An adaptive nonparametric model for the systematic factors of portfolio credit risk premia*

Flavia Giammarino[a†], Pauline Barrieu [b]

November 26, 2008

[a] Department of Statistics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom. E-mail address: F.Giammarino@lse.ac.uk.

[b] Department of Statistics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom. E-mail address: P.M.Barrieu@lse.ac.uk.

Abstract

The aim of this paper is to investigate the empirical relationship between daily fluctuations in the risk premium for holding a large diversified credit portfolio, which we approximate by a benchmark credit index, and some tradeable market factors which capture systematic risk. The analysis is based on an adaptive nonparametric modelling approach which allows for the data-driven estimation of the nonlinear dynamic relationship between portfolio credit risk premia and their hypothetical components. Our main finding is that the empirical weights of the systematic factors display sudden jumps during market crises and a less intense time-dependent behaviour during normal market conditions. In addition, we find that during market crises the directions of the empirical relationships are often inconsistent with ordinary economic intuition, as they are influenced by the specific circumstances of financial markets distress.

JEL Classification: G01, G10, G14, C14, C22.
Keywords: Credit Risk; Financial Crises; Nonstationary time series; Nonparametric modelling.

1 Introduction

Identifying the systematic and idiosyncratic components of the fluctuations in the risk premium of a portfolio of credit-related products is particularly relevant for the construction of hedging strategies and for the effective diversification of risk. Furthermore, investigating the relationship between the overall risk of a credit portfolio and its systematic determinants is also significant for the development of credit risk management and basket credit derivatives pricing models. Indeed, in credit models the dependence structure of a portfolio is often modelled assuming the existence

*We would like to thank Gary Kendall for providing the data on credit default swap spreads and for his useful suggestions. We are also grateful to Qiwei Yao for his valuable advices and to Rudiger Kiesel for his helpful comments.

†Corresponding author. Tel.: +44 (0)20 7955 6013. Fax: +44 (0)20 7955 7416.

of some common systematic risk factors having a linear influence on the default risk of all the individual counterparties (see, for instance, McNeil et al. (2005) and Schönbucher (2003)).

Although their practical importance, the development of empirical credit risk models has been hindered by the limited availability of credit data. In fact, historical default data are insufficient and inadequate for the purpose of statistical modelling. As an alternative, information on credit risk can be inferred from financial market data, such as corporate bond credit spreads and credit default swap (CDS) spreads. In particular, the credit default swap market represents a more reliable source of credit data, as it is characterized by higher trading volume, liquidity, and informational efficiency than the corporate bond market (Blanco et al., 2005), where prices are often distorted by tax and liquidity issues. Furthermore, the credit default swap market provides forward-looking information, as it reflects investors' expectations on the evolution of default risk over future time periods. Additionally, the information on default risk is provided both at the firm-specific level, by means of single-name credit default swaps, and at the portfolio level, through CDS indices. Specifically, a single-name credit default swap is a financial product leading an investor to pay a periodic premium, referred to as CDS spreads, in return for coverage against the loss deriving from the default of a specific reference entity over a pre-determined time horizon; the entity whose default is being insured can be either a financial or non-financial company or a sovereign counterparty, while common default events covered by CDS contracts are bankruptcy, failure to make payments connected to debt obligations, and corporate restructuring. Nevertheless, market liquidity tends to be more concentrated on the recently launched credit indices, such as the ones of the iTraxx and CDX families, which are traded in higher volumes than the individual CDS products. Credit indices are standardized portfolios of single-name credit default swaps; hence, the observed price of a credit index, which is also referred to as spread, can be regarded as representative of investors' view on the aggregate default risk of a pre-specified credit portfolio over a given time period. It follows that from the point of view of empirical modelling, credit indices offer some advantages compared to single-name CDS products: firstly, their market is more liquid, therefore they are likely to be more reactive to the arrival of new information than the individual CDS products; secondly, credit indices provide an aggregate measure of the default risk associated to a portfolio; therefore, the cross correlations between the individual components are already implied in the observed market prices.

As a consequence, for the purpose of the analysis of this paper we approximate fluctuations in portfolio credit risk premia with the daily returns of the iTraxx Europe index, of which we aim to investigate the systematic risk factors. The choice of iTraxx Europe is due to the fact that, being a benchmark credit index including more than one hundred credit default swaps on corporate entities operating in different industry sectors, its returns, intended as the first differences of the daily spreads, may be a good approximation of the fluctuations in the default risk of a large diversified portfolio of corporate credit exposures, as the one usually hold by main financial institutions. However, while the empirical determinants of transaction prices of single-name credit default swaps were investigated by quite a few studies (see Cossin et al. (2002), Longstaff et al. (2003), Blanco et al. (2005), Ericsson et al. (2004), Norden and Weber (2007) and Das et al. (2008)), the components of daily spread changes of CDS indices were previously analysed only by Bystrom (2008) and Alexander and Kaeck (2008). Bystrom (2008) found a negative relationship between daily spread changes of iTraxx Europe sector indices, current stock returns and lagged stock returns. The same author also found a tendency of credit default swap spreads to widen when stock price volatility increases. Alexander and Kaeck (2008) estimated a Markov-switching model for iTraxx Europe sector indices considering two different regimes corresponding to a higher and a lower level of credit default spreads volatility. They found that interest rates, stock returns and stock volatility have a higher impact on credit default swap spreads in the high-volatility regime.

A common characteristic of earlier empirical studies is that the relationship between movements in CDS spreads and their hypothesized determinants is assumed to be described by a particular functional form, often specified as linear. However, the revealing information on the systematic risk factors is likely to affect the default risk of a credit portfolio, as for instance a credit index, in a nonlinear way, and no prior knowledge on the specific functional form of the relationship is available. As a consequence, in this paper we do not make any assumption on the structure of the relationship between the returns of the credit index and the considered risk factors, which we specify as movement in interest rates, stock index returns and changes in an equity volatility index, but we estimate the regression function nonparametrically. Furthermore, using an adaptive estimation method, we take into account the possibility that the impact of the systematic factors to the overall credit risk of the portfolio may be time-varying, reflecting either a smooth adjustment to the evolution of the credit and economic scenario, or sudden jumps corresponding to extreme and unexpected negative developments in credit markets. Additionally, we allow the volatility of daily credit index fluctuations to be time-varying, reflecting the different degrees of markets uncertainty on the evaluation of default risk.

Estimating the proposed heteroskedastic nonparametric regression model on daily data from November 2004 to January 2008, we find that the European credit market went through several different phases during the considered time period. In particular, the estimation results indicate the presence of prologued tranquil phases interrupted by shorter periods of unusual tensions associated to shocks to the perception of default risk at the systematic level, such as the downgrade of Ford and General Motors in 2005, the slowdown of the US housing market in 2006 and the credit crisis started in 2007. As a consequence, the estimated regression and volatility functions display pronounced time-varying behaviour and sudden discontinuities. In particular, while in normal market conditions the risk factors weights are relatively weak and their signs are coherent with economic intuition and with earlier empirical findings, during periods of market crisis the magnitudes of the estimated relations are significantly modified and their directions are not always consistent with ordinary economic insights, as they reflect the specific issues affecting financial markets during each crisis.

The proposed adaptive nonparametric model results particularly suitable for modelling the features of portfolio default risk premia and in terms of goodness of fit it outperforms both its parametric and non-adaptive counterparts. However, the resulting empirical evidence on the time-inhomogeneous behaviour of the weights of the systematic risk factors poses new issues for risk and portfolio management, in particular regarding the performance and the effectiveness of hedging and diversification strategies.

The remainder of the paper is organized as follows: section 2 describes the adaptive nonparametric modelling methodology, while section 3 presents the details regarding its application to the context of this paper and the criteria used to select the systematic risk factors. The employed dataset is described in section 4 and the estimation results are presented in section 5, while their detailed interpretation is discussed in section 6. Section 7 examines the model performance, while section 8 concludes the paper.

2 Adaptive nonparametric methodology

Adaptive nonparametric methods are particularly suitable to estimate economic relationships characterized by instability and sudden shifts as well as for modelling time-inhomogeneous properties of financial time series. The problem of adaptive nonparametric estimation of a regression function was initially considered by Spokoiny (1998) and Polzehl and Spokoiny (2006). The methodology was successively extended and applied to different areas of financial econometrics, such as volatility modelling (Härdle et al. (2003), Mercurio and Spokoiny (2004) and Spokoiny (2007)), risk management (Chen and Spokoiny, 2007), estimation of copula parameters (Giacomini et al., 2007), nonstationary time series (Čižek et al., 2007) and estimation of the tail of a distribution function (Grama and Spokoiny, 2008).

The development and the application of adaptive nonparametric modelling techniques in financial econometrics is motivated by the growing empirical evidence of the presence of structural breaks and parameter instability in financial and macroeconomic time series (see, for instance, Stock and Watson (1996), Andreou and Ghysels (2002) and Morana and Beltratti (2004)). Indeed, ignoring structural breaks not only leads to neglect information about how economic and financial relations are modified according to the evolution of the economic scenario, political and institutional changes and financial crises, but it also negatively affects parametric statistical inference (Lamoureux and Lastrapes (1990), Mikosch and Stărică (2000) and Pesaran and Timmermann (2004), among others).

On the contrary, adaptive nonparametric methods take automatically into account the time-dependent properties of financial time series and economic relationships, as they are developed to model regression functions and volatility processes characterized by inhomogeneous smoothness properties and unknown number and location of jumps. In particular, in contrast with classical parametric modelling, in which it is assumed that a unique parametric model is adequate to describe a long time series of observations, the adaptive nonparametric approach requires the fulfilment of a parametric assumption only locally; in particular, the degree of locality of the

parametric approximation is selected by means of a change point analysis. In fact, the values of the regression and volatility functions are approximated by fitting simple parametric models to the data belonging to the maximal past interval of each sample point in which no structural break is found. Furthermore, since at each time point the approximating parametric models are estimated using different subsamples of observations, the adaptive estimation naturally accounts for time-varying coefficients. Moreover, adaptive nonparametric estimators are characterized by optimal properties; in particular, if the local parametric models are not seriously misspecified, the quality of the adaptive estimation is equivalent to the one obtained with prior knowledge of the locations of the change points.
Details on the local parametric approximation and on the selection of the estimation intervals are presented in the remaining of this section, in particular in subsections 2.1 and 2.2 respectively, while subsection 2.3 summarizes the adaptive pointwise estimation procedure.

2.1 Local parametric approximation

Consider the following nonparametric regression model with heteroskedastic noise

$$y_t = \mu(t, \mathbf{X}_t) + \sigma(t)\epsilon_t \qquad t = t_0, \ldots, n \tag{2.1}$$

where y_t is the observation of the dependent variable at time t, $\mathbf{X}_t$ is a row vector of observations on explanatory variables at time t, possibly including lagged values of y_t, and the ϵ_t's are i.i.d. random errors with zero mean and unit variance. The functions $\mu(.)$ and $\sigma(.)$ are respectively the unknown mean and volatility processes, on which no assumption is made; the functions are therefore allowed to be nonlinear and to present inhomogeneous degree of smoothness and eventually discontinuities.
The only assumption which is needed for the adaptive nonparametric estimation is that $\mu(.)$ and $\sigma(.)$ can be *locally* approximated by simple parametric models. Precisely, it is assumed that for each t, $t = t_0, \ldots, n$, there exists a past time interval of the form $I(t) = [t - \Delta_t, t]$, $\Delta_t \in \mathbf{N}$, indicated as *interval of time-homogeneity*, in which the time series of the dependent variable y_t nearly follows a simple parametric model $\mathcal{P}_{\theta,\psi}$ with constant parameters θ and ψ (Čižek et al., 2007). For instance, in the intervals of time-homogeneity the regression function may be assumed to belong to the class of linear models with constant coefficient vector θ, while the conditional variance may be described by a local ARCH (Engle (1982)) or GARCH (Bollerslev (1986)) model with parameter vector ψ. Indeed, although ARCH and GARCH models are not robust to structural breaks (see, for instance, Mikosch and Stărică (2000)), when fitted to homogeneous observations they still provide a good description of the stylized properties of financial time-series.
After having specified the form of the local parametric model, the pointwise estimates of $\mu(.)$ and $\sigma(.)$ at time t, indicated by $\hat{\mu}(t, \mathbf{X}_t)$ and $\hat{\sigma}(t)$, are obtained evaluating the approximating parametric model $\mathcal{P}_{\theta,\psi}$ at the maximum likelihood estimates of its coefficients computed on the data belonging to the selected interval of time-homogeneity at time t, namely $\hat{I}(t) = [t - \hat{\Delta}_t, t]$.

It follows that even if the structural form of the approximating parametric model is required to be the same for each t, its estimated parameters will depend on the selected interval of time-homogeneity $\hat{I}(t)$, and therefore on t. Hence, the adaptive pointwise estimation of model (2.1) is naturally associated to time-varying coefficient estimates $\hat{\theta}_{\hat{I}(t)}$ and $\hat{\psi}_{\hat{I}(t)}$.

2.2 Multiscale local change point analysis

The aim of the *multiscale local change point analysis* in the context of adaptive nonparametric estimation is the identification of the longest past interval of every sample point t, of the form $I(t) = [t - \Delta_t, t]$, $\Delta_t \in \mathbf{N}$, in which the time series of the dependent variable can be appropriately approximated by a simple parametric model $\mathcal{P}_{\theta,\psi}$ with constant coefficients θ and ψ.
Precisely, the analysis identifies the intervals in which no statistically significant jump or movement in the parameters θ and ψ of the model is observed, which correspond to the intervals in which the null hypothesis of no structural break is not rejected by a statistical test. Precisely, for each t, the identified longest interval of time-homogeneity, indicated by $\hat{I}(t) = [t - \hat{\Delta}_t, t]$, $\hat{\Delta}_t \in \mathbf{N}$, is selected among the members of a pre-determined increasing family of possible interval candidates which are denoted by $I_0(t), I_1(t), \ldots, I_K(t)$, where $I_K(t)$ is the longest possible past interval of t. The selection is based on the outcomes of Andrew's (Andrews (1993)) supremum likelihood ratio test for a change point of unknown location in nonlinear models, which is applied sequentially to the tested intervals $I_1(t), \ldots, I_K(t)$. The first interval candidate $I_0(t)$ is excluded from the tested intervals, since for technical reasons it is assumed that $I_0(t)$ is always time-homogeneous. The change point test is initially performed on the candidate intervals including the observations near the estimation point t, such as $I_1(t), I_2(t), \ldots$ and then extended to the subsequent intervals of increasing length. This implies that the change points are initially searched among the observations in a *local* past neighbourhood of t, while observations distant from t are progressively included if and only if no structural break was found near t. Indeed, the analysis is discontinued when the interval containing the first change point is detected, or equivalently, the first time that an interval candidate is rejected. Therefore, the selected time-homogeneous interval at time t, namely $\hat{I}(t) = [t - \hat{\Delta}_t, t]$, $\hat{\Delta}_t \in \mathbf{N}$, is the latest non-rejected interval and it corresponds to the longest past interval of t in which no change point was found; this is exactly the interval on which the local parametric model $\mathcal{P}_{\theta,\psi}$ is fitted to obtain $\hat{\theta}_{\hat{I}(t)}$ and $\hat{\psi}_{\hat{I}(t)}$.

2.2.1 Definitions and notation

Before introducing the details of the test procedure it is necessary to clarify the distinction between the *non-adaptive* estimates their *adaptive* counterparts. For a fixed t and a fixed k, $k = 1, \ldots, K$, assume that the multiscale local change point analysis can be discontinued either at $I_k(t)$, or at a shorter interval $I_{k'}(t)$, $k' < k$, as if there were no further intervals to test after $I_k(t)$. The non-adaptive estimates at step k, indicated by $\tilde{\theta}_{I_k(t)}$ and $\tilde{\psi}_{I_k(t)}$, are defined as the estimates computed using all the observations in the interval $I_k(t)$, irrespective of the outcome of the multiscale local change point analysis on $I_k(t)$ and on the preceding shorter candidate intervals $I_{k'}(t)$, $k' < k$. Differently, the adaptive estimates, denoted by $\hat{\theta}_{I_k(t)}$ and $\hat{\psi}_{I_k(t)}$, are always computed using the data belonging to the selected interval of time-homogeneity $\hat{I}(t)$, which might be one of the shorter intervals $I_{k'}(t)$, $k' < k$ and therefore be shorter than $I_k(t)$.

2.2.2 Testing algorithm

First of all, for each t the multiscale local change point analysis requires to specify a family of interval candidates. For this purpose, a geometric grid of points such as $\Delta_k = \Delta_0 a^k$, with $\Delta_0 \in \mathbf{N}$, $a > 1$ and $k = 0, 1, \ldots, K$, is defined (see Čižek et al. (2007)). The specified grid uniquely determines an increasing family of subsets of $[t_0, t]$, as follows

$$I_0(t) = [t - \Delta_0, t], I_1(t) = [t - \Delta_1, t], \ldots, I_K(t) = [t - \Delta_K, t]$$

with $\Delta_k \in \mathbf{N} \quad \forall \quad k = 0, 1, \ldots K$ and where $I_K = [t - \Delta_K, t]$ is the largest past interval of t defined by the grid. Excluding the shortest interval $I_0(t)$, which is always assumed to be time-homogeneous and it is therefore not tested for the presence of change points, the multiscale local change point analysis is implemented sequentially to the increasing intervals $I_1(t)$, $I_2(t)$, ..., $I_K(t)$ until the first structural break is found.

To clarify the testing procedure, assume that the multiscale local change point analysis detected no structural break in $I_1(t), I_2(t), \ldots, I_{k-1}(t)$ and that we want to assess the time-homogeneity of $I_k(t)$. However, $I_k(t)$ is partially overlapping with the shortest interval $I_{k-1}(t)$ since the definition of the grid implies $\Delta_k > \Delta_{k-1}$ and therefore $I_k(t) \supset I_{k-1}(t)$. Moreover, we assumed that the time-homogeneity of $I_{k-1}(t)$ was not-rejected, and therefore no structural break was found the interval $[t - \Delta_{k-1}, t]$. It follows that it is only needed to check whether there is a change point τ in the subinterval $I_k(t) \setminus I_{k-1}(t) = [t - \Delta_k, t - \Delta_{k-1}[$ (see Figure 2.1). To

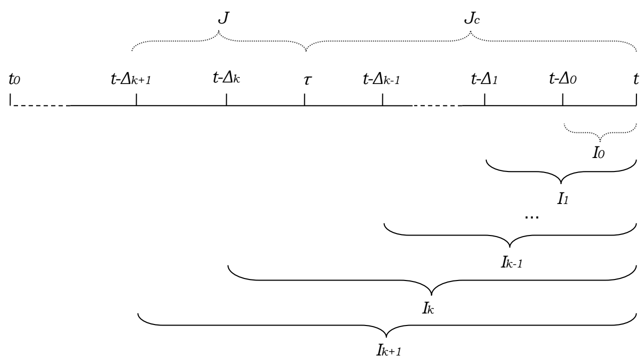


Figure 2.1: Multiscale Local Change Point Analysis

investigate the presence of a structural break τ in $I_k(t) \setminus I_{k-1}(t)$, the multiscale local change point analysis uses the data belonging to the larger testing interval $I_{k+1}(t)$. Precisely, for each potential change point $\tau \in [t-\Delta_k, t-\Delta_{k-1}[$, the testing interval $I_{k+1}(t)$ is divided into two smaller subintervals $J(t) = [t - \Delta_{k+1}, \tau]$ and $J_c(t) = [\tau + 1, t]$. Afterwards, the the local parametric model $\mathcal{P}_{\theta,\psi}$ is fitted on $I_{k+1}(t)$, $J(t)$ and $J_c(t)$ independently, leading to the estimates $\tilde{\theta}_{J(t)}$, $\tilde{\psi}_{J(t)}$, $\tilde{\theta}_{J^c(t)}$, $\tilde{\psi}_{J^c(t)}$, $\tilde{\theta}_{I_{k+1}(t)}$ and $\tilde{\psi}_{I_{k+1}(t)}$ and the associated maximized log-likelihoods $L_{J(t)}(\tilde{\theta}_{J(t)}, \tilde{\psi}_{J(t)})$, $L_{J^c(t)}(\tilde{\theta}_{J^c(t)}, \tilde{\psi}_{J^c(t)})$ and $L_{I_{k+1}(t)}(\tilde{\theta}_{I_{k+1}(t)}, \tilde{\psi}_{I_{k+1}(t)})$. If there is no structural break in $I_k(t)$, fitting the local parametric model on the entire $I_{k+1}(t)$ and fitting it on its two subsamples $J(t)$ and $J^c(t)$ should lead to very similar parameter estimates. As a consequence, the likelihood ratio $L_J(t)(\tilde{\theta}_{J(t)}, \tilde{\psi}_{J(t)}) + L_{J^c(t)}(\tilde{\theta}_{J^c(t)}, \tilde{\psi}_{J^c(t)}) - L_{I_{k+1}(t)}(\tilde{\theta}_{I_{k+1}(t)}, \tilde{\psi}_{I_{k+1}(t)})$ is expected to be small for each $\tau \in [t - \Delta_k, t - \Delta_{k-1}[$. This argument underlies the use of Andrew's test statistic, which corresponds to the supremum of the log-likelihood ratio over all the possible $\tau \in [t-\Delta_k, t-\Delta_{k-1}[$, as defined in equation (2.2).

$$\mathcal{T}_{I_k(t)} = \max_{\tau \in I_k(t) \setminus I_{k-1}(t)} \left\{ L_{J(t)}(\tilde{\theta}_{J(t)}, \tilde{\psi}_{J(t)}) + L_{J^c(t)}(\tilde{\theta}_{J^c(t)}, \tilde{\psi}_{J^c(t)}) - L_{I_{k+1}(t)}(\tilde{\theta}_{I_{k+1}(t)}, \tilde{\psi}_{I_{k+1}(t)}) \right\} \quad (2.2)$$

Accordingly with what stated before, the test rejects for large values of $\mathcal{T}_{I_k(t)}$. Specifically, the null hypothesis of no structural break is rejected when $\mathcal{T}_{I_k(t)} > \xi_k$, where ξ_k is an appropriate critical value depending on the length of the tested interval $I_k(t)$.

2.2.3 Simulation of critical values

Although Andrew's (Andrews (1993)) asymptotic critical values for the supremum likelihood ratio statistic are not appropriate when the test is applied to arbitrary short intervals, as in the case of the adaptive nonparametric estimation, the finite-sample critical values necessary for the multiscale local change point analysis can be generated by simulation. In view of the fact that the arguments supporting the simulation procedure pertain the behaviour of the adaptive pointwise estimator under the null hypothesis of time-homogeneity of the tested interval $I_k(t)$, for all k and for all t, from now on we suppress the dependence of the different quantities on the estimation point t. In fact, under the hypothesis of time-homogeneity the values of the parameter estimates and of the associated maximized log-likelihoods depend only on the number of observations used for the estimation and not on the specific choice of the estimation point t. As a consequence, the critical values $\xi_1, \xi_2, \ldots, \xi_K$ do not depend on t, but only on the lengths of the corresponding tested intervals $I_1, I_2, \ldots, I_K$.

The simulation algorithm is aimed at selecting critical values ξ_k, $k = 1, \ldots, K$ such that the risk of wrongly rejecting a time-homogeneous interval is minimized for all k, $k = 1, \ldots, K$. This is obtained exploiting a non-asymptotic bound for the log-likelihoods of exponential families initially derived by Polzehl and Spokoiny (2006). In particular, the simulation procedure relies on the fact that under the assumption of time-homogeneity the adaptive $\hat{\theta}, \hat{\psi}$ and non-adaptive $\tilde{\theta}, \tilde{\psi}$ estimates, and therefore their associated maximized log-likelihoods $L_{I_k}(\hat{\theta}_{I_k}, \hat{\psi}_{I_k})$ and $L_{I_k}(\tilde{\theta}_{I_k}, \tilde{\psi}_{I_k})$, should coincide with high probability in every interval I_k, $k = 1, \ldots, K$. Precisely, the following condition holds

$$\mathbb{E}_{\theta,\psi}|L_{I_k}(\tilde{\theta}_{I_k},\tilde{\psi}_{I_k}) - L_{I_k}(\hat{\theta}_{I_k},\hat{\psi}_{I_k})|^r \leq \rho\frac{\mathfrak{R}_r(\theta,\psi)}{K} \tag{2.3}$$

for $k = 1, 2, \ldots, K$, where r and ρ are given positive constants and $\mathfrak{R}_r(\theta,\psi)$ is a measure of the risk of estimation defined as follows

$$\mathfrak{R}_r(\theta,\psi) = \max_{k\leq K}\Big\{\mathbb{E}_{\theta,\psi}|L_{I_k}(\tilde{\theta}_{I_k},\tilde{\psi}_{I_k}) - L_{I_k}(\theta_{I_k},\psi_{I_k})|^r\Big\} \tag{2.4}$$

Therefore, the critical values for the supremum likelihood ratio test can be simulated such that they satisfy the inequality in (2.3). In practice, the critical values can be fixed sequentially: considering $\xi_1,\ldots,\xi_{k-1}$ as already fixed and assuming $\xi_{k+1},\ldots,\xi_K = \infty$, the k^{th} critical value ξ_k can be selected as the minimum value which satisfies the following inequality

$$\mathbb{E}_{\theta,\psi}|L_{I_l}(\tilde{\theta}_{I_l},\tilde{\psi}_{I_l}) - L_{I_l}(\hat{\theta}_{I_l}(\xi_1,\ldots,\xi_k),\hat{\psi}_{I_l}(\xi_1,\ldots,\xi_k))|^r \leq \rho\frac{k\mathfrak{R}_r(\theta,\psi)}{K} \tag{2.5}$$

for $l = k,\ldots,K$.

2.3 Adaptive pointwise estimation

The adaptive pointwise estimates of the regression function and of the volatility process of the time series described in equation 2.1 are obtained combining the local parametric approximation presented in section 2.1 with the multiscale local change point analysis explained in section 2.2. In practice, the final adaptive estimates are obtained implementing the following steps for each estimation point $t = 1,\ldots,n$.

- Define an increasing family of past intervals of t as $I_0(t) = [t-\Delta_0, t]$, $I_1(t) = [t-\Delta_1, t]$, $\ldots$, $I_K(t) = [t-\Delta_K, t]$, where $\Delta_k = \Delta_0 a^k$, with $\Delta_0 \in \mathbf{N}$, $a > 1$ and $k = 0, 1, \ldots, K$ where K is the width of the maximal past interval of t.
- Assuming that $I_0(t)$ is a time-homogeneous interval, start the multiscale change point analysis from $I_1(t)$, i.e. compute $\mathcal{T}_{I_1(t)}$.
- If $\mathcal{T}_{I_1(t)} > \xi_1$, reject $I_1(t)$ and compute the maximum likelihood estimates of the coefficients of the local parametric model $\mathcal{P}_{\theta,\psi}$ using only the data on y_t and $\mathbf{X}_t$ belonging to $I_0(t) = [t-\Delta_0, t]$. In this case $I_0(t)$ is the selected interval of time-homogeneity, i.e. $\hat{I}(t)$ with $\hat{\Delta}_t = \Delta_0$.
- Otherwise, if $\mathcal{T}_{I_1(t)} \leq \xi_1$, do not reject $I_1(t)$ and search for change points in the subsequent longer intervals $I_2(t), I_3(t), \ldots$ in the same way.
- If $I_k(t)$, $k = 2,\ldots,K$, is the first rejected interval, i.e. $\mathcal{T}_{I_j(t)} \leq \xi_j \quad \forall \quad j < k$ and $\mathcal{T}_{I_k(t)} > \xi_k$, fit the local parametric model $\mathcal{P}_{\theta,\psi}$ on the observations belonging to $I_{k-1}(t) = [t-\Delta_{k-1}, t]$, since $I_{k-1}(t)$ is the latest not-rejected interval and therefore the selected interval of time-homogeneity $\hat{I}(t)$ with $\hat{\Delta}_t = \Delta_{k-1}$.

- If no past interval of t is rejected, i.e. $\mathcal{T}_{I_k(t)} > \xi_k \ \forall \quad k = 1, \ldots, K$, fit $\mathcal{P}_{\theta,\psi}$ to the longest possible past interval of t, namely $I_K(t)$. In this case the estimated interval of time-homogeneity $\hat{I}(t)$ corresponds to the longest available interval $I_K(t)$, implying $\hat{\Delta}_t = \Delta_K$.

- The adaptive pointwise estimate of the values of the regression function $\hat{\mu}(t, \mathbf{X}_t)$ and of the volatility process $\hat{\sigma}(t)$ at time t are obtained evaluating the local parametric model $\mathcal{P}_{\theta,\psi}$ at the maximum likelihood estimates of its coefficients computed on the data belonging to $\hat{I}(t)$, i.e. $\hat{\theta}_{\hat{I}(t)}$ and $\hat{\psi}_{\hat{I}(t)}$.

3 Model specification and implementation

In this section we describe the actual application of the adaptive nonparametric methodology to the problem considered in this paper, namely estimating the possibly time-varying and nonlinear relationship between daily fluctuations in a credit index and some other financial variables representing sources of systematic risk. In particular, in subsection 3.1 we describe the motivations leading to the selection of the variables which we expect to explain the evolution of default risk, in subsection 3.2 we describe the employed econometric model, while the last subsection (section 3.3) is dedicated to the discussion of the issues pertaining the implementation of the estimation method.

3.1 Selection of the risk factors

The selection of the financial variables which may hypothetically explain the daily fluctuations in credit risk premia relies on the theoretical arguments of Merton's model (Merton (1974)), according to which the credit spread on corporate debt is a decreasing function of the value of the firm, a decreasing function of the default-free interest rate and an increasing function of the business risk of the company as measured by the volatility of firm returns. Although Merton's model is aimed at explaining the credit spread of an individual corporate entity, it is often regarded as a general benchmark model for credit risk, therefore we consider its predictions still applicable in the context of portfolio credit spreads. However, in order to apply its insight to our specific problem, which is explaining daily spread changes of a credit index, instead of using firm-specific stock returns and stock volatilities to approximate changes in firm's value and in the business risk affecting the company, we use market indices. Therefore, in our model changes in firm value are represented by a main stock index, while the overall business risk is represented by an option-implied volatility index. In particular, we prefer implied volatilities to historical volatility measures since they reflects near-term market expectations. Furthermore, earlier empirical evidence supports the fact that option-implied volatility outperforms historical volatility in explaining variation in credit default swap spreads (Benkert (2004)). Regarding the default-free interest rate, in our model it is approximated by a swap rate for 1-year maturity. The motivation for the choice of a swap rate as a proxy for the default-free interest rate is the empirical evidence that the most commonly used government bond yields are often distorted by the incorporation risk premia associated with the low liquidity of the bond market, short sale constraints and tax effects, and that as a consequence the government curve is not considered any more by financial markets as the benchmark for default-free interest rates (Houweling and Vorst (2005)). The specific choice of the swap rate for the shortest available maturity of 1 year is based on the fact that we want our interest rate variable to reflect expectations on the short-term evolution of the economic environment; indeed, longer maturities such as 5 or 10 years may reflect the perspective of an inversion of the current business cycle, and therefore their values may not be consistent with the information on the credit and economic scenario currently reflected by the credit index. Indeed, although the modelled credit index has a relatively long maturity, specifically of 5 years, we expect it to reflect near-term expectations on the economic and credit environment. In fact, the credit index modelled in this paper is less liquidly traded at other maturities, therefore every new information or shock on the present economic scenario is likely to be reflected on the index traded for the longer maturity of 5-year.

3.2 Specification of the local parametric model

In what follows we indicate by $y_t = log(Y_t/Y_{t-1})$ the logarithmic returns of the the credit index at time t, where Y_t is the spread of the credit index on day t; by $r_t = log(R_t/R_{t-1})$ the logarithmic daily changes of the swap rate, where R_t is the level of the swap rate for day t; by $p_t = log(P_t/P_{t-1})$ the daily returns of the stock index, where P_t indicates the price of the index on day t, and by $v_t = log(V_t/V_{t-1})$ the logarithmic first differences of the daily values of the volatility index, denoted by V_t. In addition, we define a row vector containing the values of the selected explanatory variables at time t as follows $\mathbf{X}_t = [r_t \quad s_t \quad v_t]$. To model the relationship between the daily fluctuations of the credit default swap index and their economic determinants we specify the following nonparametric regression with heteroskedastic noise

$$y_t = \mu(t, y_{t-1}, \mathbf{X}_t) + \sigma(t)\epsilon_t \qquad t = t_0, \ldots, n \tag{3.1}$$

where the ϵ_t's are i.i.d. random errors with zero mean and unit variance. Therefore, in addition to the systematic factors defined in section 3.1, we also include among the explanatory variables the first lag of the credit index; indeed, we want to investigate the presence of significant first order serial correlation in the time series of the iTraxx Europe, as this phenomenon was detected in earlier empirical studies (Bystrom (2008) and Alexander and Kaeck (2008)). Regarding the approximating parametric model $\mathcal{P}_{\theta,\psi}$, we select a stationary GARCH(1,1) regression model with Gaussian errors (Bollerslev (1986))

$$\begin{aligned} y_t &= \varphi y_{t-1} + \mathbf{X}_t \boldsymbol{\gamma} + \eta_t \\ \eta_t &= h_t \epsilon_t \\ h_t^2 &= \omega + \alpha \eta_{t-1}^2 + \beta h_{t-1}^2 \end{aligned} \tag{3.2}$$

with $-1 < \varphi < 1$, $\boldsymbol{\gamma} \in \mathbb{R}^3$, $\omega > 0$, $\alpha, \beta \geq 0$, $\alpha + \beta < 1$. The model coefficients $\theta = [\varphi \quad \boldsymbol{\gamma}]'$ and $\psi = [\omega \quad \alpha \quad \beta]'$ are estimated by maximum likelihood under the assumption $\epsilon_t \stackrel{iid}{\sim} \mathrm{N}(0, 1)$. The choice of the stationary GARCH(1,1) regression model is based on the fact that, being the parametric approximation only local and therefore possibly applied to a small number of homogeneous observations, a simple model is enough to describe the properties of the data. Based on the parametric approximation described in equation (3.2), the adaptive estimates at time t are obtained fitting the model in (3.2) to the data belonging to the maximal past interval of t in which the multiscale local change point analysis (described in section 2.2) did not reject the hypothesis of time-homogeneity, namely $\hat{I}(t)$. Hence, the adaptive estimates obtained at time t are as follows

$$\hat{\mu}(t, y_{t-1}, \mathbf{X}_t) = \hat{\varphi}_{\hat{I}(t)} y_{t-1} + \mathbf{X}_t \hat{\boldsymbol{\gamma}}_{\hat{I}(t)} \tag{3.3}$$

$$\hat{\sigma}^2(t) = \hat{\omega}_{\hat{I}(t)} + \hat{\alpha}_{\hat{I}(t)} (y_{t-1} - \hat{\varphi}_{\hat{I}(t)} y_{t-2} - \mathbf{X}_{t-1} \hat{\boldsymbol{\gamma}}_{\hat{I}(t)})^2 + \hat{\beta}_{\hat{I}(t)} \hat{h}^2_{\hat{I}(t)} (t-1) \tag{3.4}$$

An important remark is that the adaptive estimation of $\hat{\mu}(t, \mathbf{X}_t)$ and $\hat{\sigma}(t)$ is independent at each point in time t. Therefore, the dependence structure imposed to the conditional variance by the GARCH(1,1) equation in (3.2) is not extended to the full sample, since the adaptive estimate of the conditional variance at time t, namely $\hat{\sigma}^2(t)$, does not depend on its past value $\hat{\sigma}^2(t-1)$. The independence of the pointwise estimation at each sample point t implies that eventually a global linear parametric model with constant error variance may be recovered as a special case of the nonparametric model in (3.1); in particular, this is obtained when the estimated parameters of the mean equation take on the same value for all t ($\hat{\varphi}_{\hat{I}(t)} = \hat{\varphi}$ and $\hat{\boldsymbol{\gamma}}_{\hat{I}(t)} = \hat{\boldsymbol{\gamma}} \quad \forall t$) and fitting the local GARCH leads to the same estimate of the conditional variance for all t ($\hat{\sigma}^2(t) = \hat{\sigma}^2 \quad \forall t$).

3.3 Implementation details

The actual implementation of the adaptive nonparametric estimation method and of the associated multiscale local change point analysis requires to make some assumptions and to fix some parameters. First of all, in order to obtain a set of testable subintervals for each point in time t, a grid of points, such as $\Delta_k = \Delta_0 a^k$, needs to be defined, and the parameters Δ_0 and a have to be fixed. In particular, the choice of Δ_0 is characterized by a trade-off between smoothness of the parameter estimates and immediacy in the detection of the change points. Indeed, a large value of Δ_0 leads to smoother parameter estimates, but at the same time increases the delay with which change points are detected; on the contrary, a small value of Δ_0 makes the procedure more reactive to the changing structure of the data, but the parameter estimates will be characterized by higher variability. We therefore fix $\Delta_0 = 20$, leading to an average delay of about one month in the detection of structural breaks, and at the same time providing a reasonable number of observations to estimate the local model specified in (3.2). After having fixed $\Delta_0 = 20$, we choose $a = 1.2595$; the particular choice for the value of a is aimed at minimizing the number of neglected observations when defining the candidate intervals of time-homogeneity by means of the geometric grid of points.

Additionally, in order to initialize the change point test procedure described in section 2.2.2, we need to discard some of the first observations. We therefore choose to start the estimation algorithm at $t_0 = 40$. The choice is based on the fact that discarding only a small number of observations makes impossible testing time-homogeneity on past data, while omitting more data leads to neglect more information in the estimation. Therefore, we select a starting point leading to discard approximately two months of data, but providing three testable subintervals.

Furthermore, the critical values for the multiscale local change point analysis are generated according to the method described in section 2.2.3 performing $N = 100$ Monte Carlo simulations. In order to select the parameters r and ρ, we estimate the model using the different values in the sets $r = \{0.5, 1\}$, $\rho = \{0.5, 1, 1.5\}$ (as in Čižek et al. (2007)) and we choose the combination which provides the best fit of the data. In particular, we observe that the choice $r = 1$ and $\rho = 1$ leads to the best performance in terms of the estimated root mean squared error of both the mean and the variance equation. However, as will be shown in section 7.3, table 2, the estimation results are quite insensitive to the choice of r and ρ.

Moreover, in order to simulate critical values it is required to make some hypotheses on the underlying parameter values of the approximating parametric model. Hence, we estimate model (3.2) on moving windows of 100 observations[1] and we set φ, $\boldsymbol{\gamma}$ and ω equal to the their most extreme rolling estimates and α and β equal to their mean estimates[2]. Choosing the most extreme values for φ, $\boldsymbol{\gamma}$ and ω makes the procedure more conservative, since the critical values are generated taking already into account some possible extreme values of the coefficients.

The local GARCH(1,1) regression models (as in equation (3.2)) are estimated using the "garchpq.m" function of the UCSD GARCH toolbox[3] appropriately modified in order to obtain the simultaneous maximum likelihood estimation of both the mean and the variance equations. The various steps of the multiscale local change point analysis, the adaptive pointwise estimation and all the other results presented in the paper are produced with our MATLAB codes.

4 Data

Our data set consists of 804 daily observations on main European credit, equity and volatility indices and on an Euro interest rate; it covers the period from the 22^{nd} of November 2004 to the 21^{st} of January 2008. Although the analysed credit index was launched in June 2004, our available dataset contains only a few discontinuous observations on it before November 2004. Therefore, for the purpose of this analysis we use data from the forth week of November 2004

[1]We start estimating the model at $t = 100$, using the past 100 observations. Then, at $t + 1$ the newest (101^{st}) observation is added to the estimation sample and the oldest (the 1^{st}) observation is removed. We proceed in this way until we reach the end of the sample $t = 804$.

[2]It is not possible to set α and β equal to their most extreme values because this would lead to a violation of the local stationarity assumption.

[3]Downloadable at http://www.kevinsheppard.com/wiki/UCSD_GARCH.

onward. Furthermore, the considered 804 observations refer to trading days only, since holidays were removed before the analysis. After this correction the considered financial time series present no missing values.

4.1 Credit index

The credit index that we aim to model is the iTraxx Europe index. iTraxx Europe is the benchmark credit index in Europe and it is constructed as an equally weighted portfolio of 125 liquidly traded single-name credit default swaps. The CDSs included in the index are related to the debt of European companies belonging to different industry sectors[4]. Moreover, the companies included in iTraxx Europe are investment grade rated, which means that it is expected that they will meet the payment obligations connected to their outstanding debt with very high probability. iTraxx indices are managed by Markit Group Ltd[5], a financial information services company which provides daily credit data. The development, composition and pricing of credit indices is based on Markit's close cooperation with a network of partner banks[6], which also act as market makers for Markit's indices. The components of iTraxx Europe are in fact determined by the contributing banks, which after having identified the CDSs satisfying the requirements of the credit index, determine the inclusion of the products characterized by the highest trading volume over the preceding six months. The index composition is updated every half-year, on March 20^{th} and September 20^{th} (rolling dates), and the basket resulting from the revision is labelled as a new *series* of iTraxx Europe. After the launch of a new series, the earlier series of the index continue to exist, but market liquidity tends to be concentrated on the most recent series which is often referred to as *on the run*. Regarding the pricing of iTraxx Europe, the daily spread of the index is computed by Markit. The spread's calculation is based on the bid and ask quotes provided every day by the contributing banks. If one of the companies included in the index defaults, which means that it either fails to make payments connected to its debt exposure, or it goes bankruptcy or it undergoes a restructuring process, the corresponding CDS is removed from the portfolio. Hence, after a default event iTraxx Europe continues to trade as a reduced basket which is labelled as a new *version* of the index. The iTraxx Europe index is available for 3, 5, 7 and 10-year maturities. However, the index trading for 5-year maturity is the most liquid, therefore we concentrate on it for the analysis. Furthermore, the time series of credit index spreads modelled in this paper is constructed as a concatenation of subsequent *on the run* series of iTraxx Europe; this means that we let each available series of the credit index cover the period from its launch date to the rolling date of the next series. This construction is motivated by the fact that the *on the run* series of iTraxx Europe are the most liquid and therefore more

[4] The following sectors are represented in iTraxx Europe: Autos, Consumers, Energy, Industrials, TMT and Financials.

[5] www.markit.com.

[6] The banks currently contributing to the pricing of iTraxx indices are: ABN AMRO, Bank of America, BNP Paribas, Credit Suisse, Deutsche Bank, Dresdner Klienwort, Goldman Sachs, HSBC, Morgan Stanley, JPMorgan, RBOS and UBS.

informative. We do not notice any significant discontinuity in the time series of credit index spreads (plotted in Figure 4.4 (a)) and of their daily changes (plotted in Figure 5.1 (a)) due to the aggregation of the different series of the credit index.

4.2 Interest rate

To measure fluctuations in interest rates in the Euro area we select the Euro Swap Rate versus Euribor for 1-year maturity. We use daily middle rates downloaded from Datastream (code: ICEIB1Y). Interest rate swaps are among the most widely traded over-the-counter (OTC) derivatives. The swap rate represents the fixed rate that investors are willing to exchange against a floating rate, in our case the Euribor, over a fixed time-horizon, in this case 1-year. Hence, our interest rate variable reflects investors' views about the evolution of the Euribor over the next year and their perception of default risk in the interest rate swap market over the same time-horizon. Indeed, being interest rate swaps OTC products, it is not guaranteed that each counterparty will satisfy the obligations of the contract, namely making regular interest payments; as a consequence, investors face default risk is some extent.

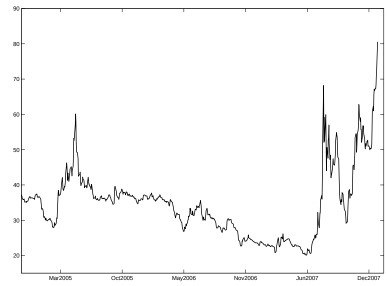
4.3 Stock index

In order to represent the performance of the Euro zone equity market we use the Dow Jones EURO STOXX 50 index. The time series of index closing prices in Euro was downloaded from Datastream (code: DJES50I). The Dow Jones EURO STOXX 50 is a blue-chip index; in fact, its value reflects the performance of 50 large sector leaders characterized by the highest market capitalization. The composition of the index is reviewed every year in September and the components are weighted according to their market capitalization. The choice of the Dow Jones EURO STOXX 50 instead of a broader equity index, such as the Dow Jones STOXX 600, is motivated by the fact that there exists a volatility index based on options on the Dow Jones EURO STOXX 50 which we can use as a further explanatory variable. This allows us to include explanatory variables representing the return and the volatility of the same basket of stocks.

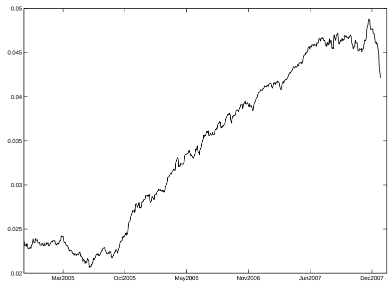
4.4 Volatility index

As option-implied volatility index we choose the Dow Jones VSTOXX 50, which measures the volatility implied in options on the Dow Jones EURO STOXX 50. Option contracts on the Dow Jones EURO STOXX 50 are negotiated on the Eurex derivatives exchange and they are characterized by high trading volume. The value of the Dow Jones VSTOXX 50 is computed by STOXX Ltd[7] such that it has a constant remaining time to expiry of 30 days. Therefore, the value of the Dow Jones VSTOXX 50 reflects near term market expectations on the evolution of the volatility of the European equity market conveyed by market option prices. The closing price of the volatility index were obtained from Datastream (code: VSTOXXI).

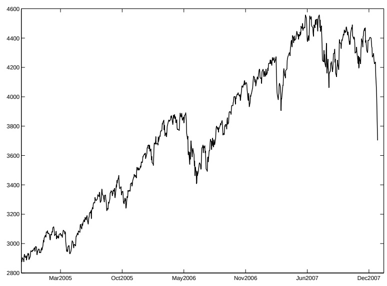
[7] www.stoxx.com.



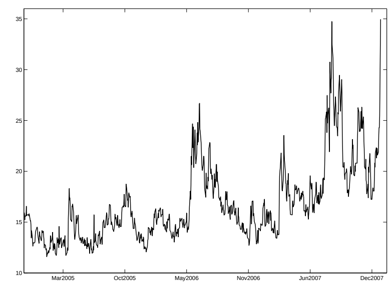
(a) iTraxx Europe (5-Year)



(b) Euro Swap rate vs Euribor (1-Year)



(c) Dow Jones Euro Stoxx 50



(d) Dow Jones VStoxx 50

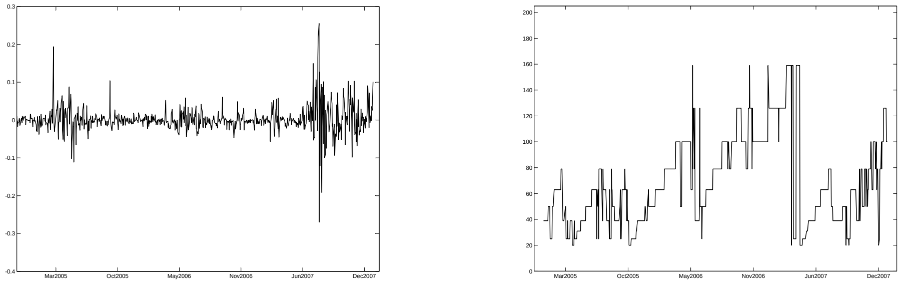
Figure 4.1: Daily levels of the CDS index (a), interest rate (b), stock index (c) and volatility index (d).

5 Empirical results

In this section we present the results of the adaptive pointwise estimation of the nonparametric model in 3.1 by means of a local parametric approximation with a GARCH(1,1) regression (as in 3.2) and the results of the associated change point analysis. In particular, we focus on the presentation and interpretation of the time-dependent impact of the hypothesised systematic risk factors on the daily fluctuations of portfolio credit spreads.

5.1 Detected change points

The multiscale local change point analysis detects several structural breaks delimiting different phases of the credit market. In particular, the results of the testing algorithm highlight the presence of prologued tranquil phases interrupted by shorter periods of unusual tensions associated to significant shocks to the credit market. The duration and the time periods covered by the different regimes of the credit default index are visible in Figure 5.1 (b). In particular, in Figure 5.1 (b) the phases characterized by homogeneous market conditions are represented by the periods in which the width of the selected estimation interval $\hat{I}(t)$ increases with t. When the length of the estimation intervals falls approaching the minimum possible length ($\Delta_0 = 20$), it means



(a) Daily returns of iTraxx Europe

(b) Lengths of the selected estimation intervals

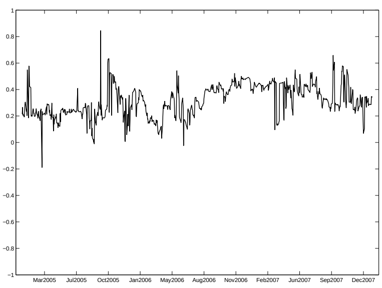
Figure 5.1: Daily returns of the CDS index (a) and lengths of the estimated intervals of time-homogeneity (b).

that a structural break was recently found and that market conditions have changed significantly. Comparing 5.1 (a) and 5.1 (b), it is evident that the detected change points are often associated to shocks to the volatility of the CDS index. In particular, in the considered sample the first change points are detected in April 2005 and October 2005; they correspond to nervous trading in the European CDS index reflecting investors' concerns about the possibly wide consequences of the downgrade of the US car makers Ford and General Motors in May 2005. A minor change point is detected in mid-2006 and it is associated to the worsening US macroeconomic scenario and the deterioration of the US housing market started in the second half of the same year. The latest detected structural breaks delimit the extreme tensions in international financial markets between April 2007 and September 2007 associated to the remarkable losses, writedowns, downgrades and bankruptcies related to the collapse of the subprime industry. The last change point is detected in December 2007 corresponding to a new shock to the interbank market and the responding central banks measures.

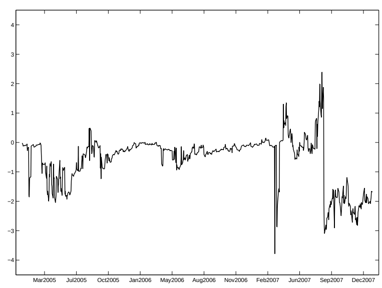
5.2 Adaptive pointwise estimates of the coefficients

The estimated regression and volatility functions of credit default index daily changes display pronounced time-varying behaviour and sudden jumps. As it is visible from Figure 5.2, the values and the dynamics of the estimated coefficients of the systematic factors obtained in normal market conditions are remarkably different from the ones observed in periods of market turbulence. In fact, during tranquil market phases the adaptive estimates of the regression coefficients take on fairly low values and their signs are consistent with the insights of structural credit models. In fact, the estimated coefficients of the interest rate variable (Figure 5.2 (b)) and of the stock market index (Figure 5.2 (c)) are overall negative, while the coefficient of the implied volatility index (Figure 5.2 (d)) is generally positive. On the contrary, during periods of market stress the magnitudes of the estimated relations are significantly modified and their directions are not always consistent with common economic intuition as they reflect the specific issues affecting financial markets during each crisis.

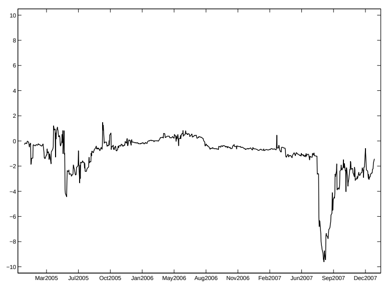
Regarding the time-series properties of the CDS index, we find that its autocorrelation coefficient is not particularly time-dependent (Figure 5.2 (a)) and it oscillates around a value of 0.3, consistently with the results of Alexander and Kaeck (2008). In particular, the first-order serial correlation observed in the daily spread changes of iTraxx Europe may be explained by the mechanism by which the daily price of the index is generated. In fact, as explained in section 4.1, the daily price of iTraxx Europe is the fair price agreed by Markit's contributing banks; therefore, the serial correlation observed in iTraxx Europe's return may be a result of the persistence of operators' expectations on the fair price of the credit index. In contrast, the estimated conditional volatility (Figure 7.1 (b)) appears very reactive to the evolving perception of systematic credit risk and it rapidly increases during periods of market stress. Furthermore, we observed that the estimated coefficients of the local GARCH(1,1) equation are pronouncedly time-varying, indicating non-stationarity in the volatility process. However, the coefficients dynamics do not lead to any relevant interpretation, as for instance we do not observe any clear pattern in the persistence of the volatility process, therefore they are not reported in the paper.



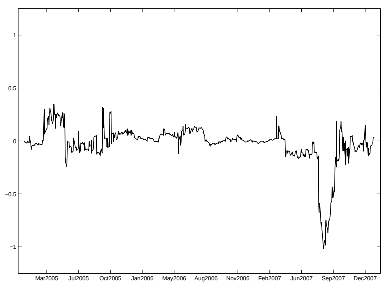
(a) 1^{st} lag of iTraxx Europe $\hat{\phi}(t)$



(b) Euro Swap Rate $\hat{\gamma}_1(t)$



(c) DJ EuroStoxx $\hat{\gamma}_2(t)$



(d) DJ VStoxx $\hat{\gamma}_3(t)$

Figure 5.2: Adaptive pointwise estimates of the risk factors' coefficients.

6 Interpretation

In this section we present a more detailed description and interpretation of the estimated empirical relations between fluctuations of the credit default index, movement in interest rates, stock index returns and changes in option-implied volatility during periods of market stress. The estimation results related to normal market conditions are not discussed in depth since they are consistent with common intuition and with the findings of earlier empirical studies (Bystrom (2008) and Alexander and Kaeck (2008)).

6.1 Downgrade of Ford and General Motors (2005)

The multiscale local change point analysis detects a first structural break in April 2005, when the downward trend of credit default swap spreads was inverted by investors' concerns about disappointing economic news and the negative business outlook on Ford and General Motors. The 5^{th} of May 2005 the two US auto makers were actually downgraded to speculative grade by Standard & Poors. Although the downgrade of Ford and General Motors was an idiosyncratic or possibly an industry-specific issue, the event caused dislocation in credit markets leading to the observation of significant "excess co-movement" (Acharya et al. (2007)) among apparently unrelated CDS spreads. Indeed, although the iTraxx Europe index includes only European companies, and at that time only 9 out of the 125 components were related to the auto sector[8], during the downgrade period the credit index widened by more than 20 basis points and its estimated volatility increased above 7%. A first explanation for the observed contagion among credit markets is the fact that the debt of Ford and General Motors was widely included in many securitized credit products. Therefore, the exposure to the credit risk of the two US car makers was not limited to their bond holders and to the securities related to the auto industry, but it was broadly diffused. A second explanation is related to the scarce liquidity of the corporate bond market, which made very difficult selling-off the riskier Ford and General Motors bonds. As a consequence, financial institutions acting as corporate bond market makers priced the higher default risk that they were forced to hold into the prices of the other securities they intermediated, and in particular into a wide set of credit default swaps (Acharya et al. (2007)), irrespective of the industry or geographical region in which the underlying companies operated their business.
The decline in interest rates and stock prices reflecting investors' concerns about the weaker macroeconomic outlook and the potential broad consequences of the downgrade of a large amount of corporate debt exacerbated the relation between CDS spreads, stock returns and equity implied volatility. Indeed, during the time period between the detected structural breaks all the estimated regression coefficients display pronounced movements. Between March and July the coefficient of the swap interest rate declined to approximately -2, signalling a stronger negative link between widening credit risk premia and declining interest rates. Between March and May the coefficient of the implied volatility index also strengthened, moving above 0.3. In May the estimated coefficient

[8] Bay Motoren Werke, Continental, DaimlerChrysler, FIAT, GKN, Peugeot, Renault, Valeo and Volkswagen.

of stock returns had a short excursion in the positive region, and it jumped back into the negative area (below -4) at the end of the month, when iTraxx Europe began tightening. The estimated positive relation between changes in CDS spreads and the returns of the stock market index is due to the fact that in May both the credit and the stock index increased, signalling opposite view about the state of the economy. In fact, while the spreads of the CDS index had a pronounced and prologued reaction to the fairly deteriorated credit and macroeconomic scenario, the stock index experienced only a short-termed downward correction in April. The change point analysis signals the end of market tensions in October 2005, when a new structural break is detected corresponding to eased economic concerns. Afterwards, the credit market enters a tranquil period characterized by progressively tightening credit default swap spreads.

6.2 Deterioration of the US housing market (2006)

An additional structural break is found in June 2006 following the spring sell-off in credit and equity markets. In particular, the upward shift in equity volatility was probably signalling the uncertainty about the status of the US economy and investors' concerns related to the weakening of the US housing market. Indeed, as measured by the S&P Case-Shiller National Home Price Index, the prices of residential houses in the United States displayed a sharp drop. The slump followed a 86% increase in real value between the end of 1996 to the beginning of 2006 (Shiller (2007)). On the contrary, at that time there were no negative signals about the economic prospects of the Euro area where financial markets were hit by unexpected positive news about economic growth.

The simultaneous decrease in CDS spreads and stock prices leads to the estimation of a positive regression coefficient (above 0.83) for the equity index. In the same period, the estimated coefficient of the swap interest rate drops below -0.93. The coefficient of the equity volatility index is about 0.15 in the same period. The estimated conditional volatility of the CDS index displays only a slight temporary increase above 3%. Therefore, even in this case the change point analysis detects a modification in the relations among EU financial markets corresponding to events related to the US economy. The spillover is likely to be explained not only by the dependence of EU fundamentals on US economic growth, but also by the broad diffusion of structured credit products deriving from the securitization of US mortgages and hence sensitive to the deterioration of the US housing market. The moderate disorder observed in financial markets in mid-2006 was followed by a new tranquil regime of decreasing credit default swap spreads, soaring stock prices and raising interest rates.

6.3 Credit and liquidity crisis (from 2007)

Following the major shocks to the subprime industry in February-March 2007, when several losses affected a number of lenders, a main structural break is detected between the end of March 2007 and the beginning of April 2007. The reassessment of the perception of default risk related to structured products pushed upward the iTraxx Europe index which widened abruptly reaching 68 bps the 30^{th} of July, while its estimated conditional volatility peaked above 30% the 1^{st} of August 2007. While the CDS index was rapidly widening, the retreat from risky assets caused a decline in the stock index, leading to a wide and sudden downward jump in the estimated regression coefficient. In fact, the 16^{th} of August all the estimated regression coefficients take on very extreme values: -9.6 for stock returns, -1.02 for the volatility index and 2.4 for the swap interest rate[9]. Furthermore, in the same period the signs of the coefficients of the interest rate variable and of the stock market volatility are in contradiction with the usual economic intuition. After the intervention of the Federal Reserve, which the 17^{th} of August reduced the discount rate by 0.5%, extended the lending horizon, and broadened the list of accepted collaterals, all the estimated coefficients begin moving back towards their pre-crisis levels, even though the relation of the CDS index with the stock market and with the swap interest rate seems to be still stronger and permanently affected until the end of our sample.

The observed inversion of the empirical relationship between CDS spreads and the swap interest rate in August 2007 is likely to be explained by the liquidity crunch observed in the same period. In particular, the uncertainty regarding banks' exposures to subprime mortgages losses increased the perception and the price of counterparty and default risk in the interbank markets, leading to upward pressures in interest rates and funding liquidity issues. Therefore, in that period the movements in the swap interest rate probably reflected the perception of higher credit risk both in the interbank market and in the interest rate swap market, leading to the estimation of a positive link between increasing interest rates and widening CDS spreads. The negative relationship between credit default swap spreads and stock volatility observed in August 2007 is also possibly related to liquidity issues. In particular, the impossibility of selling-off the riskier structured credit products combined with limited access to the interbank market probably led banks to reduce their leverage unwinding their positions in the more liquid stock market, increasing its volatility. Therefore, being the higher option-implied volatility associated to the wish of financial institutions to preserve their solvency ability, it was negatively associated to the increasing default risk signalled by CDS spreads, resulting in a negative estimated coefficient. Similar arguments explain the exacerbated relation between changes in CDS spreads and stock returns; indeed, the link between falling stock returns and peaking CDS spreads was probably intensified by the activity of leveraged investors such as investment banks and hedge funds unwinding their equity portfolios for risk-reduction purposes (Khandani and Lo, 2007). An additional change point is detected in September 2007, signalling the end of the turmoil experienced by the credit market in August. A last change point is detected in December 2007, possibly corresponding to renewed tensions in the interbank market. Indeed, the Libor peaked in mid-December and this was followed by an additional interest rate cut by the Federal Reserve.

[9] Similar jumps were also observed in a recent study on the credit crunch by the International Monetary Fund (Frank et al. (2008)). In particular, the jumps were observed in the conditional correlations of the daily CDS spreads of 12 major US financial institutions with S&P returns and with the Libor. They also found evidence of a structural break in their multivariate GARCH model at the end of July 2007.

7 Model performance and robustness

In this section we assess the performance of the proposed locally adaptive nonparametric model. First of all, in subsection 7.1 we consider the effectiveness of the local parametric approximation analysing the model residuals. Afterwards, in subsection 7.2 we compare the fit of the data obtained applying the adaptive estimation method with the fits provided by the non-adaptive and global maximum likelihood estimation of the GARCH(1,1) regression model in equation (3.2). In the last subsection (section 7.3) we examine the robustness of the estimation results to the particular selection of the parameters of the multiscale local change point analysis.

7.1 Residual analysis

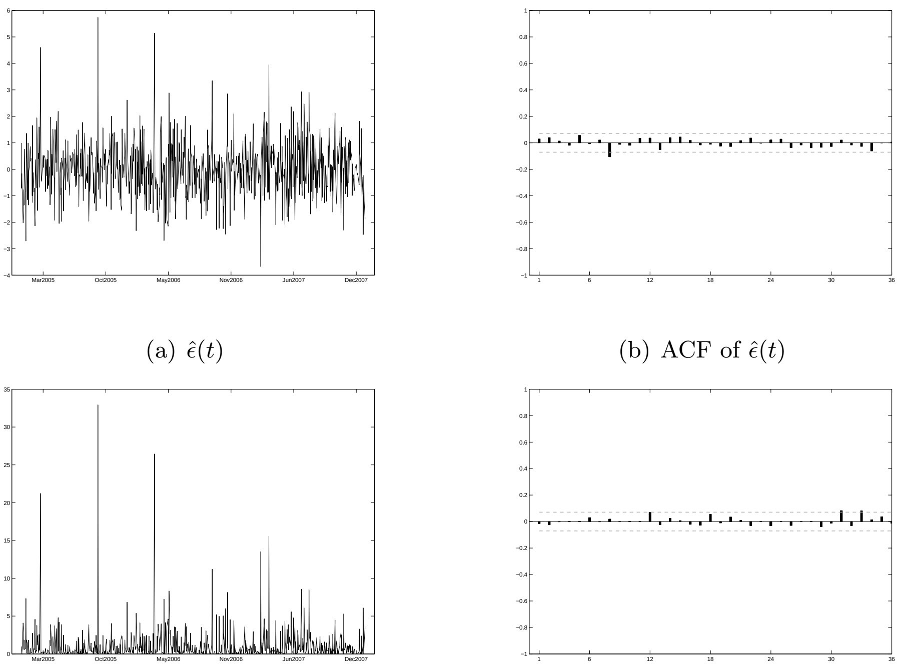
To assess the effectiveness of the pre-specified local parametric approximation, we check whether the standardized residuals resulting from the adaptive estimation behave approximately as i.i.d.. Indeed, if the employed GARCH(1,1) regression model is able to describe effectively the local structure of the time series of the daily changes of the iTraxx Europe index, after correcting its observed values y_t subtracting the estimated mean $\hat{\mu}(t, y_{t-1}, \mathbf{X}_t)$ and dividing by the estimated volatility $\hat{\sigma}(t)$, the resulting time-series of residuals $\hat{\epsilon}_t = \frac{y_t - \hat{\mu}(t, y_{t-1}, \mathbf{X}_t)}{\hat{\sigma}(t)}$ should not display the presence of any additional modelling information, such as serial correlation of their values and of their squares. The standardized residuals are plotted in Figure 7.1 (a), and their estimated autocorrelation function (ACF) is presented in Figure 7.1 (b) together with the upper and lower boundaries of their confidence interval based on asymptotic normality of the sample autocorrelation, namely $(-1.96/\sqrt{n}, 1.96/\sqrt{n})$. The squared standardized residuals and their ACFs are plotted in Figure 7.1, (c) and (d). Overall, we find that the local parametric approximation based on the GARCH(1,1) regression model is quite effective since the standardized model residuals and their squares do not display any significant serial correlation.

7.2 Goodness-of-fit

To evaluate the goodness of fit the model we measure the error magnitude by means of the root mean squared error. Hence, we measure the error of the regression function and the prediction error of the estimated variance as

$$RMSE(\hat{\mu}) = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{\mu}(t, y_{t-1}, \mathbf{X}_t) - y_t)^2} \quad RMSE(\hat{\sigma}^2) = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma}(t)^2 - \hat{\eta}_t^2)^2}$$

The adaptive nonparametric model based on a local GARCH(1,1) parametric approximation provides a good fit for the conditional mean and volatility processes of daily spread changes of the iTraxx Europe index; in fact, the RMSE of the regression function is 2.8505% while the error of the variance amounts to 0.3373%. As shown in Table 1, in terms of goodness of fit the adaptive nonparametric model outperforms its global parametric counterpart, which corresponds to a



(a) $\hat{\epsilon}(t)$

(b) ACF of $\hat{\epsilon}(t)$

(c) $\hat{\epsilon}^2(t)$

(d) ACF of $\hat{\epsilon}^2(t)$

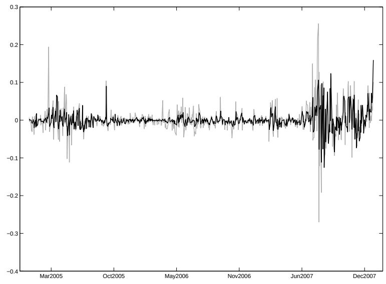
Figure 7.1: Standardized residuals and estimated autocorrelation functions.

GARCH(1,1) regression model estimated in the full sample; this indicates that taking into account the dynamic relationships between financial markets leads to a definitely better performance of the econometric model. Moreover, we compare the performance of the adaptive nonparametric

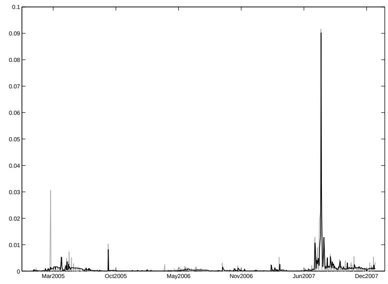
	RMSE($\hat{\mu}$)	RMSE($\hat{\sigma}^2$)
Adaptive r=1 ρ=1	2.8505%	0.3373%
Global Parametric	3.2452%	0.4671%
Non-Adaptive	3.2737%	0.4616%

Table 1: Comparison of goodness of fit measures of the adaptive nonparametric model against the global parametric and non-adaptive alternatives.

model with a non-adaptive alternative, which is a GARCH(1,1) regression model estimated on an increasing sample size. Specifically, the non-adaptive estimates are computed initially fitting the GARCH(1,1) regression model using all the observations up to $t_0 = 40$, obtaining a first vector of coefficient estimates; then at $t_0 + 1$ one more observation is added to the estimation sample and the non-adaptive estimates are computed based on this larger sample size. We proceed in the same way, adding one observation at a time, until the end of the sample at $t = 804$. The non-adaptive model can be seen as a myopic version of the adaptive nonparametric model, since it still produces time-varying coefficients, but the coefficient vector is estimated at each point in time t using all the available past data without taking into account eventual structural breaks. As shown in Table 2, the adaptive nonparametric model outperforms also its non-adaptive version.



(a) Time series of iTraxx Europe (gray line) and adaptive pointwise estimate of the conditional mean $\hat{\mu}(t, y_{t-1}, \mathbf{X}_t)$ (black line)



(b) Squared residuals η_t^2 (gray line) and adaptive pointwise estimate of the conditional variance $\hat{\sigma}(t)^2$ (black line)

Figure 7.2: Actual and fitted values of the first two conditional moments of the daily returns of iTraxx Europe.

7.3 Robustness to the assumptions of the change point test

To assess the sensitivity of the performance of the proposed model to the assumptions made on the parameters used to determine the critical values of the change point testing algorithm, namely r and ρ, we repeat the adaptive pointwise estimation using as input different parameter values. The goodness of fit measures corresponding to different choices of the parameters r and ρ

r	ρ	RMSE ($\hat{\mu}$)	RMSE ($\hat{\sigma}^2$)
0.5	0.5	2.8946%	0.3407%
0.5	1	2.8693%	0.3387%
0.5	1.5	2.8702%	0.3385%
1	0.5	2.9112%	0.3441%
1	1	2.8505%	0.3373%
1	1.5	2.8714%	0.3384%

Table 2: Comparison of goodness of fit measures for the adaptive nonparametric model corresponding to different values of the test parameters r and ρ.

are presented in Table 2. As stated in section 3.3, the combination $r = 1$ and $\rho = 1$ minimizes the RMSE. However, from the results in Table 2 it is evident that the performance of the model is not significantly affected by the choice of r and ρ. Furthermore, from the results in Table 2 it can be noticed that the performance of the adaptive nonparametric model is constantly superior than the performance of a global parametric or non-adaptive model (reported in Table 1), irrespective of the choice of r and ρ.

8 Conclusions

The aim of this paper is to investigate the systematic factors affecting the daily fluctuations in the level of default risk inherent a large diversified credit portfolio. In this empirical analysis the credit portfolio is represented by a benchmark credit index and the corresponding default risk variations are measured by its daily returns. The empirical study is motivated by its practical relevance for hedging, portfolio selection and risk management. The modelling approach is based on the adaptive nonparametric methodology developed by Spokoiny (1998) and Polzehl and Spokoiny (2006) and further extended by Čižek et al. (2007). The application of adaptive nonparametric methods to the problem considered in this paper is based on two orders of considerations. Firstly, the information on systematic risk factors is likely to affect the overall risk of a credit portfolio in a nonlinear way, and no prior information on the functional form of the relationship is available; hence, we do not impose any restriction on the structure of the regression function, which we estimate nonparametrically. Secondly, the structure of the relationship between portfolio credit risk and its systematic factors possibly evolve over time as a consequence of economic and credit cycles and tensions in financial markets, such as the current financial crisis; therefore, we employ an adaptive estimation method which naturally leads to time-varying coefficients and provide estimates of the dates at which the major modifications happened. Additionally, in order to reflect the evolving degree of uncertainty related to the fluctuations of the default risk inherent the portfolio, we allow the volatility of credit index returns to be time-varying. Based on these considerations, we describe the relationship between the daily spread changes of the main iTraxx Europe credit index for 5-year maturity, the Euro swap interest rate versus Euribor for 1-year maturity, EURO STOXX 50 returns and changes in the VSTOXX 50 volatility index, via an adaptive nonparametric regression with heteroskedastic noise. Our estimation results reveal that the relationship between the daily returns of the credit index and the considered systematic factors was affected by several structural breaks between November 2004 and January 2008. In particular, the estimated change points correspond to the dates delimiting the time periods in which the European credit market experienced unusual turbulences, such as in spring 2005, when the bonds of Ford and General Motors were downgraded to junk status, in spring 2006, when the US macroeconomic outlook and housing market unexpectedly deteriorated, and in August 2007, when the credit crisis exacerbated influencing the valuation of other asset prices not directly linked to the credit and subprime-mortgage market, such as stocks and interest rates. As a consequence, the estimated regression coefficients display pronounced time-varying behaviour and sudden jumps. In particular, the most prominent jumps are observed in August 2007, corresponding to the rapid unfolding of the present crisis. Furthermore, in the same period the observed empirical relations between credit spreads, interest rates and stock market volatility are deeply modified; the abrupt adjustment of the risk factors' weights is likely to be associated to the unusual events which characterized financial markets in summer 2007, in particular the dry-up of the interbank market and the massive unwinding of equity positions by leveraged investors trying to reduce their risk. Overall, the proposed adaptive nonparametric model results particularly suitable for describing the state-dependent dynamics of portfolio default risk premia. Further research might investigate the effect of the time-inhomogeneous behaviour of the weights of the systematic risk factors on portfolio and risk management strategies and models.

References

V. V. Acharya, S. M. Schaefer, and Y. Zhang. Liquidity Risk and Correlation Risk: A Clinical Study of the General Motors and Ford Downgrade of May 2005. CEPR Discussion Paper No. 6619, December 2007.

C. Alexander and A. Kaeck. Regimes Dependent Determinants of Credit Default Swap Spreads. *Journal of Banking & Finance*, 32:1008–1021, 2008.

E. Andreou and E. Ghysels. Detecting Multiple Breaks in Financial Market Volatility Dynamics. *Journal of Applied Econometrics*, 17:579–600, 2002.

D. W. K. Andrews. Tests for Parameter Instability and Structural Change with Unknown Change Point. *Econometrica*, 61(4):821–856, 1993.

C. Benkert. Explaining Credit Default Swap Premia. *Journal of Futures Markets*, 24(1):71–92, 2004.

R. Blanco, S. Brennan, and I. W. Marsh. An Empirical Analysis of the Dynamic Relationship between Investment–Grade Bonds and Credit Default Swaps. *Journal of Finance*, 60(5):2255–2281, 2005.

T. Bollerslev. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31: 307–327, 1986.

H. Bystrom. Credit Default Swaps and Equity Prices: the iTraxx CDS Index Market. In N. Wagner, editor, *Credit Risk – Models, Derivatives, and Management*, volume 6 of *Financial Mathematics Series*, pages 69–84. Chapman & Hall, 2008.

Y. Chen and V. Spokoiny. Modeling and Estimation for Nonstationary Time Series with Applications to Robust Risk Management. *Econometrics Journal (to appear)*, 2007.

P. Čižek, W. Härdle, and V. Spokoiny. Adaptive Pointwise Estimation in Time-Inhomogeneous Time-Series Models. *Econometric Theory (to appear)*, 2007.

D. Cossin, T. Hricko, D. Aunon-Herin, and Z. Huang. Exploring for the Determinants of Credit Risk in Credit Default Swap Transaction Data: Is Fixed-Income Markets Information Sufficient to Evaluate Credit Risk? FAME Research Paper No. 65, December 2002.

S. R. Das, P. Hanouna, and A. Sarin. Accounting based versus market based cross–sectional models of CDS spreads. *Journal of Banking and Finance (to appear)*, 2008.

R. F. Engle. Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4):987–1008, 1982.

J. Ericsson, K. Jacobs, and R. A. Oviedo. The Determinants of Credit Default Swap Premia. CIRANO Working Paper No. 2004s-55, 2004.

N. Frank, B. Gonzàlex-Hermosillo, and H. Hesse. Transmission of Liquidity Shocks: Evidence from the 2007 Subprime Crisis. IMF Working Paper No. WP/08/200, 2008.

E. Giacomini, W. Härdle, and V. Spokoiny. Inhomogeneous Dependency Modelling with Time Varying Copulae. *Journal of Business & Economic Statistics (to appear)*, 2007.

I. Grama and V. Spokoiny. Adaptive Estimation of the Excess of a Distribution Function. *Annals of Statistics (to appear)*, 2008.

W. Härdle, H. Herwartz, and V. Spokoiny. Time Inhomogeneous Multiple Volatility Modelling. *Journal of Financial Econometrics*, 1(1):55–95, 2003.

P. Houweling and T. Vorst. Pricing Default Swaps: Empirical Evidence. *Journal of International Money and Finance*, 24:1200–1225, 2005.

A. E. Khandani and A. W. Lo. What Happened To The Quants In August 2007? *Working Paper*, 2007.

C. G. Lamoureux and W. D. Lastrapes. Persistence in Variance, Structural Change and the GARCH Model. *Journal of Business & Economic Statistics*, 8(2):225–234, 1990.

F. A. Longstaff, S. Mithal, and E. Neis. The Credit Default Swap Market: Is Credit Protection Priced Correctly? *Working Paper*, 2003.

A. J. McNeil, R. Frey, and P. Embrechts. *Quantitative Risk Management: Concepts, Techniques and Tools*. Princeton University Press, 2005.

D. Mercurio and V. Spokoiny. Statistical Inference for Time-Inhomogeneous Volatility Models. *Annals of Statistics*, 32(2):577–602, 2004.

R. Merton. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance*, 29(2):449–470, 1974.

T. Mikosch and C. Stărică. Change of structure in financial time series, long range dependence and the GARCH model. CAF Working Paper No. 58, 2000.

C. Morana and A. Beltratti. Structural change and long-range dependence in volatility of exchange rates: either, neither or both? *Journal of Empirical Finance*, 11:629–658, 2004.

L. Norden and M. Weber. The Comovement of Credit Default Swap, Bond and Stock Market: an Empirical Analysis. *European Financial Management (forthcoming)*, 2007.

M. H. Pesaran and A. Timmermann. How costly is it to ignore breaks when forecasting the direction of a time series? . *International Journal of Forecasting*, 20:411–425, 2004.

J. Polzehl and V. Spokoiny. Propagation–Separation Approach for Local Likelihood Estimation. *Probability Theory and Related Fields*, 135(3):335–362, 2006.

P. J. Schönbucher. *Credit Derivatives Pricing Models.* Wiley Finance, 2003.

R. Shiller. Understanding Recent Trends in House Prices and Home Ownership. NBER Working Paper No. 13553, 2007.

V. Spokoiny. Estimation of a Function with Discontinuities via Local Polynomial Fit with an Adaptive Window Choice. *Annals of Statistics*, 26(4):1356–1378, 1998.

V. Spokoiny. Multiscale Local Change Point Detection with Applications to Value–at–Risk. *Annals of Statistics (to appear)*, 2007.

J. H. Stock and M. W. Watson. Evidence on Structural Instability in Macroeconomic Time Series Relations. *Journal of Business & Economic Statistics*, 14(1):11–30, 1996.