

Parameter Stability and the Valuation of Mortgages & Mortgage-Backed Securities

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Abstract

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G12, G21

Key words: mortgage, model risk, prepayment

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Abstract

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1. Introduction

The recent financial crisis was triggered by large and unexpected losses on mortgages and mortgage-backed securities, suggesting widespread failure of risk management tools used in the valuation of those complex assets. Yet traditional economic analysis of portfolio choice and equilibrium asset pricing typically relies on the assumption that investors are perfectly informed as the value of parameters that affect asset returns. But in the real world investors must estimate parameters from noisy empirical data, producing imperfect estimates at any particular point in time. To make matters worse, technological change may alter the fundamental underlying relationships so that parameter values are non-stationary over time. The effect of this parameter instability on asset pricing has been a topic of wide interest in finance; however, most existing research focuses on equity markets. In this paper, we extend consideration to the mortgage market, today the largest component of the overall bond market, and the precipitating cause of the financial crisis that began to affect markets in 2007. We evaluate the effect of estimation risk, sometimes called model risk, on pricing mortgages and mortgage-backed securities. Model risk is particularly acute in the valuation of mortgages and mortgage-related assets since cash flows are to a considerable extent interest rate dependent due to the call option embedded in the typical contract¹.

Even before recent mortgage market turmoil, many commentators noted the size and growing importance of the mortgage market in the overall capital market. Indeed, the mortgage market is now larger than either the government bond market or the corporate bond market and is approaching the size of those two other segments

combined². Given the embedded call option in mortgage contracts, correctly modeling borrower refinancing behavior is a central problem in mortgage valuation. Econometric models of mortgage prepayment typically use data from prior periods to estimate the probability of refinancing conditional on interest rate levels, loan seasoning, and other factors. These models are then used to predict future prepayment rates under simulated interest rate conditions. But if market conditions are evolving over time such that model parameters are unstable, predictions may diverge widely from actual results. In the current paper, we examine this problem and price mortgages using option-adjusted spread (OAS) techniques incorporating prepayment models based on disaggregated loan level event histories. Results suggest an economically significant difference in asset values associated with parameter instability over time. We obtain comparable results across well-known term structure models. This model risk may help explain the large losses often incurred by mortgage market participants during bond market rallies.

2. Literature Review

Early theoretical research focusing on portfolio choice under parameter uncertainty includes Barry [1974], Klein and Bawa [1976], Klein and Bawa [1977], Brown [1979] and Bawa and Brown [1979], Barry and Brown [1985], Coles and Lowenstein [1988], and Coles, Lowenstein, and Suay [1995]. These papers focus on

¹ In addition, where credit risk is insured, such as in agency MBS, any defaults in the underlying pool translate into prepayments to the investor.

² According to the Securities Industry and Financial Markets Association (www.sifma.org), as of Q2 2008, the total mortgage-backed securities market was \$7.6 trillion. After adding \$2.5 trillion in asset-backed securities, which includes non-shelter credit card receivables, as well as home equity, and manufactured housing assets, the total market comes to \$10.1 trillion, about 32% of total market size. As of the same date, the Treasury bond market (excluding agency issues) stands at \$4.7 trillion and the corporate bond market total at \$6.2 trillion.

equity markets, for example, construction of portfolios when mean return vectors are known but covariance matrices are not.

The literature is much smaller when we turn to the bond market. It has become widely recognized that term structure volatility is not constant over time and, accordingly, the pricing of fixed income instruments optimally incorporates stochastic volatility. Bliss and Smith [1997] argue that choice of model and parameter stability is closely connected. They re-examine the often cited Chan, Karolyi, Longstaff, and Sanders [1992] analysis of term structure volatility and the ability of various single factor term structure diffusion processes to model rate movements over the period 1964-1989. Bliss and Smith argue that a structural break occurred during the early 1980s due to Fed policy, i.e. that parameters were not stable over the study period. A related analysis focusing on the effect of alternative interest rate processes on the value of mortgages, in particular, appears in Archer and Ling [1995]. These papers, in effect, focus on model choice and parameter stability given model choice, with respect to the term structure model. But to price mortgages, and by extension mortgage-backed securities, one needs both a term structure model and a prepayment model. Our contribution here is to focus on the prepayment component of the pricing equation.

We have not been able to find published research that specifically quantifies this aspect of model risk in mortgage pricing, although Bernardo and Cornell [1997] provide some evidence of the magnitude of the difference in mortgage and MBS valuation by major investment firms. They report dispersion in bids for 32 auctioned collateralized mortgage obligations (CMOs) of 63%, with greater dispersion for those issues that extend further into the future, noting that “CMO tranches are highly sensitive to prepayments.”

Bernardo and Cornell conclude that “sophisticated investors do not agree on some combination of the specification of the model, the underlying interest rate process, or the prepayment function” (page 798). In this paper we are able to focus on the prepayment function issue noted by Bernardo and Cornell and measure its impact controlling for choice of term structure model.

Turning now to the mortgage research literature, we note initially that the volume of work on termination risk in general, and prepayments in particular, is vast, so our survey here is necessarily abbreviated. A number of themes are, nevertheless, evident. Among the earliest to examine the topic were Dunn and McConnell ([1981a], [1981b]), Green and Shoven (1986) and Quigley (1988), all of whom identified the role of interest rates as well as borrower mobility on rates of mortgage prepayment, and Schwartz and Torous (1989) who related prepayment rates to the valuation of mortgage-backed securities. Academic interest in the topic accelerated during the early 1990s with theoretical papers by Brueckner (1992), Follain, Scott, and Yang (1992), Kau et al (1992a, 1992b), Brueckner (1994), and Stanton (1995). These papers dealt with optimal exercise of the borrower’s call option given stochastic interest rates and the implications for mortgage contract design and pricing, both of mortgages and mortgage-backed securities. Concurrently, Wall Street firms were developing proprietary prepayment models for use in valuation routines that supported their trading strategies (Richard and Roll [1989], Patruno [1994], and Hayre and Rajan [1995], Cheyette [1996], Hayre, Chaudhary, and Young [2000]).

The refinancing wave of 1993 followed by the sharp increase in rates during 1994 produced large losses for many market participants, reinforcing the business imperative

to develop better models³. As Kalotay, Yang, and Fabozzi [2004] note: “Econometric prepayment models have consistently failed during fast prepayment periods.”⁴ During the latter half of the 1990s, many researchers turned their attention to institutional constraints that might reduce prepayments, even when call options appeared to be deep in the money (Peristiani et al. [1996], Archer et al. [1996, 1997], Caplin et al. [1997], Green and LaCour-Little [1999]). These studies identified declines in collateral value, credit status, and other macroeconomic forces, such as unemployment, as significant factors inhibiting prepayment. Research in the late 1990s and into the new century re-focused on the role of borrower mobility (Clapp, Harding, and LaCour-Little [2000]), Pavlov [2001]), a more complex econometric treatment of mortgage termination risk using a competing risk framework (Deng, Quigley, and Van Order [2000], Ambrose and LaCour-Little [2001]), and the use of non-parametric methods (Maxam and LaCour-Little [2001], LaCour-Little, Marschoun, and Maxam [2002]). At this same time, researchers began to examine other factors, such as the signaling effect of borrower points (Stanton and Wallace [1998]) and the effect of house price movements on prepayment rates (Mattey and Wallace [2001])

Relatively less has been written linking institutional and technological change in the mortgage industry with borrower behavior. Arora, Heike, and Mattu (2000) review historical returns in the mortgage-backed securities sector during the 1990s. They find a “...steady improvement in refinancing efficiency over the past decade, which has caused

³ A more recent example of major losses involving the valuation of mortgages is Homeside Lending, Inc. This top ten U.S. mortgage lender was acquired by National Bank of Australia in 1997 for approximately \$1.0 billion; the parent announced in September 2001 a write-off of \$1.2 billion to cover unanticipated losses in the valuation of mortgages and mortgage servicing rights.

⁴ Kalotay, Yang, and Fabozzi [2004], page 6.

prepayment models to consistently understate mortgage callability.”⁵ LaCour-Little (2001) reviews the effect of technology on the mortgage industry over the last decade, noting clear evidence of declining mortgage servicing costs but limited evidence of declining mortgage origination costs. Peristiani, Bennett, and Peach (1998) argue that the mortgage market has evolved considerably since the 1980s as a result of technological, institutional, and competitive market forces causing greater rates of prepayment during the 1990s than would otherwise have been expected. In their empirical analysis, Peristiani et al. use loan level data, including credit scores, to compare a cohort of loans originated during 1984-1990 to a cohort originated during 1991-1994, using hazard model techniques. They report an increased propensity to refinance among the latter group.

Recently a large volume of literature has also emerged to address the subprime mortgage market collapse, the effect on the housing market, and the resulting contagion to other financial market sectors (see, for example, Cutts and Sanders [2008], Demyanyk and Van Hemert [2008]; Foote et al [2008]; Gorton [2008]; or Whalen [2008]). Focusing more broadly on risk management processes, Stulz [2008] proposes six distinct types of risk management failure, the first of which is mismeasurement of known risks. In the context of mortgage asset valuation, prepayment is, of course, a known risk. We would argue that assumption of parameter stability over time constitutes an example of mismeasurement of this type.

Our contribution here is to re-focus attention on the temporal stability of the parameters of the prepayment function, using carefully constructed micro-level data from

⁵ Arora, Heike, and Mattu (page 6).

two most recent refinancing waves, and the pricing impact of this instability.

Specifically, we estimate a semi-parametric Cox hazard models for each of the two cohorts and find the response to the refinancing incentive to be greater in the 1997 cohort, compared to the 1992 cohort, when measured over comparable duration. We find an additional interesting effect in the decline in the constraining effect of contemporaneous LTV, apparent in the 1992 cohort but significantly diminished in the 1997 cohort.

3. A Theory of Mortgage Prepayment

Following Archer, Ling, and McGill [1996], one may represent the total probability of mortgage termination, λ_{Tt} , for all reasons, as:

$$\lambda_{Tt} = \lambda_{Dt} + (1 - \lambda_{Dt}) [\lambda_{Mt} + (1 - \lambda_{Mt}) \lambda_{P.NM_t}] \quad (1)$$

where λ_{Dt} is the probability of termination due to default at time t , λ_{Mt} is the probability of terminating due to moving at time t , and $\lambda_{P.NM_t}$ is the probability of terminating due refinancing in place (prepaying and not moving) at time t . Since we employ a very short time horizon in our data (essentially the first 36 months of loan life), we make the simplifying assumption that $\lambda_{Dt} = \lambda_{Mt} = 0$. Clapp, Harding, and LaCour-Little (2000), using data in which movers could be separated from refinancers, found that the cumulative probability of mobility for FRM borrowers was approximately .045 over a three and one-half year period during the mid-1990s, so our assumption regarding mobility seems reasonable.

The potential savings that can be realized over the remaining life of the loan drive refinancing decisions. Consider a mortgage with book value $BV(r_c, t)$, where r_c is the contract rate of interest. The value of $BV(r_c, t)$ declines over time according to its amortization schedule, with a loan term of n . At any point in time, t , household wealth is given by

$$W_t = FA_t + (H_t - MV_t) \quad (2)$$

where FA_t are other net financial assets at time t , H_t is house value at time t , and MV_t is the market value of the mortgage at time t . MV_t is a function of contractual loan payments, remaining loan term, $n-t$, and the market rate of interest, r_m .

Household wealth maximization implies a pure refinancing strategy, namely, minimizing the value of MV_t . This is accomplished by following the rule:

$$\text{Prepay when } (MV(r_m, t) / BV(r_c, t)) > 1 \quad (3)$$

In other words, the borrower should prepay the loan as soon as the option goes into-the-money, that is, as soon as the market rate of interest declines below the contract rate on $BV(r_c, t)$. In the presence of transaction costs, TC , however, the rule becomes:⁶

$$\text{Prepay when } (MV(r_m, t) / (BV(r_c, t) + TC)) > 1 \quad (4)$$

⁶ Transactions costs will also sometimes lead borrowers to postpone prepayment even when the option is in the money, because of the hysteresis effect described in Dixit and Pindyck (1998).

If transaction costs were declining over time, due to institutional and technological change, we would expect to see rates of prepayment increasing over time for a given level of refinancing incentive, i.e. parameters of the prepayment function would be not be stable over time. In the econometric specification that follows, we use MV/BV as the measure of refinancing incentive.

Given the computational burden of evaluating partial differential equations and issues of path dependency, pricing of mortgages and mortgage-backed securities given a term structure diffusion and prepayment and default models is typically done via Monte Carlo methods. Credit risk of minimal concern at the security level at least for conforming loans that are packaged into agency securities, since those bear the guaranty of Fannie Mae, Freddie Mac, or Ginnie Mae. Accordingly, default is often ignored and pricing algorithms focus on cash flows subject to prepayment (call) risk. Multiple interest rate paths are generated using the term structure model selected and along each path the prepayment model generates cash flows as a function of simulated interest levels and other factors.⁷ For a given price, the option adjusted spread (OAS) is calculated as the spread over simulated treasury rates that equates price to the average present value of expected cash flows over all simulated paths. Alternatively, for a given OAS, we may solve for price by averaging the present value of expected cash flows discounted at treasury plus OAS over all simulated paths. In the analysis that follows, we use the commercially available WINOAS software to generate simulated interest rate paths and an empirically derived prepayment model to generate cash flows given simulated interest rates. We then discount the cash flows generated at simulated treasury spot rates plus a

typical OAS value to compute pseudo asset prices and examine how those vary given the prepayment model data used.

4. Data

Data is drawn from the servicing records of a major mortgage market participant that prefers anonymity. Samples were drawn from two origination years: 1992 and 1997, henceforth referred to as the 92 cohort and the 97 cohort. The mortgage market changed in a fairly significant way in between these two years, since credit scoring and then automated underwriting was widely adopted during 1995. For a detailed discussion of the evolution of the mortgage market over the decade of the 1990s, including the role of technology, see LaCour-Little [2000], which documents industry consolidation, driven by scale economies, especially in loan servicing. As a result, real origination costs per loan have been held constant, producing declining average costs per dollar of loan volume, as loan size has increased with housing prices over time. This greater efficiency in the mortgage industry would be expected to pass through to borrowers in the form of reduced transaction costs, leading to greater refinancing sensitivity.

Figure 1 shows the path of mortgage and treasury rates interest rates between 1992-2003. As can be seen from this graph in both cohort years, mortgage interest rates were predominately in the 7.50-8.50% range, followed by market rallies in 1993 and 1998 that produced rates about 100 basis points lower, in the 6.50-7.50% range.

[Insert Figure 1 about here]

⁷ Typical factors included in Wall Street and proprietary prepayment models include loan age, loan balance, contract type, coupon, some measure of path-dependent coupon burnout and, possibly, loan-to-value ratio, geography, origination channel, and credit score.

We select 30 year fixed rate mortgage loans from both cohorts with note rates ranging from 8.00-9.00%, so as to have loans that clearly would experience incentives to refinance within 24 months of loan origination. Since, with this data set, we cannot distinguish mobility from refinancing behavior, we are effectively assuming that mobility within 24 months of home purchase is negligible. All of the loans used are 30-year conventional fixed rate mortgages that were subsequently sold to Fannie Mae. All were originated for home purchase, as opposed to refinancing an existing loan. This is an important distinction, since with refinancing loans, mortgage age is not equal to time at current residence⁸. Many prepayment analyses pool these distinct loan types together applying a single seasoning factor, a questionable procedure given the measurement error in the age variable. Table 1 presents descriptive statistics for the two cohorts.

5. Empirical Methodology for Prepayment Model

Hazard models have been widely used in the mortgage termination literature, so we provide only a brief overview here. The set-up for the Cox proportional hazard model begins by defining the time to prepayment, T , as a random variable, which has a continuous probability distribution, $f(t)$, with t a realization of T . The cumulative probability is defined as

$$F(t) = \int_0^t f(s) ds = \Pr(T \leq t) \quad (5)$$

⁸ In econometric terms, the borrower's tenure in the house, when measured by loan age, is a truncated variable, whereas for purchase money loans, housing tenure is equal to loan age. For refinancing loans, we

from which the survival function follows as

$$S(t) = 1 - F(t) = \Pr\{T \geq t\} \quad (6)$$

The survival function describes the probability that the time to prepayment will be of length at least t . The probability (l) that a prepayment will occur in the next short interval of time, Δt , given that the borrower has not prepaid prior to time t is defined as:

$$l(\Delta t) = \Pr\{t \leq T \leq t + \Delta t \mid T \geq t\} \quad (7)$$

From this probability, the hazard rate is defined as

$$h(t) = \lim_{\Delta t \rightarrow 0^+} \frac{\Pr\{t \leq T < t + \Delta t \mid T \geq t\}}{\Delta t} = \frac{f(t)}{S(t)} \quad (8)$$

Adding k covariates, some of which may be time-varying, we can write the model as:

$$H_i(t) = \lambda_0(t) \exp(\beta_1 X_{i1} + \beta_2 X_{i2}(t) + \dots + \beta_k X_{ik}) \quad (9)$$

In (5), $\lambda_0(t)$ is the baseline hazard rate, \mathbf{X} is a vector of covariates, and $\boldsymbol{\beta}$ is the vector of parameters to be estimated. Finally, taking logs of both sides, we have the model to be estimated by the Cox method of partial likelihood:

know only that the homeowner has been in the house for at least the length of the current loan.

$$\log H_i(t) = \alpha(t) + \beta_1 X_{i1} + \beta_2 X_{i2}(t) + \dots \beta_k X_{ik} \quad (10)$$

One of the attractions of the Cox model is that the baseline hazard factors out in the estimation of the likelihood function; accordingly, results for the covariates are independent of any particular baseline hazard rate. While this is useful for researchers interested in assessing the effect of a particular covariate on the hazard, it makes model predictions more difficult, since one must assume some particular baseline hazard form.

Again, for consistency with Wall Street approaches, we simply assume 100% PSA as the baseline hazard rate.⁹ Such a pattern is consistent with the household mobility interpretation of the baseline hazard. Once in a new home, given transaction costs, households are unlikely to move during the first two-three years, but as time passes, the probability of a move increases, eventually leveling off at a constant rate of 6% annually.

Turning now to model specification, which we will hold constant across the two classes of models, we have a number of covariates available. Loan characteristics include original loan size (ORIBAL), original and time-varying loan-to-value ratio (ORIGLTV, CLTV), and loan origination date. The note rate on the mortgage is captured in the time-varying ratio of the market value of the loan to the book value of the loan (VALPO), a proxy for the value of the borrower's prepayment option. Borrower characteristics at

⁹ The Public Securities Administration (PSA), now the Bond Market Association, established the PSA standard for measuring prepayment speeds. 100% PSA implies a linear monthly increase in prepayment speed over the first 30 months of mortgage life, followed by a constant rate of 6% thereafter. Wall Street conventions often describe a pool as having speeds equivalent to some multiple of PSA, e.g. 200 PSA. This can, and usually does, mean not a constant prepayment speed but a time-varying speed that is equivalent to the particular constant PSA speed, in terms of either weighted average life or bond yield.

⁹ Loan balances were amortized based on stated term and contract rate and house values updated using the publicly available OFHEO state level house price indices. In future versions of this paper, we hope to use zip code level CSW house price indices for collateral valuation updating.

time of loan origination include age (BRWAGE) and income (INCOME), measures often used to proxy for household wealth and level of financial sophistication.

Loan balances were amortized based on stated term and contract rate. House values updated using the publicly available OFHEO state level house price indices. We also use MSA level OFHEO house price indices for collateral valuation updating. The Public Securities Administration (PSA), now the Bond Market Association, established the PSA standard for measuring prepayment speeds. 100% PSA implies a linear monthly increase in prepayment speed over the first 30 months of mortgage life, followed by a constant rate of 6% thereafter. Wall Street conventions often describe a pool as having speeds equivalent to some multiple of PSA, e.g. 200 PSA. This can, and usually does, mean not a constant prepayment speed but a time-varying speed that is equivalent to the particular constant PSA speed, in terms of either weighted average life or bond yield.

We tried several specifications but, in the interest of brevity and to focus on pricing effects, only report the best results, which do not include either of the borrower characteristics. This is consistent with LaCour-Little [1999] that finds very limited effects of borrower characteristics, after eliminating prepayments produced by borrower mobility and with Archer, Ling, and McGill [1997] who argue that demographic factors often simply proxy for mobility differences, not real differences in refinancing sensitivity. Our final specification includes only amortized loan balance (AMORTBAL), contemporaneous LTV (CLTV), and refinancing incentive (VALPO). We also include a dummy variable (DUM80) equal to one, if the CLTV is greater than 80% and zero otherwise, since such mortgages may be relatively more difficult to refinance. Finally,

we add the house price volatility (PROBNEG) as a measure of negative equity in the house.

Consistent with much of the previous literature, the refinancing incentive is defined as the present value of the mortgage at market rates divided by current book value. We estimate the volatility of house prices using the standard deviation of the corresponding MSA OFHEO house price index in the previous 36 months. We use the house price volatility (PROBNEG) as a measure of negative equity in the house. We expect that a household with negative equity in the house is less likely to take advantage of the positive prepayment option value of the loan. In rare events where we do not have the corresponding MSA OFHEO house price index we use the corresponding state level OFHEO price index.

Obviously, the 1992 cohort has had a much longer history than the 1997 cohort. In order to control for this difference, we only examine prepayment performance over the first 42 months of loan life, i.e. 1992-1995 (roughly) for the 92 cohort and 1997-2000 for the 97 cohort. We chose 42 months, since this was the longest duration available from the 97 cohort. In each case, the 42-month period incorporates a subsequent refinancing wave (the 1993 rate drop for the 1992 cohort and the 1998 rate drop for the 1997 cohort). For the 1997 cohort, the observation date, therefore, the censoring date is June, 2000 while that for the 1992 cohort is June, 1995.

6. Results

We show the descriptive statistics of the loan characteristics and borrower characteristics of both the 1992 cohort and the 1997 cohort in Table 1. Overall we find

that sampled loans are representative of the period geographically and calendar-wise while at the same time showing wide variations in important loan characteristics and borrower characteristics.

Panel A of Table 1 shows the descriptive statistics of the 1992 cohort. The sample size is 1215. Sample loans are originated throughout the year. Note rates of loans range between 8.00% and 9.00% with the mean and the median note rates of 8.50%. Loans are originated from all states except Alaska. Loan to value ratios vary widely ranging from 16.00% to 90.00%. The mean and the median loans have loan to value ratios of 71.81% and 77.60%, respectively. Most loans have a term of 360 months. The mean and median borrower wages are \$39,207 and \$37,000, respectively. Borrowers' wage and income vary widely. The mean and median borrower wages are \$64,223 and \$58,704, respectively. Finally, FICO scores vary widely as well with the mean and the median scores of 741.05 and 758.

Panel B shows the corresponding figures for the 1997 cohort. The sample size is slightly smaller (N=1065). Overall, we find that the sample loans exhibit comparable characteristics to those of the 1992 cohort. We conduct a univariate analysis of the loan characteristics variables and borrower characteristics variables between the two cohorts. As shown in Panel C, differences are economically minor for all variables except loan to value ratios and FICO scores. These small differences are statistically significant only for note rates and discount points. The increase in loan to balance ratio may reflect the enhanced capability of the mortgage market to underwrite high risk loans, which is consistent with the technological innovation in the mortgage market during the study period.

To see whether indeed the borrowers of the 1997 cohort prepaid faster than those of the 1992 cohort, we compare the proportion of prepaid loans and the average age of prepaid loans at prepayment and report the results in Table 2. The univariate analysis is consistent with the mortgage market innovation hypothesis. Of the 1992 cohort, about 38% prepay within 42 months window (by June 1995) while about 51% of the 1997 cohort prepay within 42 months window (by June 2000). The chi-square test statistic is 38.08 with the p-value of essentially zero. This is consistent with the hypothesis that innovations in the mortgage market in the 90s led to a faster prepayment.

However, we find that the mean age of the prepaid loans of the 1997 cohort is 19.14 months while that for the 1992 cohort is 19.69. There is little difference in how fast these prepayments occurred. It appears that the decrease in transaction costs helped those at the margin to prepay, i.e., those who would not have taken advantage of the prepayment option otherwise.

We show the descriptive statistics of the regression variables of both the 1992 cohort and the 1997 cohort in Table 3. Again, overall the regression variables have similar distribution between the two samples. We note that the present value of payments to balance ratios ranges between 0.955 to 1.227 for the 1992 cohort and from 0.902 to 1.223 for the 1997 cohort. The house price volatility of the 1997 cohort is greater with the standard deviation rising up to 6% reflecting the house price run-up that starts around 1997.

[Insert Table 3 about here.]

We investigate the sensitivity of the prepayment decision to the prepayment option value for the 1992 cohort and the 1997 cohort using the Cox proportional hazards

model and the logit model and the regression estimates are reported in Table 4. We present the results of the Cox proportional hazards model in Panel A of Table 4. As expected, the coefficient of the refinancing incentive is clearly larger for the 97 cohort (24.195) than the 92 cohort (17.997). In other words, the 97 cohort was considerably more sensitive to refinancing opportunities, relative to the 92 cohort. We present the results of the logit model of prepayment in Panel A of Table 4. Similarly, the coefficient on the refinancing incentive is clearly larger for the 97 cohort (50.320) than the 92 cohort (18.609).

[Insert Table 4 about here.]

As expected, mortgage balance has a positive influence on prepayment decision in both cohorts. Coefficients on amortized loan balance are essentially identical across the two cohorts (1.113 for the 92 cohort; 0.593 for the 97 cohort). The coefficients on the contemporaneous loan-to-value ratio are negative for both cohorts, but significant only for the 1992 cohort. This is consistent with the explanation that refinancing incentive apparent in a high loan-to-value ratio is outweighed by other countervailing consideration of a high loan-to-value ratio including refinancing constraint on a borrower with a high loan-to-value ratio. On the other hand, the coefficient on the variable CLTV over 80% is positive and significant for the 97 cohort, whereas it is positive and not significant for the 92 cohort. We do not have a coherent explanation for this result. The house price volatility variable is negative and significant only for the 1992 cohort as expected from the negative equity argument.

In Table 5, we present results of the Cox proportional hazards model where we use the entire event history of individual observations up to the observation date by

calculating all the time-varying regression variables to the origination date. In Panel A, we use an equal event history period of 42 months for both cohorts. As result of the time series generated for each loan in the sample the number of observations of the 1992 cohort rises to 36,980 while that for the 1997 cohort rises to 29,828. We find that results of the hazard models are similar to those using the data only at the end of each loan observation for both the 92 cohort and the 97 cohort. One notable difference is that the house price volatility variable is negative and significant even for the 1997 cohort as expected from the negative equity argument.

The difference in the interest rate component of the refinancing sensitivity between the two cohorts narrows with the full event history data. The interest rate component of the prepayment propensity as shown by the coefficient of for the present value of payments to balance ratio, which is a measure of the call option value of the loan, is 18.9 for the 1992 cohort and 20.0 for the 1997 cohort. It indicates that if we use the parameter value of the interest rate component of prepayment propensity of the 1992 cohort for the estimation of the prepayment behavior of the 1997 cohort, the probability of prepayment will be understated roughly three times, which is the ratio of the exponentials of the coefficients, i.e., $\exp(20.0-18.9)$. While not reported as a table, we find that the interest rate component of the prepayment propensity of the 1997 cohort is about twice as large as that of the 1992 cohort when we use the difference between the note rate of the mortgage loan and the current mortgage rate as the proxy for the moneyness of the call option of the mortgage loan. The estimation of the change in the prepayment sensitivity to interest rate drop suggests that the doubling of prepayment speed is feasible over the study period.

[Insert Table 5 about here.]

In Panel B, we consider the scenario where we estimate the prepayment function using the event history of the 1992 cohort up to June, 2000 where the number of observations rises to 69,108. The result shows that we would again underestimate the prepayment propensity of the 1997 cohort if we use the coefficient of 17.8 for the present value of payments to balance ratio, where the correct value would be 20.0. Finally, we consider the scenario where we estimate the prepayment function using the event history of the combined cohorts so that the number of observations is 98,936. We find that we would underestimate the prepayment propensity even more using the coefficient of 12.9 for the present value of payments to balance ratio.

7. Model Error and the Pricing of Mortgages

We now turn to the main question we wish to consider; namely, what effect do these differences have on mortgage pricing?

To evaluate the effect of the prepayment speed on the mortgage valuation, we price mortgages with premium mortgage interest rates. Premium mortgage loans should price above par and lose value as prepayment speed rises. We first price mortgage loans statically where we use a flat static term structure and then we price mortgage loans dynamically where we use stochastic term structure models. Static pricing results are tabulated in Table 6 and the corresponding graphs are in Figure 3 while dynamic pricing results are tabulated in Table 7 and the corresponding graphs are in Figure 4. For both static and dynamic pricing, the current mortgage rates are 8%, 8.5% and 5.25% on the valuation dates of June 30, 1997; June 30, 1997; June 30, 1997, respectively.

In Table 6, we show the static prices of premium mortgage loans as a function of both prepayment speed expressed as multiples of 100 PSA (columns) and premiums in note rates (rows). The responses of the value of 8% premium mortgages to the varying prepayment speeds expressed as PSA multiples are shown in Panel A. The drop in value of a mortgage with 40 basis points premium rate, as the prepayment speed rises from 200 PSA to 400 PSA, is about 63 basis points. The drop in value of a mortgage loan with 80 basis points premium rate is about 127.71 basis points as the prepayment speed rises from 200 PSA to 400 PSA. The impact of doubling of prepayment speed is clearly economically significant where losses measured as drop in the value of a mortgage loan can be up to a few hundred basis points. As we vary the base mortgage rates, we find similar results as shown in Panel B where the base mortgage rate is 8%, which is the rate as of June 30, 2000 and in Panel C where the base mortgage rate is 5.25%, which is the rate as of June 30, 2003.

[Insert Table 6 about here]

[Insert Figure 2 about here]

Next, we price mortgages with discount mortgage interest rates. Discount mortgage loans should price below par and gain value as prepayment speed rises. We price discount mortgage loans statically where we use a flat static term structure and pricing results are tabulated in Table 7 and the corresponding graphs are in Figure 4. Results are as expected.

[Insert Table 7 about here]

[Insert Figure 3 about here]

Next, we price mortgages dynamically, that is, we model the borrower's refinancing behavior in valuing the mortgages and mortgage-backed securities allowing the interest term structure to behave stochastically. The dynamic aspect of the pricing is implemented using the Monte Carlo simulation where interest rate processes are implemented using the Black-Karasinski model and the Black-Derman-Toy model, whose main characteristics are explained briefly in Appendix 2. We simulate an appropriate number of interest rate paths and price a current coupon mortgages using a series of different prepayment hazard models in different yield curve environments (flat, inverted, and steep). At each pricing date, we present results varying parameters of the term structure simulation, as well as parameters of the prepayment function, so that we have effectively controlled for differences in term structure model.

When we model and price a mortgage-backed security we deal with two dimensions of complexity: (1) modeling the dynamic behavior of the term structure of interest rates, and (2) modeling the prepayment behavior of mortgage holders. The standard way of solving this valuation problem has been to assume a stochastic process for term structure movements and to employ either an option pricing approach (e.g., Brennan and Schwartz, [1985], Dunn and McConnell [1981], Stanton [1995]) or an empirical model (e.g., Schwartz and Torous [1989] and Waldman [1992]) for prepayment behavior. We follow the empirical valuation approach.

To preview the results: the cost of model error (using historical experience from the 92 cohort to price mortgages experiencing the 1998 refinancing wave) ranges from 3-356 basis points, depending on other term structure model choices and parameters. For example, pricing an 8% mortgage pool as of June 30, 1997 using the 1992 prepayment

model parameters and an OAS of 80 basis points produces a value of 105.3638, compared to a value of 103.8552, using the parameters of the 1997 model and setting other covariate values to overall means, a difference of 85 basis points, when volatility is set at 12% and mean reversion at 0.05.¹⁰ Additional results are shown in Table 8. In what follows, we describe our method in more detail.

Since we value a whole mortgage loan in the same way as a mortgage backed security, which is simply a weighted average of the market values of the underlying mortgage loans, we discuss the valuation of a mortgage loan only. We solve the standard partial differential equation of the value of a mortgage loan using simulation method as outlined in Schwartz and Torous [1989]. We do not impose an optimal call condition on the part of the mortgagor to value the mortgage-backed security. Rather, the value of the mortgage backed security reflects the fact that at each point in time there exists a probability of prepayment, which is estimated separately. We price the mortgage loan by discounting the cash flows to the investor in the mortgage loan, where the discount rate is the sum of the simulated forward rates on a riskless security and an appropriate option adjusted spread. The relationship between the discount rate and the forward rates is as follows:

$$Z_T(n) = \frac{1}{[1 + f_1(n)][1 + f_2(n)] \dots [1 + f_T(n)]} e^{-\int_0^T r} - 1 \quad (11)$$

where $Z_T(n)$ is the discount rate at time T on the simulated interest rate path n and $f_t(n)$ is the forward interest rate at time t on the simulated interest rate path n.

¹⁰ Arora, Heike, and Mattu [2000] report average 30-year mortgage returns for the years 1989-1999 using

Therefore, the present value of the cash flow at time t on the forward interest rate path n is:

$$PV [CF_t(n)] = \frac{CF_t(n)}{[1 + Z_t(n) + K]} \quad (12)$$

where $PV [CF_t(n)]$ is the present value of the cash flow at time t on the simulated interest rate path n, $CF_t(n)$ is the cash flow at time t on the simulated interest rate path n, $Z_t(n)$ is the discount rate at time t on the simulated interest rate path n, K is the option adjusted spread (OAS), and the cash flow in each month is the sum of interest payment, scheduled principal repayment and predicted principal repayment.

The present value of all cash flows on the simulated interest path n for a mortgage-backed security with 30 years time to maturity is then:

$$PV [MBS (n)] = PV [CF_1(n)] + PV [CF_2(n)] + \dots + PV [CF_{360}(n)] \quad (13)$$

The present value of a mortgage-backed security is then evaluated as:

$$PV [MBS] = \frac{1}{N} \sum_{n=1}^N PV [MBS (n)] \quad (14)$$

where N is the number of simulated interest rate paths.

Theoretically, the option adjusted spread is estimated by setting:

OAS values from 74-84 basis points and the Lehman Brothers prepayment model.

$$P_0 = PV [MBS] = \frac{1}{N} \sum_{n=1}^N PV [MBS (n)] \quad (15)$$

where P_0 is the observed market price of the mortgage-backed security.

We report the results based on the implementation of the Black-Karasinski model where we generate random interest rate paths using a lognormal diffusion process with input parameters for mean reversion and spot rate volatility. We estimate the mean reversion parameter using the interest rate time series of 3 month treasuries. We use the mean reversion parameter of 0.05 estimated from the sample period of January, 1992 to December, 2003 to generate the Monte Carlo interest rate simulation paths. We use quadratic spline method and Svensson method to fit the initial term structure.

We select dates of June 30, 1997, June 30, 2000, and June 30, 2003 as pricing dates, representing different term structure environments and take the current coupon mortgage rate at each date. As shown in Figure 2, as of June 1997, the yield curve is quite flat, as of June 2000 it is inverted (predicting the upcoming recession), and as of June 2003 it is normal. In Figure 4, we also show the fitting results of the initial term structure using quadratic spline method and Svensson method.

[Insert Figure 4 about here]

We assume default rates of zero for all three valuation dates. For the valuation on June 30, 1997, we assume a constant mortgage-treasury spread of 119 basis points, which is the mortgage spread over Treasury on that date. Similarly, for the valuation on June 30, 2000, we assumed a constant mortgage-treasury spread of 247 basis points; for the valuation on June 30, 2003, we assumed a constant mortgage-treasury spread of 170 basis

points. We then price a normalized \$1 mortgage loan subject to (partial) prepayment risk described by the hazard models based on either the 92 cohort or the 97 cohort of data, by taking the present value of the cash flows at an 80 basis point OAS over the simulated treasury spot rates. In this case, our approach controls completely for differences in the parameters of term structure model choice and, of course, for the choice of prepayment model. Accordingly, residual pricing differences are entirely attributable to prepayment model parameter differences.

At each pricing date, we price the current coupon 30-year fixed rate mortgage with 3 different term structure assumptions. First, we use zero volatility, expecting this assumption to produce the largest mortgage value. Next, we price at 12% and then 16% volatility, with mean reversion set to 0.05, expecting the value of the mortgage to decline and to decline more at the higher volatility level, consistent with an increase in the value of the prepayment option under higher volatility.¹¹ In Table 7 we do find that, as expected, the higher volatility, the lower are the values of the mortgage loans.

[Insert Table 7 about here]

As of June 1997, we price an 8.0% 30-year fixed rate mortgage. At that point in time, the 10-year Treasury stands at 6.51%, the 1-year Treasury is at 5.67%, and three-month Treasury is at 5.25%, a much flatter yield curve environment than has been the case in recent years. In each of the term structure scenarios, the price of the mortgage is higher using the 92 model parameters, compared to the 97 model parameters, with differences ranging from 85-101 basis points as we double the interest rate sensitivity of prepayment from 12 to 24.

We do not find it surprising that in the flat yield curve environment, faster prepayment leads to lower prices for premium mortgages. Because borrowers prepay more quickly when the value of the outstanding loan balance is above par, investors' expected capital gains fall. This was also shown in Table 6 where we use the static pricing.

Next we price an 8.50% 30 year fixed rate mortgage as of June 2000, a date on which the yield curve is inverted. In particular, the 10-year treasury rate is 6.03% (implying a much wider mortgage-treasury spread than was the case in June 1997), the one-year treasury is at 6.44%, and 3-month treasury is at 6.02%. Here pricing effects are of the same sign but larger in magnitude. Again, prices are higher using the 92 model parameters compared to the 97 model parameters, across all three scenarios, with price differences reaching a few hundred basis points under realistic scenarios.

As a third test, we price a 5.25% 30-year fixed rate mortgage as of June 30, 2003, a date on which the yield curve is normal. 3-month treasury and the one-year treasury both stand at 0.90% and 1.09%; the 10-year treasury is at only 3.54%, a near-historic low. In this case, pricing differences are more reduced. The reduction in the decrease in value as the prepayment speed rises is likely to result from the secondary effect that the earlier prepayments are made where interest rates are lower compared to later periods dampening the effect of the faster prepayment of premium mortgage loans.

In general, however, our results support the notion that model risk does matter and that increased refinancing sensitivity has, in effect, reduced the value of otherwise

¹¹ Zero volatility in this context does not imply a flat term structure; rather, rates evolve over time consistent with the actual yield curve as of that date, producing mortgage rates that increase initially then decline and vary within a tight range of 8-9% over the entire simulation period.

identical mortgage assets over the last decade or so. We attribute this result to technological change and reduction in transaction costs.

It is worth contemplating the extent to which this matters. When one considers how leveraged Fannie Mae and Freddie Mac are, it may well matter a lot. The firms have equity of around three percent of assets.¹² This means that if the price of the securities they own falls by 50 basis points, the value of their equity falls by 15 percent—not enough to threaten solvency, but a substantial amount, nevertheless. While firms can use a variety of hedging instruments to insulate themselves against interest rate risk, model risk is almost impossible to hedge against.

Archer and Ling [1995] evaluate the effect of alternative interest rate processes on the value of mortgage-backed securities. They report different pricing characteristics of a simple lognormal process, the square root process of Cox, Ingersoll and Ross and the no-arbitrage process of Black, Derman and Toy. We use the term structure model of Black, Derman and Toy, [1990] as well. While not reported as a table, we obtained qualitatively comparable results where we obtain a significant overvaluation of mortgage loans and mortgage-backed securities using the incorrect parameters.

Conclusions

We have analyzed the prepayment response rates of loans originated in the early 1990s compared to the late 1990s, a period during which technological change and increased efficiency, together with generally robust macroeconomic conditions, made mortgage refinancing an increasingly available option to most households. We conclude

¹² Because of accounting issues, it is actually difficult to know how much on-balance sheet capital the firms have at the moment.

that interest rate sensitivity has changed significantly over the decade and that the constraints of high LTV have diminished. Moreover, the model risk associated with pricing using one response function versus another can produce economically significant differences in asset values. In our simulations, the pricing difference can be up to a few hundred basis points in asset values and most pronounced in an inverted yield curve environment. Accordingly, practitioners would be well advised to explicitly incorporate potential parameter drift into models used to price mortgages and mortgage-backed securities.

In this paper we focused on prepayment, rather than default, although the latter is arguably more relevant to the recent subprime mortgage market collapse and related spillover effects. The two risks are inter-related, however, as borrowers who are unable to refinance due credit constraints or reduced equity arising from house price declines will have an elevated risk of defaulting. Additional research on parameter stability in mortgage default models will be an important extension to the research.

References

- Ambrose, Brent W. and Michael LaCour-Little. 2001. Prepayment Risk in Adjustable Rate Mortgages Subject to Initial Year Discounts: Some New Evidence. *Real Estate Economics* 29 (2): 305-328.
- Arora, Amitabh, David K. Heike, and Ravi K. Mattu. 2000. Risk and Return in the Mortgage Market: Review and Outlook. *Journal of Fixed Income* 10(1):5-18.
- Archer, Wayne, David Ling, and Gary McGill. 1997. Demographic Versus Option-Driven Mortgage Terminations. *Journal of Housing Economics* 6(2): 137-163.
- Archer, Wayne, David Ling, and Gary McGill. 1996. The Effect of Income and Collateral Constraints on Residential Mortgage Terminations. *Regional Science and Urban Economics* 26: 235-261.
- Archer, Wayne R and David C. Ling, 1995. The Effect of Alternative Interest Rate Processes on the Value of Mortgage-backed Securities. *Journal of Housing Research* 6(2): 285-314.
- Barry, C. 1974. Portfolio Analysis under Uncertain Means, Variance, and Covariances. *Journal of Finance* 29: 515-522.
- Barry, C. 1978. Effects of Uncertain and Nonstationary Parameters upon Capital Market Equilibrium Conditions. *Journal of Financial and Quantitative Analysis* 13: 419-433.
- Bawa, V.S. and S. J. Brown. 1979. Capital Market Equilibrium: Does Estimation Risk Really Matter? In V.S. Bawa, S. J. Brown, and R. W. Klein, ed. *Estimation Risk and Optimal Portfolio Choice*. Amsterdam, North Holland.
- Bernardo, Antonio and Bradford Cornell. 1997. The Valuation of Complex Derivatives by Major Investment Firms: Empirical Evidence. *Journal of Finance* 52(2): 785-798.
- Black, F., E. Derman, and W. Toy, 1990. A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options. *Financial Analysts Journal* 46, 33-39.
- Bliss R. R. and D. C. Smith, 1997. The Stability of Interest Rate Processes. Federal Reserve Bank of Atlanta, Unpublished working Paper 97-13.
- Black, F., E. Derman, and W. Toy, 1990. A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options. *Financial Analysts Journal* 46, 33-39.
- Brennan, M.J. and E. Schwartz, 1985. Determinants of GNMA Mortgage Prices. *AREUEA Journal*, 13, 209-228.

- Brueckner, J. K. 1992. Borrower Mobility, Self-Selection, and the Relative Prices of Fixed- and Adjustable-Rate Mortgages. *Journal of Financial Intermediation* 2: 401-421.
- Brueckner, J. K. 1994. Borrower Mobility, Adverse Selection, and Mortgage Points. *Journal of Financial Intermediation*. 3(4): 416-441.
- Caplin, A., C. Freeman, and J. Tracy. 1997. Collateral Damage: Refinancing Constraints and Regional Recessions. *Journal of Money, Credit and Banking* 29(4): 497-516.
- Chan, K.C., A. Karolyi, F. Longstaff, and A. Sanders. 1992. The Volatility of Short-Term Interest Rates: An Empirical Comparison of Alternative Models of the Term Structure of Interest Rates. *Journal of Finance* 68: 1209-1227.
- Cheyette, Oren. 1996. Implied Prepayments. *Journal of Portfolio Management* 23: 107-113.
- Clapp, J. M., J. Harding and M. LaCour-Little. 2000. Expected Mobility: Part of the Prepayment Puzzle. *The Journal of Fixed Income* 10(1): 68-78.
- Coles, J. and U. Loewenstein. 1988. Equilibrium Pricing and Portfolio Composition in the Presence of Uncertain Parameters. *Journal of Financial Economics* 22: 279-303.
- Coles, J., U. Loewenstein, and J. Suay. 1995. On Equilibrium Pricing under Parameter Uncertainty. *Journal of Financial and Quantitative Analysis* 30: 347-364.
- Cox, J., J. Ingersoll and S. Ross, 1985. A Theory of the Term Structure Model of Interest Rates: The Effect of Uncertainty. *Econometrica* 53, 385-407.
- Cutts, Amy Crews and Anthony Sanders. 2008. The Subprime Crisis and Its Role in the Financial Crisis. *Journal of Housing Economics*, forthcoming.
- Demyanyk, Yuliya and Van Hemert, Otto, Understanding the Subprime Mortgage Crisis (August 19, 2008). Unpublished working paper available at SSRN: <http://ssrn.com/abstract=1020396>
- Deng, Y., J. M. Quigley, and R. Van Order. 2000. Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options. *Econometrica* 68(2): 275-307.
- Dixit, Avinash-K. and R.S. Pindyck. 1988. Expandability, Reversibility, and Optimal Capacity Choice. National Bureau of Economic Research Working Paper: 6373, January 1998.
- Dunn, Kenneth B. and John J. McConnell. 1981a. A Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities. *Journal of Finance* 36: 471-483.

- Dunn, K. B. and John J. McConnell. 1985. Valuation of Mortgage-Backed Securities. *Journal of Finance* 36: 599-617.
- Follain, James R., Louis O. Scott, and Tyler Yang. 1992. Microfoundations of a Mortgage Prepayment Function. *The Journal of Real Estate Finance and Economics* 5(2): 197-217.
- Foote, Christopher L., Gerardi, Kristopher, Goette, Lorenz F. and Willen, Paul. 2008. Just the Facts: An Initial Analysis of Subprime's Role in the Housing Crisis. *Journal of Housing Economics*, forthcoming.
- Gorton, Gary B., The Subprime Panic (October 2008). NBER Working Paper No. W14398 Available at SSRN: <http://ssrn.com/abstract=1281890>
- Green, J. and J. B. Shoven. 1986. The Effects of Interest Rates on Mortgage Prepayments. *Journal of Money, Credit, and Banking* 18(1): 41-50.
- Green, Richard K. and Michael LaCour-Little. 1999. Some Truths about Ostriches: Who Never Refinances Their Mortgage and Why They Don't. *Journal of Housing Economics* 8: 233-248.
- Hayre, Lakhbir and Arvind Rajan. 1995. Anatomy of Prepayments: The Salomon Brothers Prepayment Model. New York: Salomon Smith Barney.
- Hayre, L. S., S. Chaudhary, and R. A. Young. 2000. Anatomy of Prepayments. *The Journal of Fixed Income* 10(1): 19-49.
- Heath, D. R. Jarrow and A. Morton. 1990. Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approximation. *Journal of Financial and Quantitative Analysis* 25: 419-440.
- Heath, D. R. Jarrow and A. Morton. 1992. Bond Pricing and Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation. *Econometrica* 60(1): 77-105.
- Hilliard, J. E., J. B. Kau and V. C. Slawson, Jr., 1998. Valuing Prepayment and Default in a Fixed Rate Mortgage. *Real Estate Economics* 26(2): 431-468.
- Kalotay, Andrew, Deane Yang, and Frank Fabozzi. 2004. An Option-Theoretic Model for Mortgages and Mortgage-Backed Securities. Unpublished working paper, Andrew Kalotay and Associates.
- Kau, James B and Taewon Kim. 1992. The Timing of Prepayment: A Theoretical Analysis. *Journal of Real Estate Finance and Economics* 7 (3): 221-228.

Kau, James B., Donald Keenan, Walter Mueller, and J. Epperson. 1992. A Generalized Valuation Model for Fixed-Rate Residential Mortgages. *The Journal of Money, Credit, and Banking*, 24 (3): 279-299.

LaCour-Little, Michael. 1999. Another Look at the Role of Borrower Characteristics in Predicting Mortgage Prepayments. *Journal of Housing Research* 10(1): 45-60.

LaCour-Little, Michael. 2001. The Evolving Role of Technology in Mortgage Finance. *Journal of Housing Research* 11(2): 173-205.

LaCour-Little, Michael and Gregory H. Chun. 1999. Third Party Originators and Mortgage Prepayment Risk: An Agency Problem? *Journal of Real Estate Research* 10(1/2): 55-70.

LaCour-Little, Michael, Michael Marschoun and Clark Maxam. 2002. Improving Parametric Mortgage Prepayment Models Using Non-parametric Kernel Regression. *Journal of Real Estate Research* 24(3): 299-327, 2002.

Mattey, Joe and Nancy Wallace. 2001. Housing-Price Cycles and Prepayment Rates of U.S. Mortgage Pools. *Journal of Real Estate Finance and Economics* 23(2): 161-184.

Maxam, Clark and Michael LaCour-Little. 2001. Applied Non-Parametric Regression Techniques: Estimating Prepayments on Fixed Rate Mortgage-Backed Securities. *Journal of Real Estate Finance and Economics* 23 (2): 139-160.

Pavlov, Andrey D. 2000. Competing Risks of Mortgage Prepayments: Who Refinances, Who Moves, and Who Defaults? *Journal of Real Estate Finance and Economics* 23(2): 185-211.

Patruno, Gregory N. 1994. Mortgage Prepayments: A New Model for a New Era. *The Journal of Fixed Income* 4(3): 42-56.

Peristiani, Stavros, Modeling the Instability of Mortgage-Backed Prepayments,” Research Paper 98-04, Federal Reserve Bank of New York, March 1998.

Peristiani, Stavros, Paul Bennett, Gordon Mosen, Richard Peach, and J. Raiff. 1996. Effects of Household Creditworthiness on Mortgage Refinancings. Federal Reserve Bank of New York Research Paper No. 9622.

Peristiani, Stavros, Paul Bennett and Richard Peach. 1998. Structural Change in the Mortgage Market and the Propensity to Refinance. Staff Report no. 45, Federal Reserve Bank of New York, September 1998.

Quigley, John M. 1988. Interest Rate Variations, Mortgage Prepayments, and Household Mobility. *The Review of Economics and Statistics* 69: 636-642.

Richard, S. F. and R. Roll. 1989. Modeling Prepayments on Fixed Rate Mortgage-Backed Securities. *Journal of Portfolio Management* 15(3): 73-82.

Schwartz, E., and W. Torous. 1989. Prepayment and the Valuation of Mortgage Backed Securities, *Journal of Finance* 44(2): 375-392.

Stanton, Richard. 1995. Rational Prepayment and Valuation of Mortgage-Backed Securities. *Review of Financial Studies* 8(3): 677-708.

Stanton, Richard, and Nancy Wallace. 1998. Mortgage Choice: What's the Point? *Real Estate Economics* 26(2): 173-206.

Stulz, Rene M., Risk Management Failures: What are They and When Do They Happen?(October 2008). Fisher College of Business Working Paper No. 2008-03-017 Available at SSRN: <http://ssrn.com/abstract=1278073>

Waldman, M. 1992. Beyond Duration: Risk Dimensions of Mortgage Securities. *Journal of Fixed Income*, 2, 5-15.

Whalen, R. Christopher, The Subprime Crisis: Cause, Effect and Consequences(March 1, 2008). Networks Financial Institute Policy Brief No. 2008-PB-04. Available at SSRN: <http://ssrn.com/abstract=1113888>

Appendix 1. Data Sources

Data	Source
30-year conforming fixed rates mortgage market interest rates (monthly time series)	FHLMC (Federal Home Loan Mortgage Corporation) website (www.freddiemac.com)
OFHEO repeat-sales house price index at both the state level and the metropolitan statistical area level (monthly and quarterly time series)	OFHEO (Office of Federal Housing Enterprise Oversight) website (www.ofheo.gov)
Metropolitan statistical area code	Census bureau website (www.census.gov)

Appendix 2. Specification of the processes for the interest rate on a riskless security used for the mortgage-backed security pricing

Black-Karasinski	Specification	$d(\ln r_t) = \left[\theta_t - a \ln r_t \right] dt + \sigma .dz$
	Mean of $d(\ln r)$ between $(t, t + \Delta t)$	$\left[\theta_t - a \ln r_t \right] \Delta t$
	Standard deviation of $d(\ln r)$ between $(t, t + \Delta t)$	$\sigma \sqrt{\Delta t}$
Black-Derman-Toy	Specification	$d(\ln r_t) = \theta_t dt + \sigma .dz$
	Mean of $d(\ln r)$ between $(t, t + \Delta t)$	$\theta_t \Delta t$
	Standard deviation of $d(\ln r)$ between $(t, t + \Delta t)$	$\sigma \sqrt{\Delta t}$

The above equations indicate that $\ln r_t$ follows a Weiner process.

Using yields from appropriate benchmark instruments, specifically, a set of treasuries, we deduce the initial forward curve. We carry out Monte Carlo simulations of the forward interest rates on riskless securities using the processes described above and the initial forward curve.

Table 1. Sample Distribution**Panel A. 1992 Cohort**

	N	Mean	Median	Standard deviation	Minimum	Maximum
Loan characteristics						
Origination month	1215	5.52	5	3.04	1	12
Note rate (%)	1215	8.50	8.50	0.30	8.00	9.00
Discount point	667	1.56	1.50	1.25	0.00	5.19
Original balance	1215	\$115,957	\$115,220	\$45,862	\$19,000	\$210,000
State code	1215	23.74	31	12.99	2	51
Loan to value ratio (%)	1215	71.81	77.60	14.38	16.00	90.00
Term	1215	358.28	360	13.60	240	360
Borrower characteristics						
Age	1210	39.21	37.00	10.94	20.00	85.00
Income	1151	\$64,223	\$58,704	\$40,017	\$12,840	\$918,504
FICO score	701	741.05	758.00	53.85	456	819

Origination month: January=1 to December=12, successively

State code: Alaska=1 to Wyoming=51, alphabetically ascending order

Panel B. 1997 Cohort

	N	Mean	Median	Standard deviation	Minimum	Maximum
Loan characteristics						
Origination month	1065	5.59	6	1.98	1	11
Note rate (%)	1065	8.21	8.13	0.21	8.00	9.00
Discount point	667	1.14	0.00	0.30	0.00	2.08
Original balance	1065	\$114,990	\$110,000	\$49,964	\$15,000	\$273,150
State code	1065	23.02	29	12.14	1	50
Loan to value ratio (%)	1065	79.31	80.00	16.59	15.10	95.00
Term	1065	358.82	360	10.71	240	360
Borrower characteristics						
Age	1060	40.65	38.00	12.34	19.00	85.00
Income	1151	\$61,775	\$53,712	\$35,958	\$12,768	\$315,780
FICO score	943	723.27	733.00	56.86	514	819

Panel C: Univariate Analysis of Sample Difference

	1992 cohort mean	1997 cohort mean	Difference	t-statistics	p-value based on parametric tests	p-value based on Wilcoxon two sample test
Loan characteristics						
Origination month	5.52	5.59	-0.07	-0.70	0.486	0.005
Note rate (%)	8.50	8.21	0.29	27.34	0.000	0.000
Discount point	1.56	1.14	1.42	28.86	0.000	0.000
Original balance	\$115,957	\$114,990	\$967	0.48	0.632	0.32
State code	23.74	23.02	0.72	1.38	0.171	0.214
Loan to balance ratio (%)	71.81	79.31	-7.50	-11.46	0.000	0.000
Term	358.28	358.82	-0.54	-1.02	0.309	0.210
Borrower characteristics						
Age	39.21	40.65	-1.44	-2.93	0.003	0.025
Income	\$64,223	\$61,775	\$2,448	1.50	0.133	0.000
FICO score	741.05	723.27	17.77	6.46	0.000	0.000

The t-statistics reported assume unequal variances. The p-values reported are two-sided.

Table 2. Univariate Analysis of the Prepaid Loans between the 1992 and 1997 Cohorts

	1992 cohort	1997 cohort	Difference	Test statistics	p-value
Total number of loans	1215	1002			
Number of loans outstanding	754	524			
Number of prepaid loans	461	541			
Proportion of loans prepaid (%)	37.94	50.80	-12.86	38.08	0.000
Average age of prepaid loans at prepayment (months)	19.69	19.14	0.55	1.38	0.167

The test statistics for comparing the proportions of the prepaid loans in the total between the two cohorts is chi square while the test statistics for comparing the average age of the prepaid loans at prepayment between the two cohorts is t-statistics. The t-statistics reported assumes unequal variances. The p-values reported are two-sided.

Table 3. Descriptive Statistics of Regression Variables**Panel A. 1992 Cohort**

	N	Mean	Median	Standard deviation	Minimum	Maximum
Loan age (months)	1,215	28.01	33	10.49	3	42
Cotemporaneous balance	1,215	\$112,741	\$107,494	\$49,204	\$14,628	\$265,746
Contemporaneous loan to value ratio (%)	1,215	68.26	71.40	15.27	12.60	93.14
Present value of payments to balance ratio	1,215	1.053	1.019	0.069	0.955	1.227
House price volatility	1,215	0.010	0.010	0.005	0.003	0.039

Panel B. 1997 Cohort

	N	Mean	Median	Standard deviation	Minimum	Maximum
Loan age (months)	1,065	30.52	34	9.43	6	42
Cotemporaneous balance	1,065	\$113,554	\$113,065	\$45,083	\$18,612	\$203,505
Contemporaneous loan to value ratio (%)	1,065	68.88	72.51	14.52	14.15	107.13
Present value of payments to balance ratio	1,065	1.096	1.098	0.047	0.902	1.223
House price volatility	1,065	0.018	0.020	0.007	0.002	0.060

Table 4. Estimation of the Sensitivity of the Prepayment Decision to the Prepayment Option Value for the 1992 Cohort and the 1997 Cohort

Panel A: Using Cox proportional hazard model

	1992 cohort	1997 cohort
Log of cotemporaneous balance	1.113 (74.57; 0.000; 3.043)	0.593 (33.34; 0.000; 1.809)
Contemporaneous loan to value ratio (%)	-0.011 (7.90; 0.005; 0.989)	-0.003 (0.52; 0.470; 0.997)
Present value of payments to balance ratio	17.997 (251.74; 0.000, 6.5E7)	24.195 (751.59; 0.000; 3.2E10)
Contemporaneous loan to value ratio >80%	0.122 (0.62; 0.43; 1.13)	0.233 (3.35; 0.067; 1.262)
House price volatility	-10.260 (2.06; 0.15; 0.00)	-0.133 (0.000; 0.989; 0.876)
N	1,210	1,065
Censored observation	754	524
Likelihood ratio (chi-square)	341.35	1112.80
p-value	0.000	0.000

Within brackets, we show chi-squares, first; p-values, next and hazard ratios, last.

Panel B: Using logit model

	1992	1997
Intercept	-33.585 (258.98; 0.000)	-58.258 (119.18; 0.000)
Log of cotemporaneous balance	1.143 (72.29; 0.000)	0.614 (6.183; 0.012)
Contemporaneous loan to value ratio (%)	-0.012 (7.68; 0.006)	-0.014 (2.394; 0.122)
Present value of payments to balance ratio	18.609 (233.02; 0.000)	50.320 (145.43; 0.000)
Contemporaneous loan to value ratio >80%	0.024 (0.02; 0.882)	0.668 (3.56; 0.059)
House price volatility	-4.583 (0.398; 0.528)	-53.99 (4.42; 0.035)
N	1,210	1,065
Prepayment observation	456	541
Likelihood ratio (chi-square)	306.20	1125.07
p-value	0.000	0.000

Within brackets, we show Wald chi-squares, first and p-values, next.

Table 5. Coefficients on Cox Proportional Hazard Prepayment Models Using the Event History Data

Panel A. Using an equal event history period of 42 months for both cohorts

	1992 cohort	1997 cohort
Log of mortgage balance in year at risk	1.480 (127.06)	0.084 (87.08)
Contemporaneous loan-to-value	-0.017 (16.92)	-0.006 (5.91)
Present value of payments to balance ratio	18.90 (553.66)	20.0 (448.89)
Contemporaneous LTV > .80	0.091 (0.36)	-.078 (0.14)
House price volatility	-35.06 (24.42)	-19.51 (3.00)
N	36,980	29,828
Likelihood ratio	865.60	637.27

We show Wald chi-squares in brackets.

Panel B. Using event histories until June, 2000 for both cohorts

	1992 cohort	1997 cohort	Both cohorts combined
Log of mortgage balance in year at risk	0.11 (0.01)	0.084 (0.009)	0.89 (0.06)
Contemporaneous loan-to-value	-0.01 (0.004)	-0.006 (0.004)	0.02 (.003)
Present value of payments to balance ratio	17.8 (0.82)	20.0 (0.95)	12.9 (0.49)
Contemporaneous LTV > .80	0.34 (0.14)	-0.078 (0.12)	0.27 (0.09)
N	69,108	29,828	98,936
Log likelihood	-4,374	-4,601	-15,074

We show standard errors in brackets. Chi-square statistic that both years combined is the correct model is 5999. To make baselines comparable, means were removed from the mortgage balance and loan-to-value variables.

Table 6. Static pricing of premium mortgages using the mortgage rates on June 30, 1997; June 30, 2000; June 30, 2003

Panel A. Base mortgage rate=8%

Premium	PSA speed						
	0% PSA	100	200	300	400	500	600
40bp	103.8259	102.7005	102.0767	101.6953	101.4421	101.2628	101.1292
80	107.7013	105.4254	104.1670	103.3990	102.8899	102.5296	102.2615
120	111.6236	108.1731	106.2698	105.1104	104.3428	103.8001	103.3966
160	115.5902	110.9420	108.3841	106.8287	105.8004	105.0740	104.5341
200	119.5985	113.7307	110.5090	108.5535	107.2622	106.3509	105.6740

Panel B. Base mortgage rate=8.5%

Premium	0% PSA		PSA speed		0% PSA		0% PSA	
	0% PSA	0% PSA	0% PSA	0% PSA	0% PSA	0% PSA	0% PSA	0% PSA
40bp	103.7097	102.6380	102.0392	101.6707	101.4247	101.2497	101.1190	
80	107.4635	105.2978	104.0906	103.3488	102.8544	102.5031	102.2408	
120	111.2589	107.9778	106.1531	105.0339	104.2888	103.7598	103.3651	
160	115.0936	110.6769	108.2260	106.7252	105.7274	105.0196	104.4917	
200	118.9652	113.3934	110.3084	108.4223	107.1698	106.2820	105.6203	

Panel C. Base mortgage rate=5.25%

Premium	PSA speed						
	0% PSA	100	200	300	400	500	600
40bp	104.5331	103.0700	102.2930	101.8349	101.5394	101.3347	101.1849
80	109.1570	106.1837	104.6097	103.6841	103.0884	102.6763	102.3749
120	113.8680	109.3388	106.9486	105.5467	104.6462	104.0241	103.5697
160	118.6626	112.5332	109.3085	107.4219	106.2122	105.3778	104.7687
200	123.5371	115.7647	111.6879	109.3086	107.7858	106.7367	105.9718

Table 7. Static pricing of discount mortgages using the mortgage rates on June 30, 1997; June 30, 2000; June 30, 2003

Panel A. Base mortgage rate=8%							
	0% PSA	100	200	PSA speed 300	400	500	600
Premium							
40bp	96.2263	97.3254	97.9378	98.3137	98.5640	98.7416	98.8741
80	92.5076	94.6785	95.8912	96.6372	97.1346	97.4881	97.7520
120	88.8466	92.0609	93.8613	94.9712	95.7125	96.2398	96.6338
160	85.2461	89.4743	91.8492	93.3166	94.2981	94.9972	95.5200
200	81.7088	86.9204	89.8561	91.6740	92.8921	93.7608	94.4109

Panel B. Base mortgage rate=8.5%							
	0% PSA	100	200	PSA speed 300	400	500	600
Premium							
40bp	96.3369	97.3853	97.9738	98.3375	98.5809	98.7543	98.884
80	92.7231	94.7954	95.9617	96.6838	97.1677	97.5129	97.7714
120	89.1610	92.2319	93.9648	95.0397	95.7612	96.2764	96.6624
160	85.6533	89.6964	91.9839	93.4059	94.3617	95.0450	95.5574
200	82.2027	87.1906	90.0204	91.7832	92.9699	93.8193	94.4567

Panel C. Base mortgage rate=5.25%							
	0% PSA	100	200	300	400	500	600
40bp	95.5611	96.9757	97.7321	98.1805	98.4709	98.6726	98.8206
80	91.2197	93.9994	95.4908	96.3774	96.9527	97.3531	97.6472
120	86.9792	91.0731	93.2774	94.5915	95.4463	96.0421	96.4802
160	82.8425	88.1989	91.0933	92.8241	93.9523	94.7400	95.3201
200	78.8126	85.3788	88.9400	91.0759	92.4715	93.4476	94.1673

Table 8. Dynamic pricing of bond prices using the term structures on June 30, 1997; June 30, 2000; June 30, 2003

Panel A. Valuation on June 30, 1997

	k = 12	k = 24	k = 36	k(12)-k(24)	k(12)-k(36)
zero vol; zero mean rev.	105.3638	104.5173	103.8552	0.85	1.51
12 vol; 0.05 mean rev.	102.5034	101.4975	101.0044	1.01	1.50
16 vol; 0.05 mean rev.	101.3556	100.3825	99.7968	0.97	1.56

Term Structure :

T = [3; 6; 12; 24; 36; 60; 120; 360];

Y = [5.25; 5.34; 5.67; 6.08; 6.25; 6.4; 6.51; 6.8]

Prepayment Assumption : 100PSA + k*(coupon rate – refinancing rate)

Other Conditions : 8% Coupon, 80bp OAS, 119 Initial Spread, 8000 Simulation

Panel B. Valuation on June 30, 2000

	k = 12	k = 24	k = 36	k(12)-k(24)	k(12)-k(36)
zero vol; zero mean rev.	109.8570	106.2997	104.4597	3.56	5.40
12 vol; 0.05 mean rev.	108.7972	106.4004	105.0836	2.40	3.71
16 vol; 0.05 mean rev.	106.7151	105.2810	104.1720	1.43	2.54

Term Structure :

T = [3; 6; 12; 24; 36; 60; 120; 360];

Y = [6.02; 6.51; 6.44; 6.36; 6.30; 6.18; 6.03; 5.90]

Prepayment Assumption : 100PSA + k*(coupon rate – refinancing rate)

Other Conditions : 8.5% Coupon, 80bp OAS, 247 Initial Spread, 8000 Simulation

Panel C. Valuation on June 30, 2003					
	k = 12	k = 24	k = 36	k(12)-k(24)	k(12)-k(36)
zero vol; zero mean rev.	108.2042	108.1732	108.1419	0.03	0.06
12 vol; 0.05 mean rev.	107.1025	106.5360	106.0105	0.57	1.09
16 vol; 0.05 mean rev.	106.5540	105.5834	105.1021	0.97	1.45

Term Structure :

T = [3; 6; 12; 24; 36; 60; 120; 240];

Y = [0.9; 0.98; 1.09; 1.32; 1.66; 2.46; 3.54; 4.52]

Prepayment Assumption : 100PSA + k*(coupon rate – refinancing rate)

Other Conditions : 5.25% Coupon, 80bp OAS, 170 Initial Spread, 8000 Simulation

Figure 1: 30 Year Mortgage Rate and 10-Year Treasury 1992-2003

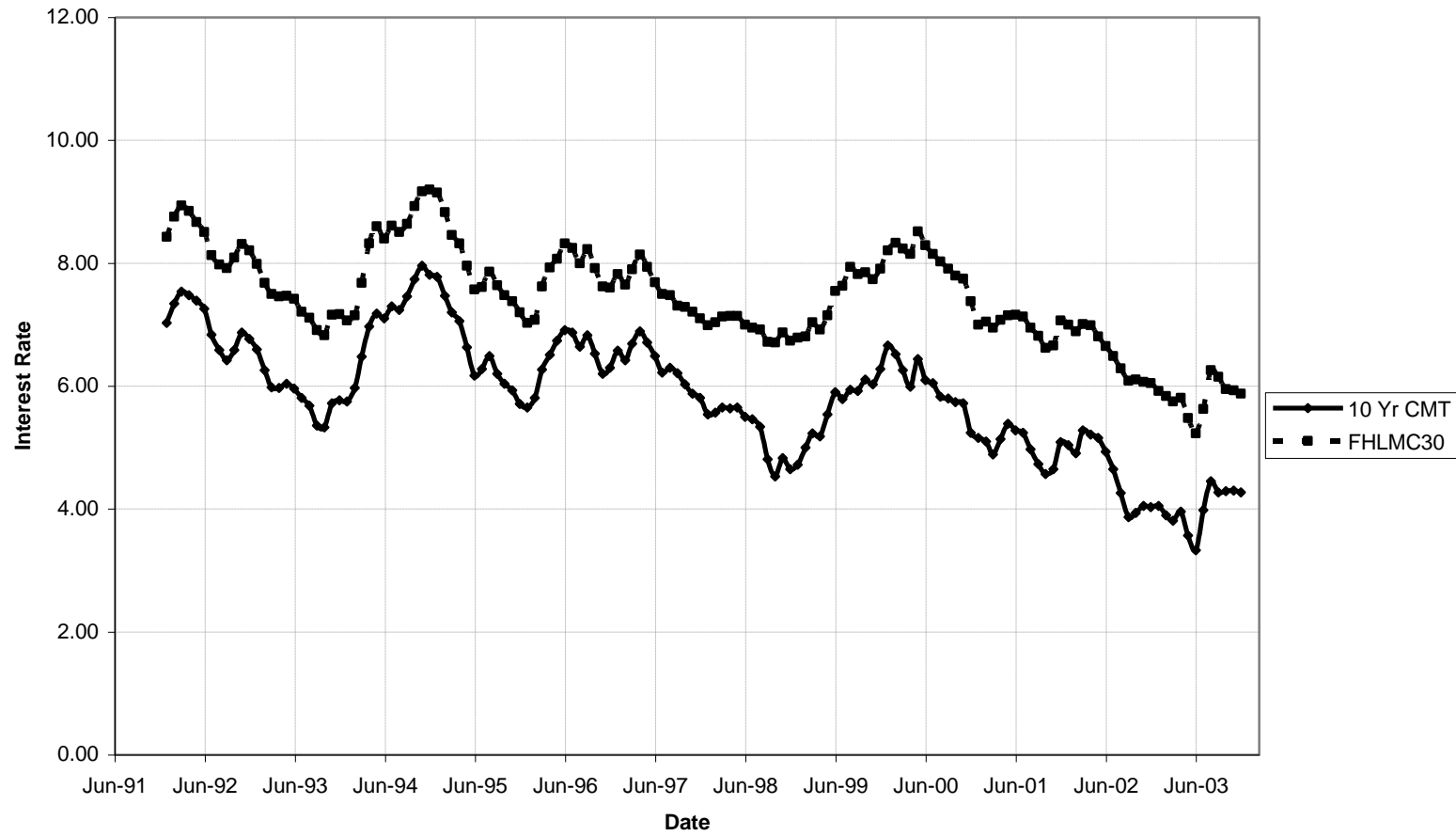
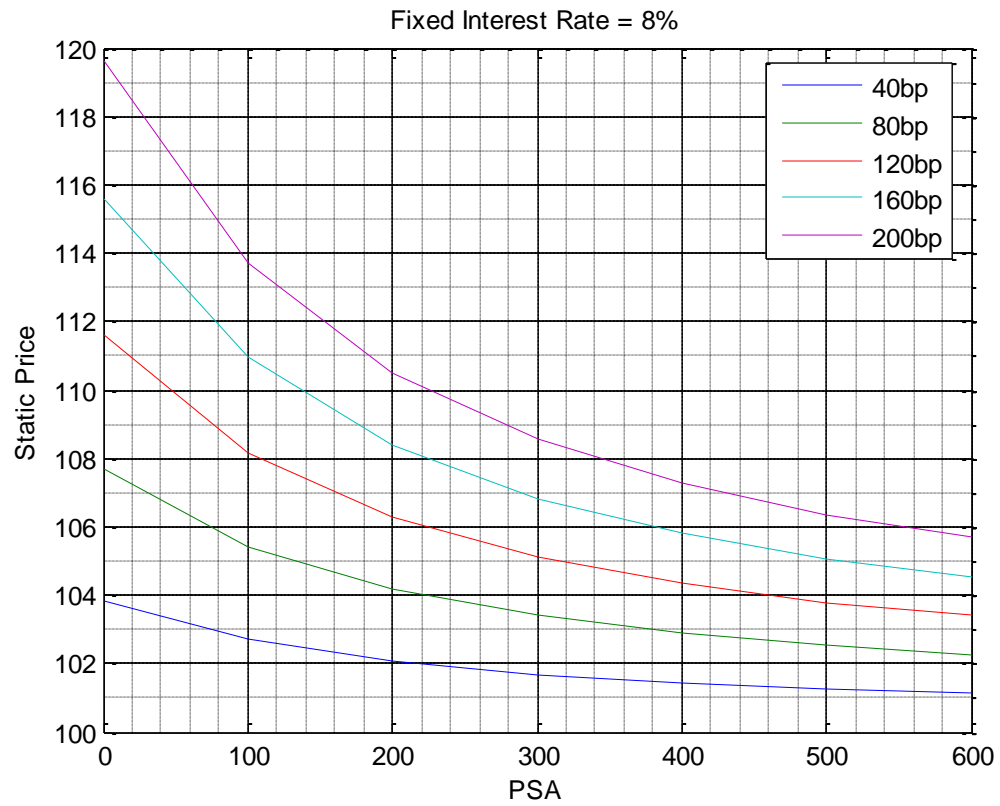
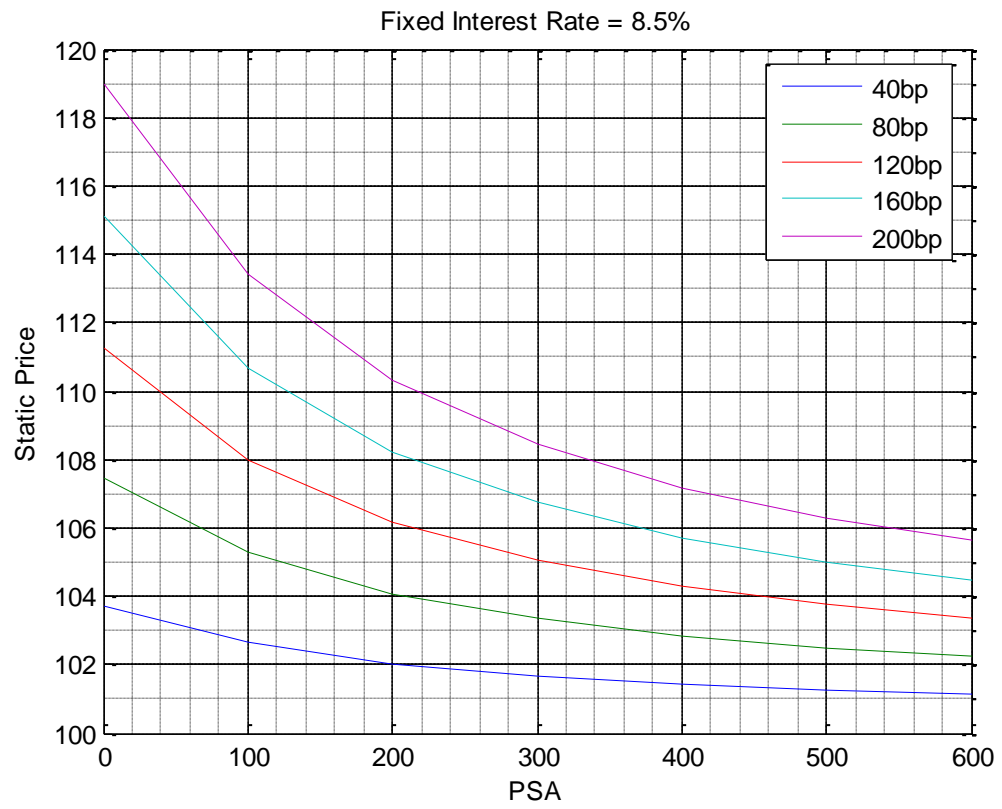


Figure 2. Static pricing of premium bond prices using the mortgage rates on June 30, 1997; June 30, 2000; June 30, 2003





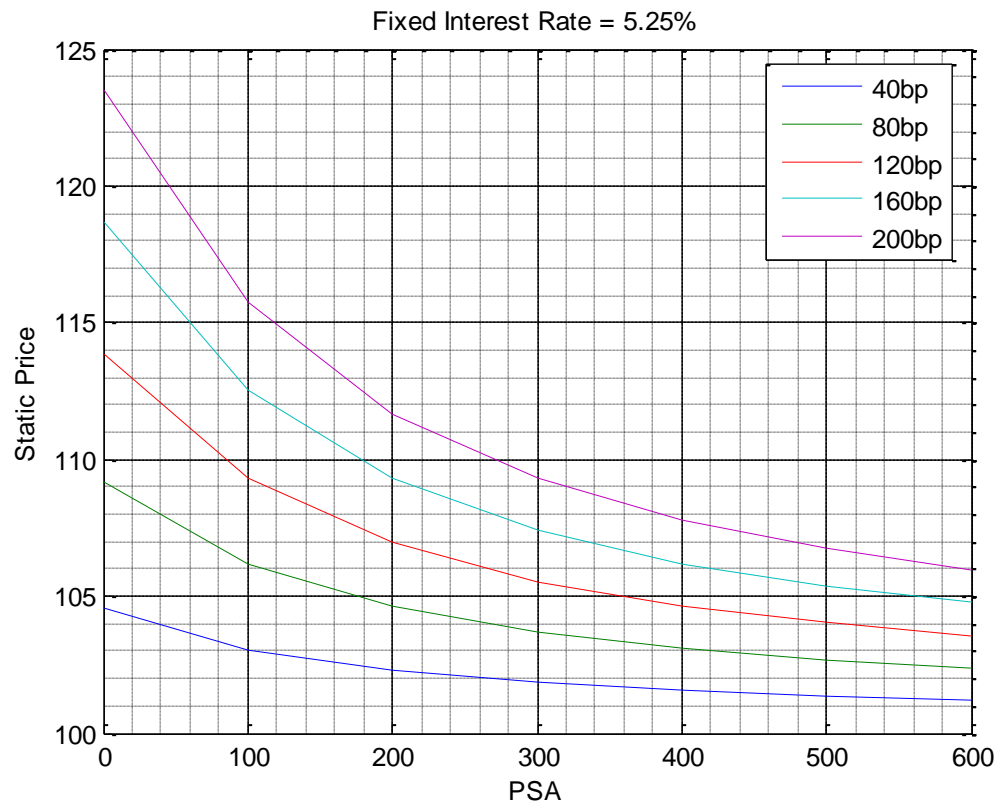
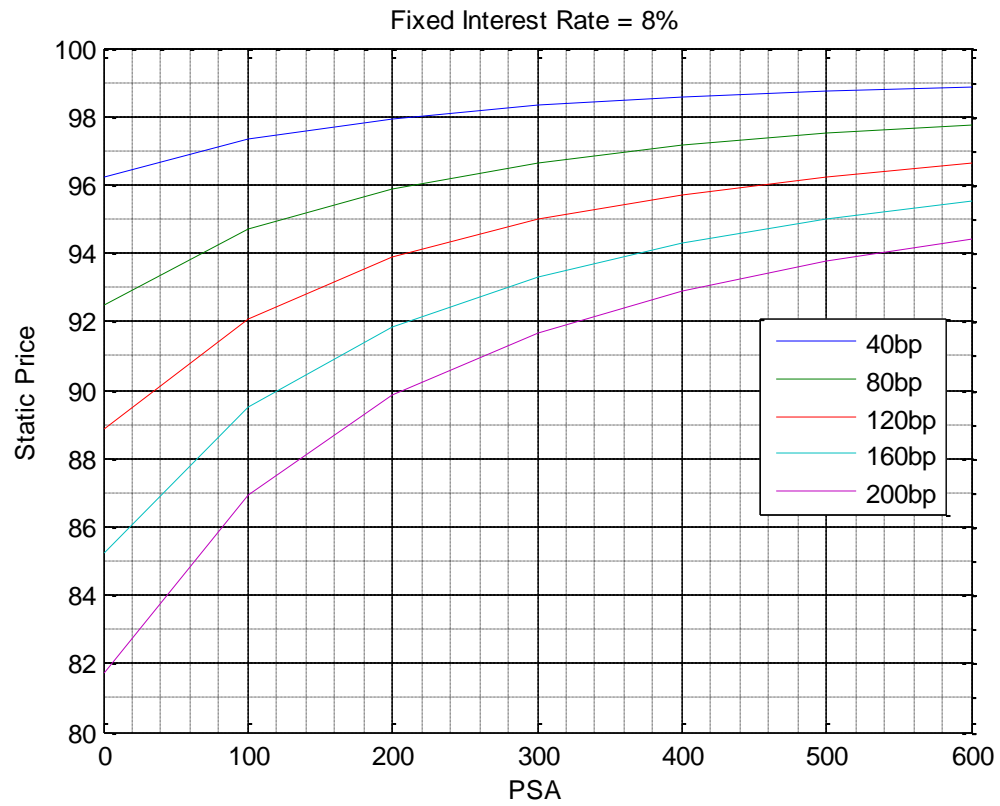
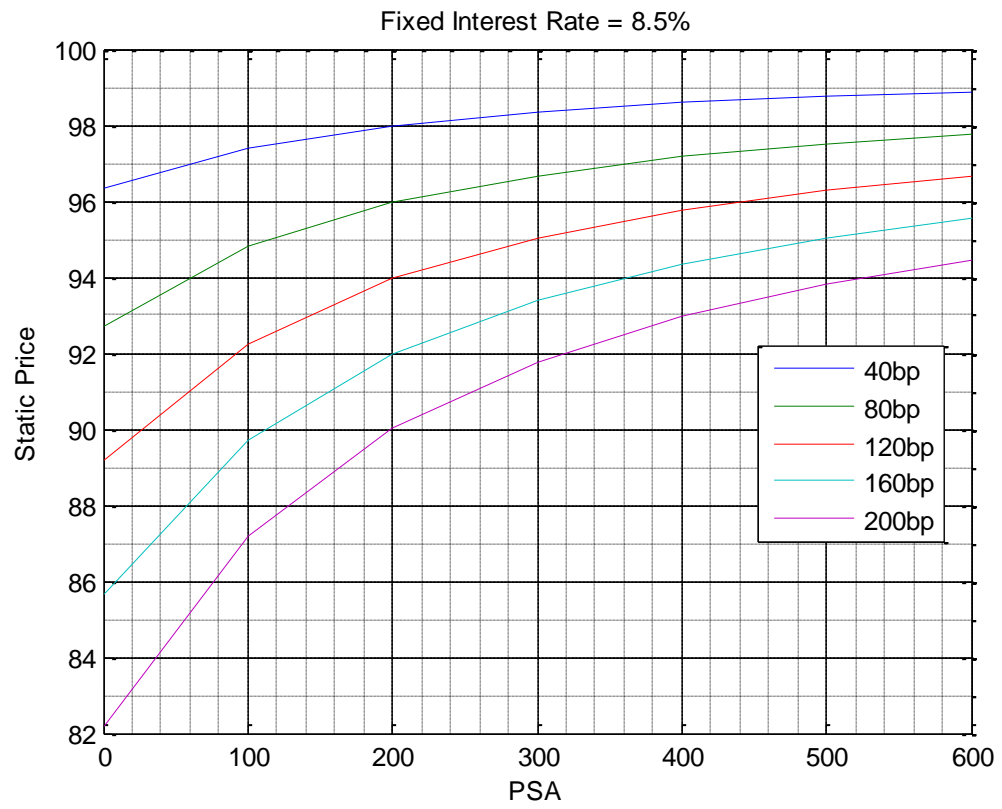


Figure 3. Static pricing of discount bond prices using the mortgage rates on June 30, 1997; June 30, 2000; June 30, 2003





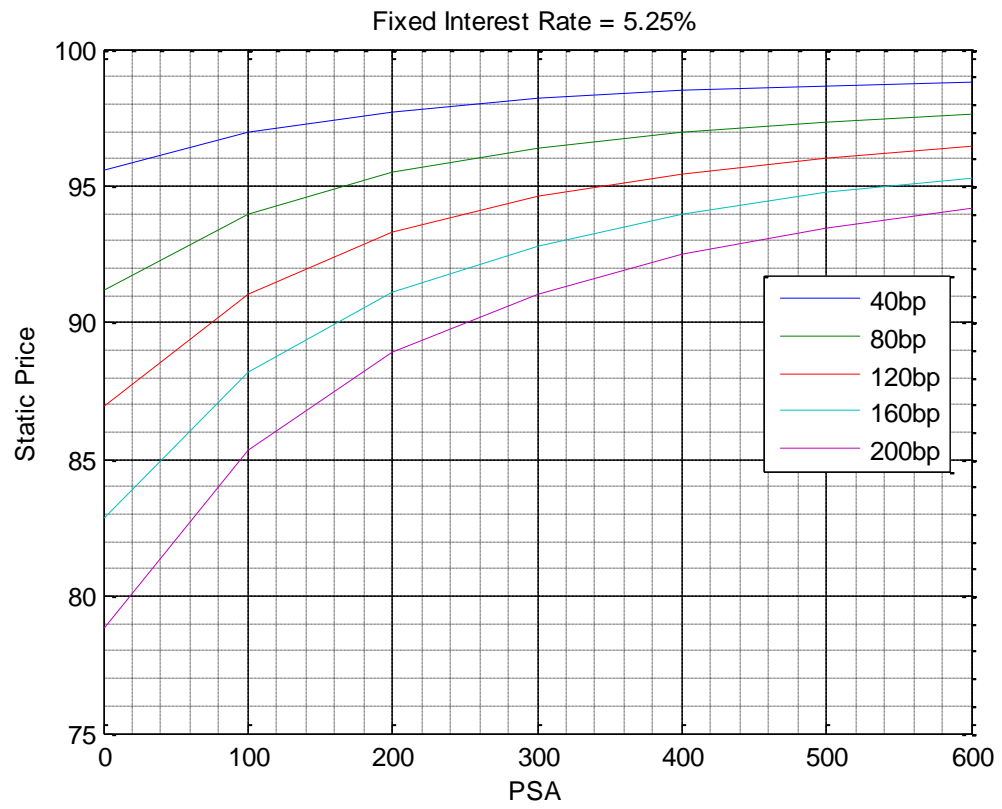


Figure 4. Term structures on June 30, 1997; June 30, 2000; June 30, 2003 and fitted term structures using the quad-spline method as well as Svensson method

