding the performance a credit CPPI strategy in an attempt to fill the gap.

Within a credit CPPI structure, the risky exposure is managed based on its mark-to-market portfolio value, where returns are driven by spread premium received through selling protection and movements in the market spread. As the portfolio is dynamically rebalanced according to realized index spreads, an analytical formula for a portfolio's terminal value is not readily available. I therefore empirically study the performance of the credit CPPI strategy by examining if a portfolio's net asset value can always stay above the present value of the required floor level under various CPPI features.

The empirical results show that the performance of a credit CPPI portfolio is highly dependent on the aggressiveness of the leverage strategy employed. The larger the multiplier assigned to a CPPI strategy, the greater the net asset value of a CPPI portfolio, but also the higher the incidence of lower net asset value when the market performs badly, especially since the financial market turmoil beginning in June 2007. Moreover, for a given multiplier, a CPPI portfolio with risky investment on an equity tranche will offer a more variable risk return profile than that with risky investment on mezzanine tranche of index portfolio. This is because iTraxx tranches themselves are already leveraged to the underlying reference portfolio, investment on iTraxx tranches will then introduce additional leverages into a credit CPPI portfolio. The equity tranche associated with a higher implicit leverage will therefore induce larger changes in mark-to-market portfolio value than those tranches associated with smaller implicit leverages. Finally the empirical evidence shows that in general, a credit CPPI strategy performs poorly in highly volatile markets. In particular, if the value of the risky investment falls sharply before the portfolio can be readjusted, the CPPI mechanism may even fail to provide the principal protection. The so-called gap risk related to a sharp decline in portfolio values that are too sudden to allow for a rebalancing is hard to be managed for most of principal-protected CPPI portfolios.

The remainder of the paper is organized as follows. Section 2 briefly introduces the iTraxx tranches and provides valuation formulae for them. Section 3 first gives an illustrative example to show how a quarterly-rebalanced CPPI portfolio adjusts its risky exposure on an iTraxx equity tranche over time. It then proposes investment rules for governing the dynamic switch of investment mix between risk-free and risky credit asset of a credit CPPI portfolio. Section 4 presents an empirical analysis of the risk/return profile of credit CPPI portfolios. The final section offers some concluding remarks.

2 Valuation of Credit Index Tranches

A credit CPPI strategy is a dynamic asset allocation strategy as the amount invested in the credit portfolio is readjusted over time, depending on the actual performance of the credit portfolio. Increases or decreases of risky exposures on iTraxx tranches depend on spread movements of the underlying iTraxx indices. Therefore, to study the performance of a credit CPPI portfolio, we need to assess the mark-to-market value of risky exposures driven by changes in credit spread.

2.1 Introduction of iTraxx Tranches

The market for standard index tranches has developed with the introduction of the two most actively traded CDS indices: the North American Investment Grade Index (CDX NA IG) and the iTraxx Europe Index (iTraxx EUR). Through the purchase of a credit tranche, an investor can gain a specific exposure to the credit risk of an underlying portfolio, and in return receive coupon payments. The tranche size is determined by the tranche's attachment point, which defines the loss amount at which losses in the underlying portfolio begin to accrue to the tranche; and the detachment point, which defines the maximum amount of losses in the underlying portfolio that can be absorbed by the tranche. The difference between the tranche's detachment point and attachment point together determines the notional amount of the specific tranche. For example, the standardized tranches of iTraxx indices have a structure defined by the attachment and detachment points of 0%-3%, 3%-6%, 6%-9%, 9%-12%, 12%-22%. The 0%-3% equity tranche would absorb the first 3% of losses in the portfolio due to credit events, the 3%-6% mezzanine tranche would absorb losses from 3% to 6% of the notional in the portfolio, and so on. The standardization of tranched products lies on both the composition of an underlying reference pool and the structure of a tranche.

2.2 Calculation of Tranche Spread

A synthetic CDO tranche j is specified by the attachment $C^{(j)}$ and detachment point $D^{(j)}$. $N^{(j)}$, the size of tranche j, is equal to $\left(D^{(j)} - C^{(j)}\right)$. A tranche investor receives periodic spread premium from the issuer, and makes contingent default payment in case of default events. Therefore the value of a specific tranche, from the perspective of the investor, is the difference between the expected premiums that a tranche investor will receive and the expected losses he will suffer over the life of the contract.

Let $t_1 < \ldots < t_n = T$ be the set of spread payment dates, with T the maturity of the CDO tranche. Denote L_i as the pool's cumulative losses up to time t_i, then $L_i^{(j)}$, the losses absorbed by tranche j up to time t_i, is calculated as

$$L_i^{(j)} = \min\left[N^{(j)}, \max\left(L_i - C^{(j)}, 0\right)\right].$$

I further denote $B(t_i) = e^{-\int_0^{t_i} r_u du}$ as the risk-free discount factor, where r_u is the risk-free interest rate; Δ_i as the tenor between payment dates t_{i-1} and t_i in the unit of years; and $s^{(j)}$ as the annualized fair spread for tranche j. The present value of the premium leg of tranche j is computed as the discount value of all expected spread fees received:

$$PV\left(\text{premium-leg}\,(j)\right) = s^{(j)} \sum_{i=1}^{n} B(t_i)\,\Delta_i \left[N^{(j)} - E\left(L_i^{(j)}\right)\right].$$

If there were no defaults by time t_i, then the premium is paid on the notional of the tranche as $s^{(j)} N^{(j)}$; otherwise, the premium is paid on the remaining amount $N^{(j)} - L_i^{(j)}$ as $s^{(j)}\left(N^{(j)} - L_i^{(j)}\right)$.

The present value of the default leg of tranche j is equal to the sum of discounted expected contingent payments that a tranche investor has to pay if a default event has affected the tranche's principal, that is:[1]

$$PV\left(\text{default-leg}\,(j)\right) = \sum_{i=1}^{n} B(t_i) \times \left[E\left(L_i^{(j)}\right) - E\left(L_{i-1}^{(j)}\right)\right].$$

The fair annualized tranche spread $s^{(j)}$ is determined so that the premium leg and default

[1] Though default payments are made immediately after default, here for simplicity I ignore the accrued payment and assume that the default payment is paid only at spread payment dates t_i.

leg have equal values:

$$s^{(j)} = \frac{\sum_{i=1}^{n} B(t_i) \times \left[E\left(L_i^{(j)}\right) - E\left(L_{i-1}^{(j)}\right) \right]}{\sum_{i=1}^{n} B(t_i) \Delta_i \left[N^{(j)} - E\left(L_i^{(j)}\right) \right]}.$$

The illustration above shows that the CDO valuation problem may be reduced to the computation of tranche j's expected cumulative losses up to time t_i, $E\left(L_i^{(j)}\right)$. Each tranche can accordingly be valued once the cumulative joint loss distribution is known. Thus the key step in pricing a CDO tranche is the derivation of the joint loss distribution of the underlying reference portfolio. Currently, the industry standard for pricing iTraxx index tranches is the one-factor Gaussian copula model proposed by Li (2000), Laurent and Gregory (2005). Implementing the model involves the computation a discretised version of the conditional loss distribution by means of a recursion formula, and integration over the common factor to obtain the unconditional loss distribution. I summarize the implementation of the Gaussian copula model in the Appendix. For a review of the recursive approach to constructing portfolio loss distribution under a Gaussian copula model, see Andersen, Sidenius and Basu (2003), Gregory and Laurent (2004). For a review of recent advances in pricing tranches of a CDO, see Wang, Rachev and Fabozzi (2006).

To dynamically rebalance a CPPI portfolio consisting of iTraxx index tranches, it is required to assess the changes in values of these tranches over time. In order to determine the mark-to-market (MtM) of a given tranche j, I define a measure $\text{DV01}^{(j)}(t)$ as the ratio of expected present value change of tranche j to the notional amount of underlying portfolio due to the change of 1 basis point in the underlying reference portfolio at time t. The MtM of tranche j is calculated as

$$MtM^{(j)}(t_i) = \left(\tilde{s}^{(j)}(t_i) - \tilde{s}^{(j)}(t_{i-1})\right) \times DV01^{(j)}(t_i) \times \frac{N^{(j)}}{N}, \tag{1}$$

where $\tilde{s}^{(j)}(t_i)$ is the time t_i realized spread for tranche j, $N = \sum_j N^{(j)}$, and N is the total notional amount of the reference portfolio.

3 Credit CPPI Strategy

The concept of a CPPI strategy is very general and not restricted to any specific risky assets. The strategy has become increasingly commercially feasible for developing structured products, and been applied to credit portfolios. The credit CPPI portfolio is just a highly leveraged trade on credit portfolios. Assume an investor wants protection of a prespecified percentage of his initial investment at maturity date T. To allow repayments of guaranteed principal, part of the notional is allocated to risk-free assets to ensure that the value of the insured portfolio will not fall below a specified value. The amount to be allocated to the risk-free assets is called floor. In order to generate higher returns, the remaining amount to be invested in risky assets is kept as a multiple of the reserve (also called cushion), which is defined as the difference between the whole portfolio value and the floor level. That is, an investor's notional is levered up by a leverage factor and invested in a set of eligible credit products in a credit CPPI structure. The sale of credit protection on a notional portfolio of credit indices via leveraged exposure constitutes an unfunded risky strategy because the risk is synthetically transferred through credit derivatives like CDOs.

The CPPI strategy is highly path dependent and explicit computations become very involved. In this section, after describing the data, I will first use an equity tranche as the risky investment within a credit CPPI strategy to illustrate how the adjustment of the exposure varies with the spread movements of the equity tranche. Then I will construct a set of investment rules to demonstrate how to dynamically manage a credit CPPI portfolio which claims to provide a principal protection at the end of the investment horizon.

3.1 Data Description

The data used in this study consists of daily closing quotes (the midpoint quotes between quoted ask and bid spreads) for the iTraxx index spreads over the period from June 2004 to June 2008, which covers the entire history of the iTraxx index. For comparison, the iTraxx HiVol index, an equally weighted portfolio of the 30 entities with the highest CDS spreads from the iTraxx index, is also used in this study. All index quotes have been made available by the International Index Company Ltd. (IIC). The iTraxx indices typically trade with a five-year maturity and are issued by series.[2] Table 1 provides some basic statistics on the spreads of both iTraxx and iTraxx HiVol indices.

[2]A new series of iTraxx indices is launched every six months when the composition of reference entities is rebalanced based on a dealer liquidity poll. This occurs twice a year on the roll dates: 20 March or 20 September. A new version index will then be on-the-run for the next six months. Currently, the on-the-run iTraxx index is series 10. In this study, I use all the on-the-run series as the iTraxx indices.

[Insert Table 1 Here]

Figure 1 plots the time series of the credit spreads of both iTraxx and iTraxx HiVol indice over the four-year period.

[Insert Figure 1 Here]

In figure 1, both indices demonstrate a general narrowing of the spread over the first nine months without exceeding 60 bp. For iTraxx indices, there is a sharp widening after May 2005 following the downgrade of General Motors (GM) and Ford. Then the indices spreads tightened by about 22 bp to around 35 bp, remaining quite stable floating at the range of 20 bp to 40 bp until the start of the market turmoil in mid-2007. Turmoil in credit markets deepened in the summer of 2007, setting the stage for the volatile shift of credit spreads afterwards. Spreads of both indices reached to the widest levels since their inception. To the extent that the spread is a compensation for credit risk, this figure also indicates that the market after the summer of 2007 is much riskier than that before the credit crunch.

3.2 Illustrative Example: CPPI on Equity Tranche

Before presenting the investment rules, for a better understanding of the mechanism of a credit-linked CPPI strategy, I will first use a 0%-3% standard iTraxx equity tranche as the risky investment within a CPPI strategy to illustrate how the adjustment of the exposure varies with the spread movements of the equity tranche. Because I don't have the quoted spreads of each tranche, I use the market quoted iTraxx spreads as the input to the model described in previous section to construct the portfolio loss distribution, and according to which to compute the model implied tranche spreads as the realized tranche spreads.[3] Column 3 of Table 2 lists the calculated spreads of the equity tranche.[4]

For convenience, I define the following notations used throughout the paper:

[3] I compare the calculated spreads with market quoted spreads for those days data are available for me. I find the calculated spreads are very closed to the quoted spreads most of the time. Although there still exist differences between them for some days due to constant correlation and recovery rate assumed when implementing the model, the pricing differences will not affect the results presented here.

[4] In the market, the equity tranche is quoted by an upfront payment (as a percentage of contract notional) plus 500 basis points running premium. For simplicity, I report spreads of the equity tranche purely in basis points.

N	initial investment amount
$R(t)$	reserve at time t
$G(t)$	cost of guarantee at time t
$RE(t)$	risky exposure of credit portfolio at time t
$CD(t)$	cash deposit; account value at time t
$MM_{RE(t)}(t)$	mark-to-market value of credit portfolio at time t with exposure $RE(t)$
$DV01^{(j)}(t)$	the percentage change of present value of the invested tranche j with respect to the change of 1 bp. of underlying market credit index
$PI_{RE(t)}(t)$	premium income earned at time t from credit investment with value of $RE(t)$
$II(t)$	interest income from cash deposit account at time t
$NAV(t)$	net asset value of portfolio at time t
$\overline{m}$	target multiplier
$m(t)$	realized multiplier at time t
$\overline{m}_{\max}$	maximum target multiplier allowed
$\overline{m}_{\min}$	minimum target multiplier allowed
r	constant risk-free interest rate
$s(t)$	market quoted credit spread
δ_t	tenor between two rebalancing periods time $t-1$ and t in unit of year
T	investment horizon

I assume a credit CPPI portfolio for an investment of US \$ 50 million ($N = \$50,000 \times 10^3$) over 4 years ($T = 4$). The guarantee equals to 100% of the initial principal. The underlying iTraxx index portfolio is composed of 125 single CDSs with a total swap volume of \$1.25 billion (10 million swap volume on each of the 125 names). Further, I assume risk-free rate $r = 5\%$, target multiplier $\overline{m} = 4$, minimum and maximum multipliers allowed to be $\overline{m}_{\min} = 2$ and $\overline{m}_{\max} = 6$, respectively. The portfolio is rebalanced every three months, so $\delta_t = 0.25$.

Table 2 presents the empirical results to show how a quarterly-rebalanced CPPI portfolio with risky exposure to the iTraxx equity tranche adjusts its allocation according to the performance of the risky exposure.

[Insert Table 2 Here]

At $t = 0$, the principal investment of \$50 million is placed in the cash deposit account, $CD(0) = \$50,000 \times 10^3$. The guarantee of 100% of principal $G(0)$ costs $\$41,135.12 \times 10^3$, (i.e. the present value of $\$50,000 \times 10^3$ received in 4 years' time at a risk-free rate of 5%) so the initial reserve $R(0)$ is $\$8,864.88 \times 10^3$ (i.e. $CD(0) - G(0)$). With a multiplier of 4, this implies a risky investment $RE(0)$ of $\$35,459.51 \times 10^3$ in the equity tranche (i.e. $RE(0) = R(0) \times \overline{m}$), which represents an initial leverage of 0.71 $\left(= \frac{35.459}{50}\right)$.

At $t = 1$, three months later, the cost of guarantee is recalculated as $G(1) = N \times (1+r)^{-(4-(1\times 0.25))}$

$= \$41,639.94 \times 10^3$. The reserve account is updated according to

$$R(1) + G(1) = R(0) + G(0) + II(1) + PI_{RE(0)}(1) + MM_{RE(0)}(1), \tag{2}$$

which means the sum of both reserve and guarantee account balances at $t = 1$ is equal to the sum of: (i) their account values at the previous period $t = 0$, (ii) interest accrued on the cash deposit ($II(1)$), (iii) premium income from the credit portfolio ($PI_{RE(0)}(1)$), and (iv) any mark-to-market profit or loss incurred on the credit portfolio ($MM_{RE(0)}(1)$), over the three months.

At $t = 1$, the interest income earned on the highly liquid cash deposit account is $II(1) = r \times \delta_1 \times CD(0) = \$625 \times 10^3$. The premium income earned from investing in the equity tranche is equal to the product of contract spread premium and risky exposure, which equals to $PI_{RE(0)}(1) = RE(0) \times \widehat{s} \times \delta_1 = \$35,459.51 \times 10^3 \times 1164.83\text{bp} \times \frac{1}{4} = \$1,032.61 \times 10^3$. According to equation (1), the mark-to-market value of risky exposure $MM_{RE(0)}(1)$ is equal to $MM_{RE(0)}(1) = -(s(1) - s(0)) \times \frac{DV01(1)}{N} \times RE(0) = \$1,654.95 \times 10^3 \Big(= -(37.07143 - 44.66667) \times \frac{307.243}{50{,}000} \times \$$ After working out the values of these three terms, according to equation (2), the reserve account $R(1)$ is updated to be $\$11,672.62 \times 10^3$. The realized multiplier $m(1)$ then becomes $\frac{RE(0)}{R(1)} = 3.04$, which lies between the allowable minimum and maximum multipliers, therefore the risky exposure is left unadjusted as $RE(1) = RE(0) = \$35,459.51 \times 10^3$. Next, the cash deposit account is updated to be $\$51,657.61 \times 10^3$ using the following equation:

$$CD(1) = CD(0) + II(1) + PI_{RE(0)}(1) + \Big[MM_{RE(0)}(1) - MM_{RE(1)}(1)\Big],$$

where the term in the bracket is the difference of mark-to-market values of risky exposures between previous and this time periods, and $MM_{RE(1)}(1)$ is $\$1,654.95 \times 10^3$ $(= -(s(1) - s(0)) \times \frac{DV01(1)}{N} \times RE(1))$. Finally, the net asset value, defined as the sum of balances of reserve and bond floor accounts, is derived as $NAV(1) = R(1) + G(1) = \$53,312.56 \times 10^3$.

3.3 Basic Algorithm for Credit CPPI Strategy

To formulate the above procedures more generally, I now present the investment rules for maintaining a credit CPPI portfolio in the following algorithm. Given the relevant account values of the portfolio at time $(t-1)$, at the next rebalancing period t: (i) Calculate the bond floor $G(t)$ as

$$G(t) = N \times (1+r)^{-(T-(t\times\delta_t))}.$$

(ii) Update the reserve account value $R(t)$ as

$$R(t) + G(t) = R(t-1) + G(t-1) + II(t) + PI_{RE(t-1)}(t) + MM_{RE(t-1)}(t),$$

where $II(t)$, the interest earned on the cash deposit account, is

$$II(t) = r \times \delta_t \times CD(t-1);$$

$PI_{RE(t-1)}(t)$, the premium income earned from the sale of credit protection, is

$$PI_{RE(t-1)}(t) = RE(t-1) \times \widehat{s} \times \delta_t;$$

and $MM_{RE(t-1)}(t)$, the mark-to-market value, is

$$MM_{RE(t-1)}(t) = RE(t-1) \times \frac{DV01(t)}{N} \times (s(t) - s(t-1)).$$

(iii) Calculate the new risky exposure $RE(t)$ by multiplying the reserve account with the target multiplier as

$$RE(t) = R(t) \times \overline{m}.$$

(iv) Compute the realized multiplier $m(t)$ as

$$m(t) = \frac{RE(t-1)}{R(t)},$$

and compare it with the target multiplier. If $\overline{m}_{\min} < m(t) < \overline{m}_{\max}$, the risky exposure $RE(t)$ is left unchanged as $RE(t) = RE(t-1)$, otherwise it is readjusted to be in accordance with the target level as $RE(t) = R(t) \times \overline{m}$. (v) Update the cash deposit account $CD(t)$ as

$$CD(t) = CD(t-1) + II(t) + PI_{RE(t-1)}(t) - \left[MM_{RE(t)}(t) - MM_{RE(t-1)}(t)\right]$$

and compute the net asset value $NAV(t)$ of the portfolio as

$$NAV(t) = R(t) + G(t).$$

4 Empirical Results - Monthly Rebalanced

To analyze the capacity of a credit CPPI strategy as a capital protection method, in this section I present the empirical risk return profile of credit CPPI portfolios under different values of the key variables within a CPPI strategy. I use the algorithm proposed in the previous section to study the performance of a monthly-rebalanced CPPI portfolio with risky exposures on CDO tranches. To keep things as simple as possible, I also assume that standard iTraxx index tranches (i.e. 0%-3%, 3%-6%,...etc) are the underlying risky investment of the credit CPPI strategy. All parameter settings for cases studied here are assumed to be the same as those in the illustrative example. To save space, the empirical results are shown in figures and detailed values of relevant variables regarding the portfolios being studied have been omitted and are available from the author upon request.

A CPPI strategy maintains a highly leveraged exposure to maximize the return from the risky investments while guaranteeing a sufficient amount is available to assure principal protection at maturity. To this end, the leveraged risky exposure is designed to expand and contract in sync with the performance of the risky asset. In order to study the relationship between risky exposure and credit spread, figure 2 plots the historical iTraxx index spread and risky exposures to three different iTraxx index tranches for a given multiplier $m = 4$.

[Insert Figure 2 Here]

Figure 2 shows the leveraged exposure is adjusted based on the credit spread in a negatively correlated way. The leveraged risky exposure grows in response to credit spread tightening, a favorable portfolio performance; and reduces in response to credit spread widening, a poor portfolio performance.

For a CPPI method, the multiplier is the key parameter that determines the amount to be invested in the risky asset. It therefore plays a crucial role in balancing risk versus return of a portfolio. To study the effect of a multiplier on the performance of a credit CPPI portfolio, figure 3 plots the net asset value of a CPPI portfolio at different levels of multipliers for the risky investment to be iTraxx equity, mezzanine tranches and iTraxx portfolio respectively.

[Insert Figure 3 Here]

Figure 3 shows that the higher the multiplier, the larger the net asset value of a CPPI portfolio, but also the higher the incidence of lower net asset value in the period of financial market turmoil beginning in June 2007. For a given level of bond floor, the greater the multiplier is, the larger the amount invested in the risky asset is. Therefore, the probability of a larger change in the net asset value also increases. This explains why a CPPI portfolio with greater multiplier has a higher net asset value and also is more likely to hit the floor.

One feature that distinguishes a credit CPPI strategy from a CPPI with risky exposures to other asset classes is the leverages embedded in the most complex forms of credit derivatives. For CDO tranched products, Gibson (2004) shows that the equity and mezzanine tranches can be viewed as leveraged exposures to the underlying CDO's reference portfolio. Although these tranches contain a small fraction of the notional amount of the CDO's reference portfolio, the leverage they possess implies they bear a majority of the credit risk. For example, a mezzanize tranche with the implied leverage of six means that a widening of the underlying iTraxx index spread would lead to a spread widening on a long mezzanize position six times as big as that on a long iTraxx portfolio position. Therefore, apart from multiplier, a credit CPPI portfolio can seek additional leverage via its investment on iTraxx tranches, the already leveraged financial products. In order to understand how various degrees of leverage of iTraxx tranches affect the net asset value of a CPPI portfolio, I plot in figure 4 the terminal value of a CPPI portfolio with risky investment on different tranches for a given multiplier.

[Insert Figure 4 Here]

Figure 4 shows that the risky investment on an equity tranche tends to increase the upside to a CPPI portfolio but also results in more frequent underperformance than its mezzanine and portfolio index counterparts, especially for cases with high target multipliers. This is because the performance of a CPPI strategy is partly driven by changes in mark-to-market value of risky investments. Since changes in the mark-to-market value of a CPPI portfolio with risky investments on an equity tranche are much larger than those on mezzanine or portfolio index for a given credit spread shift, the CPPI strategy based on an equity tranche will surely offer a more variable risk return profile than that based on a mezzanine tranche or portfolio index. Empirical results shown here also indicate that managers of credit CPPI portfolios should be aware of the additional leverages that might be introduced via investment on underlying CDO traches, apart from the conventional leverage imposed by target multipliers within a CPPI structure. Otherwise, the total leverage, inclusive of leverage implicit in CDO tranches, will become extremely large and results in poor performance during the period of market turmoil.

The performance of a CPPI strategy is also driven by the volatility of the underlying index spread. To study the impact of the volatility on the performance of a CPPI portfolio, figure 5 plots the net asset value of a credit portfolio with risky exposures to iTraxx and iTraxx HiVol tranches respectively. The iTraxx HiVol index composes of the 30 names with widest CDS spreads from the iTraxx, and has higher realized standard deviation than its iTraxx counterpart.

[Insert Figure 5 Here]

Figure 5 shows that a CPPI portfolio based on iTraxx HiVol tranches performs better than that based on iTraxx tranches most of the time. However, when markets become highly volatile after summer 2007, higher realised volatility in the underlying credit spread results in weaker performance for a CPPI portfolio. In general, a CPPI strategy performs poorly in highly volatile markets. This is because the CPPI structure requires buying the risky asset as its value rises and selling it when its value falls. A sudden market decline will thus decrease the value of a CPPI portfolio.

A CPPI strategy aims at ensuring that the portfolio value does not fall below a prespecified floor at the end of the investment horizon. To meet the insurance objective, the portfolio value must at always exceed or at least equal to the present value of the guaranteed principal. In order to check if the portfolio value at any given time is greater than the amount which, if invested in risk-free assets at any time, would accrue to the principal amount at maturity, I plot in figure 6 the bond floor and net asset value across time for CPPI portfolios with risky investments on different index tranches.

[Insert Figure 6 Here]

Figure 6 displays that when the underlying credit market performs badly, a CPPI strategy with lower multipliers will limit the downside of a portfolio value. Those with higher multipliers though tend to outperform that with lower multipliers when the underlying credit market performs strongly, gains associated with portfolios with higher leverages can still be lost if the underlying index spread becomes sharply widening. In particular, if the price of the risky asset falls so abruptly before the portfolio can be readjusted, the CPPI mechanism may even fail to provide the principal protection. The risk related to a sharp decline in portfolio values that are too sudden to allow for the rebalancing is called gap risk. Figure 6 also indicates that the gap risk rises with increases of portfolio leverages. Overall, in the case of extreme market events, portfolios with higher leverages are more likely to breach the prespecified floor level than their lower leverages counterparts.[5]

5 Conclusion

In this paper, I first describe how to implement a credit CPPI strategy to manage a principal-guaranteed portfolio, and then examine how various features of a CPPI strategy will affect the net asset value of a credit CPPI portfolio.

Using the standardized iTraxx tranches as the underlying risky investment in a CPPI structure, I find that the performance of a credit CPPI portfolio is highly dependent on the aggressiveness of the leverage strategy employed. The higher the multiplier assigned to a CPPI strategy, the larger the net asset value of a CPPI portfolio, but also the higher the incidence of lower net asset value when the market performs badly. Moreover, since iTraxx tranches themselves can be viewed as leveraged exposures to the underlying reference portfolio, a credit CPPI portfolio can therefore seek additional leverage via its investment on iTraxx tranches. That explains why for a given multiplier, a portfolio with risky investment on an equity tranche will offer a more variable risk return profile than that with risky investment on mezzanine tranche or index portfolio. The result suggests that managers of credit CPPI portfolios should be aware of the additional leverages that might be introduced via investment on underlying CDO tranches which involve highly complex techniques for valuation and hedging. The empirical evidence also shows that in general, a CPPI strategy performs poorly in highly volatile

[5]For all cases studied here, I have repeated the above analyses for a quarterly-rebalanced CPPI portfolio and obtained similar results. Due to space constraints, I do not report the results here.

markets. In particular, if the value of the risky investment falls so abruptly before the portfolio can be readjusted, the CPPI mechanism may even fail to provide the principal protection. The so-called gap risk related to a sharp decline in portfolio values that are too sudden to allow for a rebalancing becomes larger when the degree of leverages increases. To manage the gap risk, instead of keeping a constant multiplier all the time, whether a dynamic multiplier could provoke a quicker adjustment to risky exposure in response to a sudden sharp decline in the portfolio value would be an interesting issue for further investigation.

6 Appendix

The one-factor Gaussian copula model of default time has become the standard market model for valuing a synthetic CDO tranche. It assumes that the copula correlation is the same for all pairs of reference entities. For a portfolio of N reference names, I use the following parameters to describe each name in the reference portfolio: A_i is the notional amount for underlying name i, $i = 1, \ldots, N$; R_i is recovery rate for name i when default happens; τ_i is the default time of name i; $F_i(t)$ =prob$(\tau_i \leq t)$ is the risk-neutral probability that underlying name i defaults before or at time t. The risk-neutral default probabilities can be estimated using market quoted single-name credit default swap spreads with assumed constant recovery rates. For simplicity, here I assume that the loss amount of default is the same for each underlying name i, that is $A_i = A$, and $R_i = R$. Under this assumption, the portfolio loss distribution would be obtained by multiplying the number-of-default distribution with the default loss amount $A(1 - R)$. I will briefly introduce the Gaussian copula model and describe the method for implementing the model to get the number-of-default distribution in the following. The method below can also be modified to obtain the distribution for cases of $A_i \neq A_j$, or $R_i \neq R_j$. For details, see Andersen et al. (2003) , Hull (2004), Gibson (2004).

A reference name's credit quality is specified by a linear combination of the systematic common factor Z_0 and its idiosyncratic risk Z_i with systematic correlation ρ_i as

$$X_i = \rho_i Z_0 + \sqrt{1 - \rho_i^2} Z_i \,, \tag{3}$$

where X_i, Z_0, and Z_i all follow a standard normal distribution. The distribution function of X_i and Z_i is $U_i(\cdot)$ and $\Phi_i(\cdot)$ respectively. Default occurs when X_i falls below a threshold $\overline{X_i}$. It follows from equation (3) that the default threshold $\overline{X_i}$ is equal to $U_i^{-1}\left(F_i\left(t\right)\right)$. Denote $F_i\left(t\left|Z_0\right.\right)$ as the default probability conditional on the level of common factor Z_0, then

$$F_i\left(t\left|Z_0\right.\right) = \text{prob}\left(X_i < \overline{X_i}\left|Z_0\right.\right) = \Phi_i\left(\frac{\overline{X_i} - \rho_i Z_0}{\sqrt{1-\rho_i^2}}\right).$$

In a reference portfolio of K names, denote $P_t^{(K)}\left(l\left|Z_0\right.\right)$ as the conditional probability of exactly l default happened before time t, conditional on the common factor Z_0. If the number-of-default distribution $P_t^{(K)}\left(l\left|Z_0\right.\right)$, $l = 0, \ldots, K$, for a set of K names are known, then the conditional number-of-default distribution for a reference portfolio with $K+1$ names would be one of the following three:

$$P_t^{(K+1)}\left(0\left|Z_0\right.\right) = P^{(K)}\left(0\left|Z_0\right.\right)\left(1 - F_{K+1}\left(t\left|Z_0\right.\right)\right)$$

$$P_t^{(K+1)}\left(l\left|Z_0\right.\right) = P_t^{(K)}\left(l\left|Z_0\right.\right)\left(1 - F_{K+1}\left(t\left|Z_0\right.\right)\right) + P_t^{(K)}\left(l-1\left|Z_0\right.\right)F_{K+1}\left(t\left|Z_0\right.\right)$$

$$P_t^{(K+1)}\left(K+1\left|Z_0\right.\right) = P_t^{(K)}\left(K\left|Z_0\right.\right)F_{K+1}\left(t\left|Z_0\right.\right)$$

where $F_{K+1}\left(t\left|Z_0\right.\right)$ is the conditional default probability of the $(K+1)\,th$ name that we add into the portfolio of size K.

Based on the above rules, starting with $K = 0$, $P_t^{(0)}\left(0\left|Z_0\right.\right) = 1$, the conditional number-of-default distribution for the reference portfolio with N names, $P_t^{(N)}\left(l\left|Z_0\right.\right)$, $l = 0, \ldots, N$, can be solved recursively. Finally, the unconditional number-of-default distribution $P_t\left(l\right)$ can be obtained by integrating out the common factor Z_0 as

$$P_t\left(l\right) = \int_{-\infty}^{\infty} P_t^{(N)}\left(l\left|Z_0\right.\right)\phi\left(Z_0\right)dZ_0 \;,$$

where $\phi\left(Z_0\right)$ is the probability density of Z_0. The integral is calculated with numerical integration methods.

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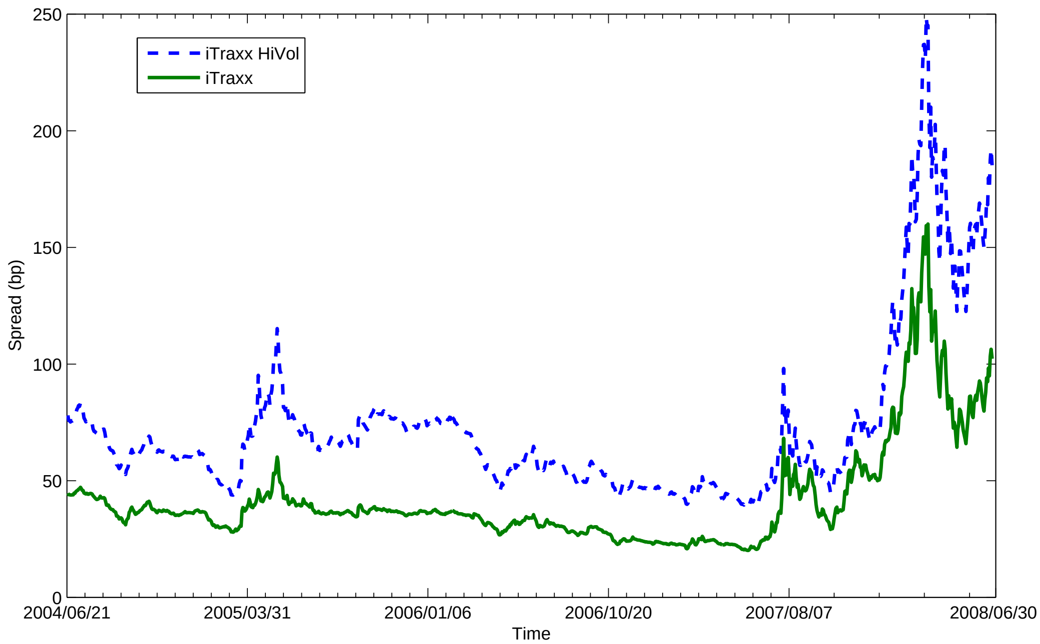

Figure 1: Credit Spreads of iTraxx and iTraxx HiVol Indices

Table 1: Descriptive Statistics for the 5-Year iTraxx and iTraxx HiVol Credit Spreads

	iTraxx	iTraxx HiVol
Mean	41.91	72.76
S.D.	22.49	35.50
Min	20.09	38.69
Max	160.00	248.75
Skew	2.47	2.35
Kurt	9.75	8.56

Units: basis point (bp).

Table 2: Quarterly-Readjusted CPPI Portfolio with Risky Exposure on iTraxx Equity Tranche

Date	iTraxx quotes (bp)	tranche spread (bp)	DV01 ($\times 10^3$)	cost of guarantee ($\times 10^3$)	premium income ($\times 10^3$)	interest income ($\times 10^3$)	MtM of portfolio ($\times 10^3$)	reserve ($\times 10^3$)	multiplier	risky exposure ($\times 10^3$)	cash deposit ($\times 10^3$)	net asset value ($\times 10^3$)	realized leverage
t	s(t)	$\widehat{s}^{(j)}(t)$	DV01(t)	G(t)	PI(t)	II(t)		R(t)	m(t)	RE(t)	CD(t)	NAV(t)	
0 : 30/06/04	44.67	1164.83	281.35	41135.12	–	–	–	8864.88	4.00	35459.51	50000.00	50000.00	0.71
1 : 30/09/04	37.07	990.94	307.24	41639.94	1032.61	625.00	1654.95	11672.62	3.04	35459.51	51657.61	53312.56	0.71
2 : 30/12/04	36.20	970.61	307.22	42150.96	1032.61	645.72	189.87	13029.79	2.72	52119.17	53246.73	55180.75	1.04
3 : 31/03/05	38.99	1035.99	299.67	42668.25	1517.75	665.58	−870.29	13825.55	3.77	52119.17	55430.06	56493.79	1.04
4 : 30/06/05	40.25	1065.35	266.81	43191.88	1517.75	692.88	−351.51	15161.03	3.44	52119.17	57640.68	58352.91	1.04
5 : 30/09/05	36.50	977.51	306.12	43721.94	1517.75	720.51	1196.60	18065.83	2.88	72263.31	59416.45	61787.77	1.45
6 : 30/12/05	36.86	986.17	306.31	44258.51	2104.36	742.71	−159.87	20216.46	3.57	72263.31	62263.51	64474.97	1.45
7 : 31/03/06	31.88	867.05	317.62	44801.66	2104.36	778.29	2288.30	24844.26	2.91	99377.05	64287.58	69645.92	1.99
8 : 30/06/06	31.06	846.80	328.71	45351.47	2893.93	803.59	536.18	28528.15	3.48	99377.05	67985.10	73879.62	1.99
9 : 29/09/06	28.21	777.18	332.68	45908.04	2893.93	849.81	1878.69	33594.02	2.96	134376.07	71067.20	79502.05	2.69
10 : 29/12/06	23.40	656.17	356.49	46471.43	3913.13	888.34	4612.51	42444.60	3.17	134376.07	75868.67	88916.03	2.69
11 : 30/03/07	23.33	654.50	352.13	47041.74	3913.13	948.36	62.11	46797.88	2.87	187191.51	80705.74	93839.62	3.74
12 : 30/06/07	24.59	686.32	349.69	47619.05	5451.15	1008.82	−1647.39	51033.16	3.67	187191.51	81714.56	98652.20	3.74
13 : 30/09/07	37.85	1009.53	300.67	48203.44	5451.15	1021.43	−14922.98	41998.36	4.46	187191.51	88187.14	90201.80	3.74
14 : 30/12/07	50.25	1291.97	271.94	48795.00	5451.15	1102.34	−12629.41	35330.87	5.30	141323.49	91646.01	84125.88	2.83
15 : 30/03/08	122.70	2825.66	182.43	49393.83	4115.44	1145.58	−37355.90	2637.16	53.59	10548.66	62339.45	52030.99	0.21
16 : 28/06/08	102.25	2409.52	191.02	50000.00	307.18	779.24	824.31	3941.73	2.68	15766.91	63018.10	53941.73	0.32

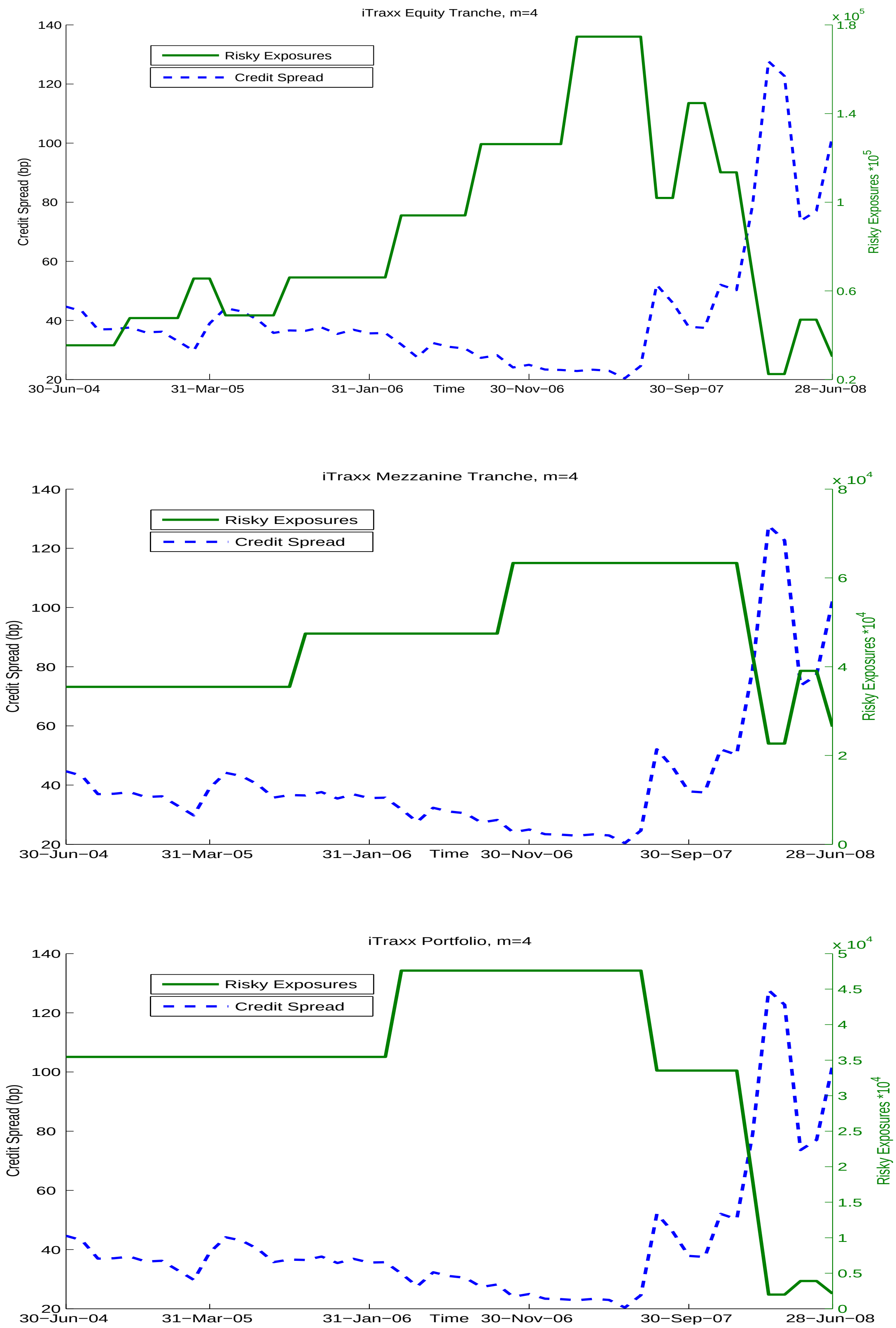

Figure 2: Relation Between Historical Spreads and Leveraged Risky Exposure

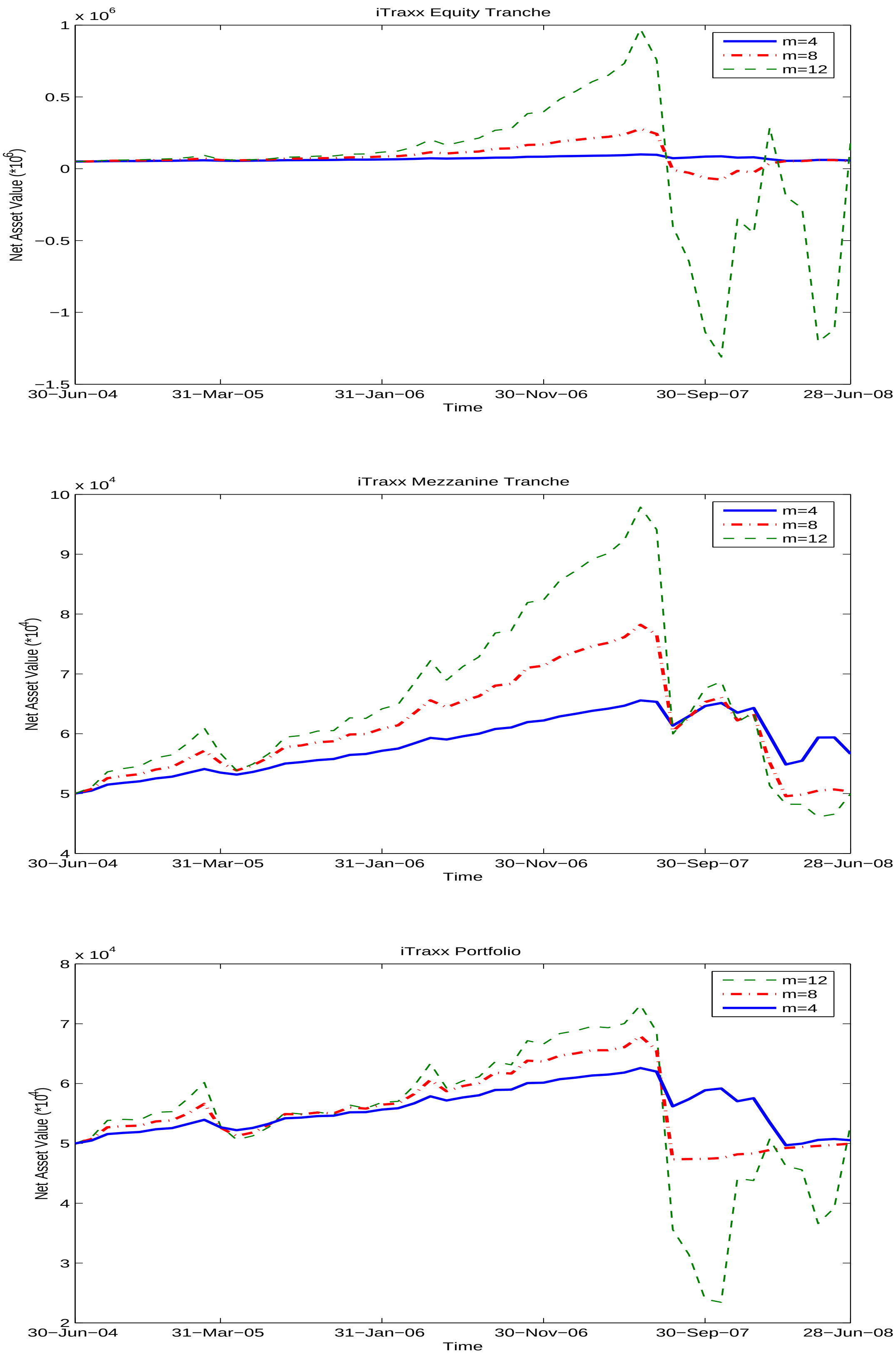

Figure 3: Effects of Multiplier on Performances of CPPI Portfolios

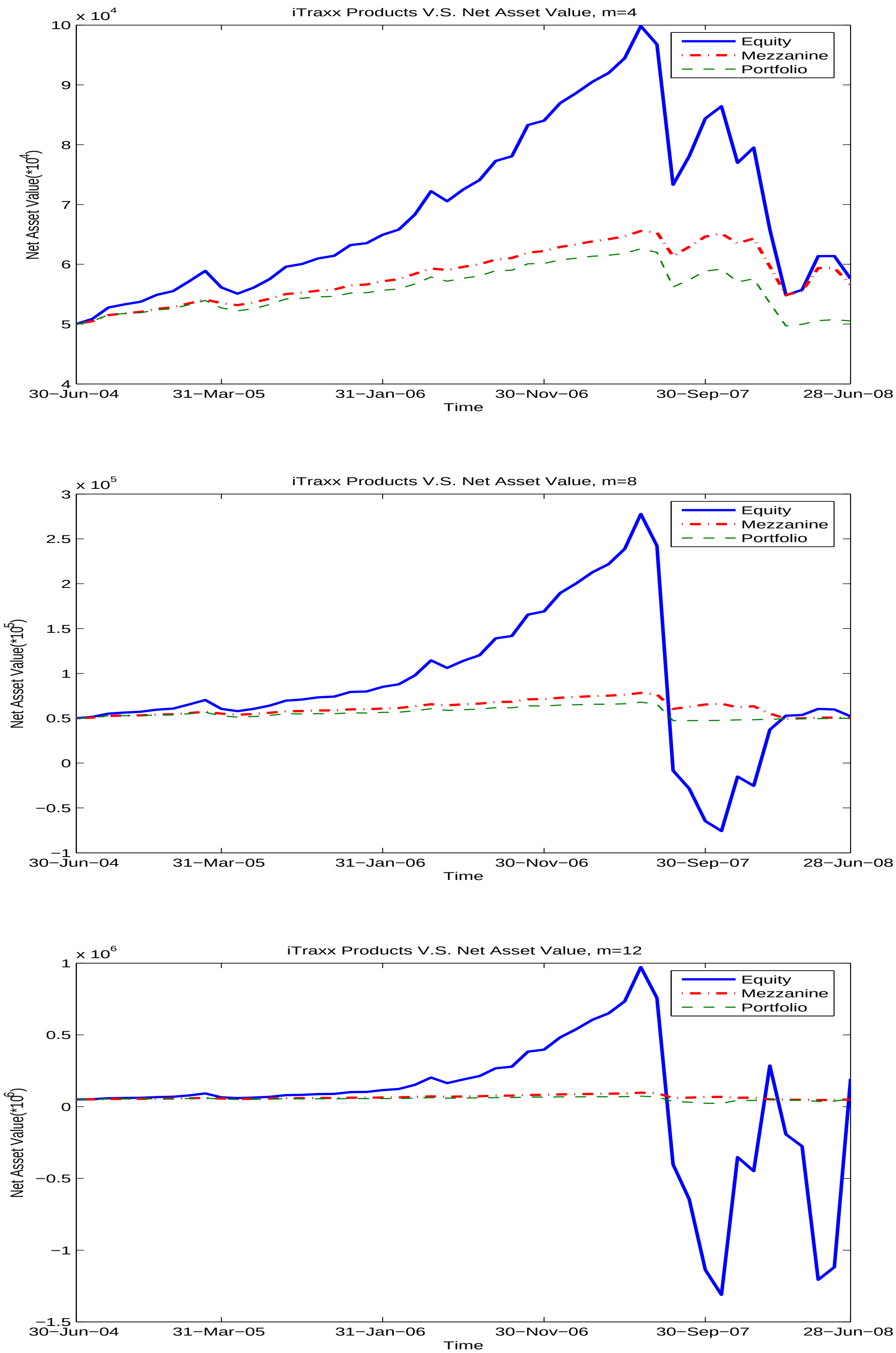

Figure 4: Effects of Tranche Products on Leveraged Risky Exposure

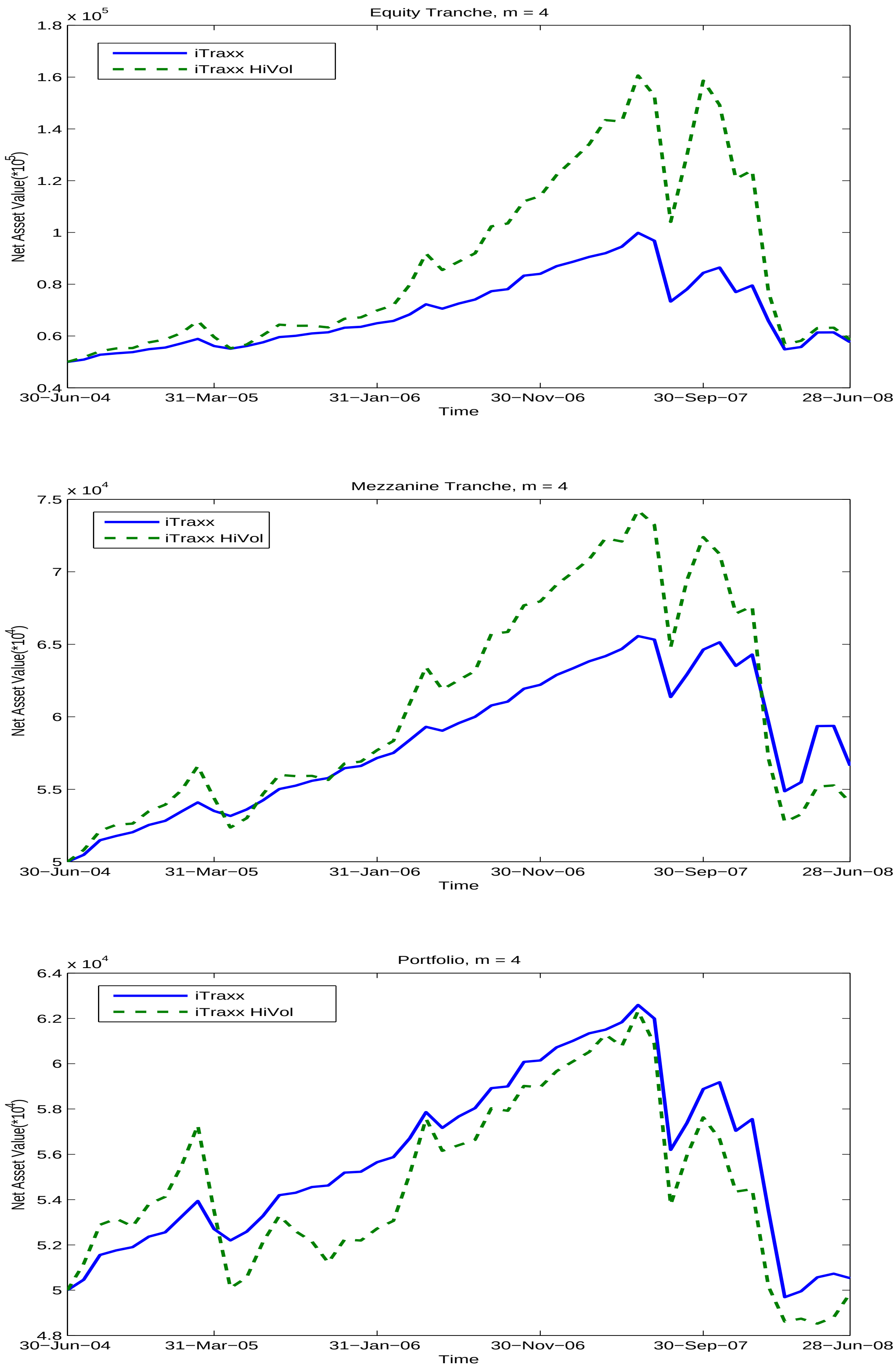

Figure 5: Performance Comparisons Between Risky Investments on iTraxx and iTraxx HiVol Tranches

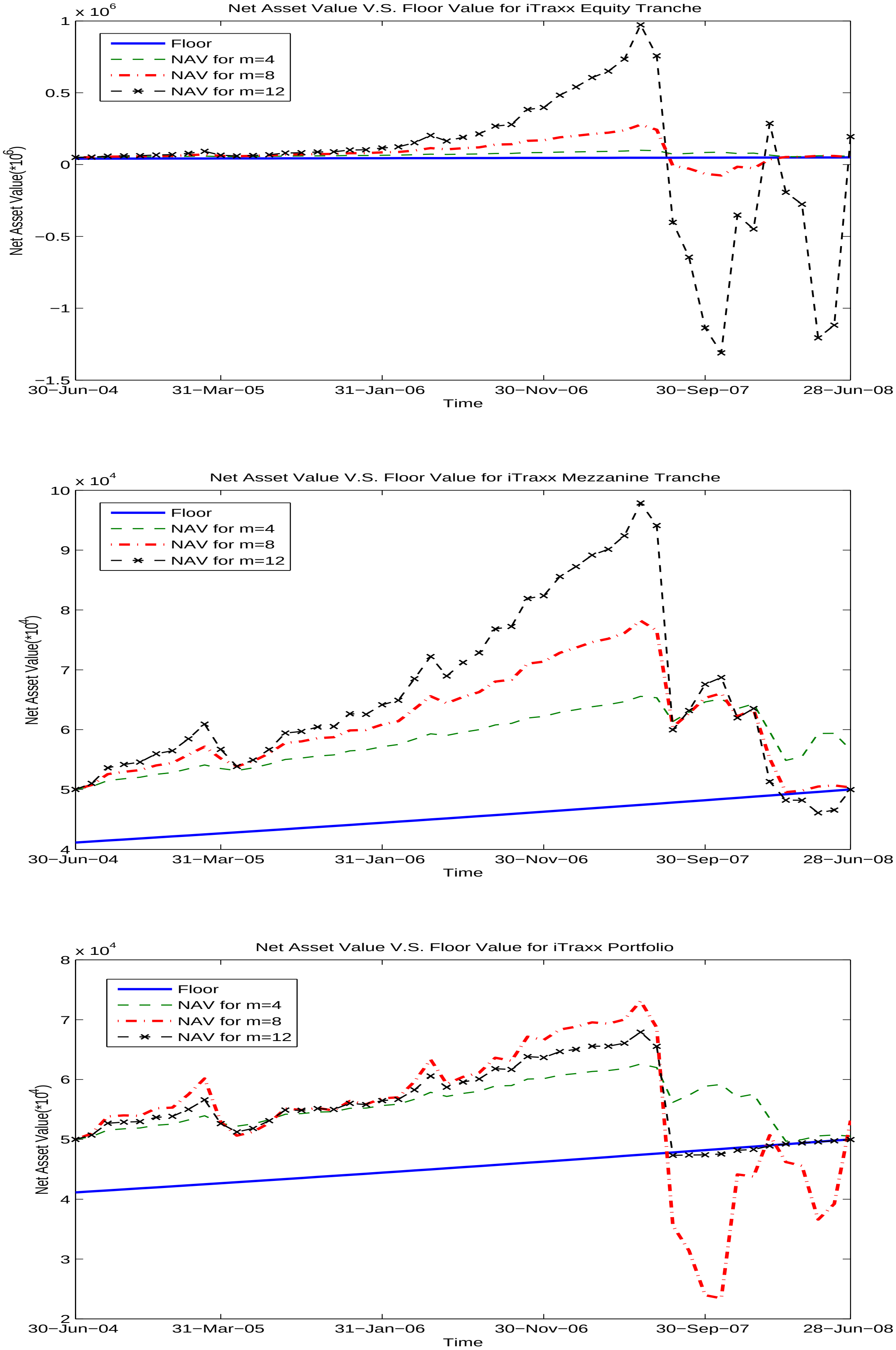

Figure 6: Efficiency of Capital Protection of Different CPPI Strategies