Extracting implied correlation matrices from index option prices: a statistical approach

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Abstract

We propose a new approach to the model calibration problem, which takes into account the multiplicity of solutions. Starting from a prior distribution on model parameters and a set of observed option prices, we propose a probabilistic construction which yields an arbitrage free pricing rule consistent with these observed option prices. Our approach yields a simple Monte Carlo algorithm for simulating from this posterior distribution, taking into account the value of liquidly traded ("vanilla") options. We also compute the sensitivity of our pricing model to the liquid option prices and provide a hedging portfolio that minimizes the risk of mispricing any exotic payoff. This algorithm can be seen as an arbitrage-free version of the Weighted Monte Carlo algorithm in Avellaneda et al [1], applicable to a wide range of pricing models and products. We illustrate our method by calibrating the correlation matrix of stocks in basket options given a set of observed call options on the basket.

Keywords: Model calibration, inverse problems, Monte Carlo simulations, model uncertainty, Bayesian statistic, convex duality, correlation matrix, basket options.

1 Introduction

The inverse problem of constructing an option pricing model (or risk neutral process) compatible with a set of given market prices of options, known in Finance as the **model calibration** problem, has been treated in the literature either as an exact inversion in presence of continuum data (as in Breeden-Litzenberger [2] or in Dupire [4]) or by applying deterministic optimization methods to minimize the difference between market prices of options and their counterparts in the model. When applied to a given set of market prices, these methods yield a single set of model parameters calibrated to the market, whereas in principle many solutions can exist. Uniqueness of the solution is not granted since the objective function involved in the minimization is typically non-convex: it can admit multiple minima and the computation of the global minimum is not easy with gradient-based methods. Finally, non-uniqueness of the solution is not simply a mathematical nuisance: it reflects **model uncertainty**. The impact of this uncertainty on valuation of exotic options is non-negligible and should be quantified instead of being ignored [3].

We propose a new approach to the model calibration problem, which takes into account the multiplicity of solutions. Starting from a prior distribution on model parameters and a set of observed option prices, we propose a probabilistic construction which yields an arbitrage free pricing rule consistent with these observed option prices. Unlike most calibration methods in the literature which involve numerical minimization of non-convex criteria, our construction only involves the solution of well-posed unconstrained minimization of a convex function, easily performed with gradient-based methods.

Under model uncertainty, the value of an exotic option is not determined by observations of market prices: it must be regarded as a random variable with a non-trivial posterior distribution. Our approach yields a simple Monte Carlo algorithm for simulating from this posterior distribution, taking into account the value of liquidly traded ("vanilla") options. This algorithm can be seen as an arbitrage-free version of Avellaneda's Weighted Monte Carlo algorithm [1], applicable to a wide range of pricing models and products. In addition to the calibration algorithm, our model enables us to compute the sensitivity of the price of any exotic payoff to the prices of liquid options. Those sensitivities can be interpreted as a position in a static portfolio composed by liquid options. We show that this portfolio minimizes the model risk. It is a way to decrease the risk of mispricing any exotic payoff due to model uncertainty.

Our calibration algorithm enables us to extract implied correlation matrices from basket options. We start from a bottom-up approach which consists in calibrating the dynamic of each underlying in the Dow Jones index. Then, we apply our Monte Carlo algorithm to calibrate the correlation matrix using only liquid basket option prices. Then, the pricing of exotic basket options is straightforward.

References

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