Heterogeneous Beliefs and the Vulnerability of Financial Innovation

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Abstract

This paper studies a dynamic equilibrium model of financial innovation with heterogeneous beliefs and imitation. We show that both volume and price of the new security after it is introduced are sensitive to the differing beliefs of participating players. We present conditions for the innovator to continue issuing the new security and for the imitator to enter the market, and identify scenarios when there is no transaction in the market. We illustrate that the market for forward-like securities is rather resilient to the underlying market movement, but the market for option-like contracts appears to be more vulnerable as the supply of the security is sensitive to the underlying market condition and may disappear with a drastic market movement.

Keywords: Financial innovation, imitation, heterogeneous beliefs

JEL Classification Codes: G01, G12, G14, G20.

1 Introduction

Financial innovation has played an important role in the development of financial markets (see, e.g., Allen and Gale (1994) and Tufano (2003)). Recent turmoils in the credit market stemming from mortgage-backed securities, especially the collateralized debt obligation (CDO), however, have raised serious questions about the efficacy and vulnerability of some new financial products. In particular, some of the problems may be attributable to the uncertainty about the underlying asset process faced by issuers, investors and rating agencies alike (Coval, Jurek, and Stafford (2008a) and Hu (2007)). This uncertainty leads to dispersed beliefs about the value of the related derivative securities and, in a sharp market downturn, causes a dramatic decline in trading activities in these securities.

This paper investigates how heterogenous beliefs about the underlying fundamental process affect the sustainability of a new financial product. We focus on the dynamics of the supply of the new security and analyze the role of competition in the presence of an imitator in determining the security offer price and trading volume. Moreover, we examine the vulnerability of different contractual forms of financial innovation subject to competition and belief heterogeneity among market participants. Our analysis in turn yields new insights into financial innovation such as tranched offerings of CDOs.

We develop a dynamic equilibrium model with an innovator who optimally determines the structure of a new financial product in the first period, an imitator who may decide to compete by offering the new product in the second period, and a representative investor who is risk averse and optimally allocates his wealth into the new security. There is uncertainty about the underlying fundamental process, which is represented by an unobservable mean of the unconditional distribution. All players have their own initial beliefs for the underlying process and revise their expectations based on new observations through Bayesian updating.

Specifically, we solve for financial equilibrium in the second period with the given contract structure and the possible entry of an imitator. The competition between the innovator and the imitator is modeled in a Cournot game. We identify the condition, based on the realization of the underlying variable that the security is contracted on, for different participation scenarios for the innovator and the imitator and for the case when the market for the security breaks down, that is when nobody offers the security for sale. In the first period, the innovator optimally chooses appropriate parameters for the contract and determines the offer price in anticipation of the potential entry of the imitator with a different belief.

Through a comparison with a benchmark model in which the innovator and the imitator share the same belief, we find that the security price can deviate from the benchmark price significantly due to the divergence in beliefs. The price is higher if the imitator is relatively more optimistic about the future payoff from the security. This benefits the innovator as her expected profit increases because she sells more of the security at a higher price. The opposite situation holds if the imitator is relatively more pessimistic about the security payoff. While the entry of an imitator does introduce competition that reduces the innovator's monopolistic profit, the potentially better-informed belief the innovator has may still confer significant first-mover advantage. Thus our analysis yields new insights into the question about the first-mover advantage that was raised in Tufano (1989).

Our work builds on the work of Allen and Gale (1991) to establish the financial equilibrium in the presence of financial innovation. We distinguish the role of an imitator by fully developing the intuition sketched in Allen and Gale (1994) and our dynamic framework allows us to examine the benefits for the innovator to be the first mover in a multi-period setting. An important advance in our model is to incorporate heterogeneous beliefs in the context of financial innovation which provides a rich texture for analyzing the vulnerability of newly designed financial securities.

We illustrate the intuition of our model through examples of basic derivative securities, which are building blocks of recent financial innovation. We show that the market for forward-like securities is rather resilient, in the sense that there is almost always a seller of the security, especially if beliefs are diverse. The market for option-like contracts, however, appears to be more vulnerable in that the supply of the security may dry up quickly when the underlying asset value experiences a drastic market movement. Even with a moderate movement in the underlying asset value, the nonlinearity of the payoff structure, coupled with heterogeneous beliefs, renders a sensitive dependence of the availability of the security on the market condition.

To the extent that a path-through security on an underlying collateral pool of assets may resemble a forward contract on the underlying asset value and tranches of the capital structure in CDOs can be viewed as various option-like contracts with claims on the underlying collateral pool, our analysis has implications for their differing behavior under changing market conditions. During the boom market for the underlying assets, both the innovator and the imitator expect earning profits selling the call option (the equity tranche), but they may have to take a loss to sell other option contracts (senior and mezzanine tranches) at prices investors would want to buy. If the loss can be offset by the profit in selling the equity tranches, in addition to the underwriting fees we ignore here, there are incentives for theses sellers to market CDOs. This may provide one potential explanation for the high yields observed for the AAA-rated senior tranches during the heyday of CDO issuance. While Coval, Jurek, and Stafford (2008a) offer an explanation based on the mis-representation of ratings and the unpriced risk of economic catastrophe (see also Coval, Jurek, and Stafford (2008b)), the implication of our analysis may indicate a different, but complementary, motive from the supply side. When the market experiences a large adverse move, expectations of security payoffs from different players can shift quite dramatically, injecting a certain degree of instability into the market and even causing a potential market breakdown. This is reminiscent of the situation in the CDO market during the 2007-2008 credit crisis.

Our work is related to a few strands of literature. In addition to the papers on financial innovation cited above, Rahi and Zigrand (2008) study the role of arbitrageurs in using financial innovation to exploit mispricing across segmented markets. Their model structure shares similar features with ours in the Cournot equilibrium. DeMarzo (2004) deals with the optimality of pooling and tranching in security design when there is information advantage to the issuer. Garmaise (2001) examines the optimal security design when many investors have diverse beliefs, and Axelson (2008) considers the situation in which investors is more informed than issuer and there is competition among investors. However, our paper do not explicitly address the issue of optimal security design. Rather, our model allows all players to have different beliefs and investigates in a dynamic framework how belief heterogeneity affects market prices and volume and impacts the vulnerability of financial innovation of various payoff structures.

Another relevant strand of literature is about conditions for no trading as well as market failure, including papers such as Ausubel (1990), Bhattacharya and Spiegel (1991), Bhattacharya, Reny, and Spiegel (1995) and Milgrom and Stokey (1982). In addition, Morris (1994) explores the structure of no trade theorem with heterogeneous prior beliefs, and presents sufficient and necessary conditions on agents' beliefs for trading to take place. Mukerji and Tallon (2001) assume that each agent is ambiguous about the market in a multi-prior model and find that if the ambiguity aversion of both buyer and seller is very large, both agents may not want to trade, and thus leading to a market breakdown.

Our work is also related to the literature on the first-mover advantage in the context of financial innovation such as Horne (1986), Silber (1983) and Tufano (1989). In particular, Tufano (1989) finds little evidence for the first-mover advantage in financial innovation, despite theoretical arguments in favor of such advantage in Gal-Or (1985) and Gal-Or (1987). Our paper shows that heterogeneous beliefs affect the first-mover advantage through the innovator's security offerings and expected profits.

The rest of paper is organized as follows. In Section 2, we describe the structure of the model and present an analysis of the effect of belief heterogeneity on the equilibrium price, volume and expected profits. We analyze in Section 3 the vulnerability of different derivative securities and discuss implications for tranched offerings of the collateralized debt obligation. We conclude in Section 4. All proofs are collected in the appendix.

2 The Model

There are three players in the model: an innovator, an imitator, and a representative investor. We use $i \in \{n, m, v\}$ to indicate the type of players. Specifically, "n" denotes the innovator, "m" the imitator, and "v" the investor, respectively.

There are three dates: $t = \{0, 1, 2\}$ in the model.[1] At time $t = 0$, the innovator produces a new financial product that has a one-period payoff, $f(x)$, based on the value of an underlying variable x at time $t = 1$ and sells it to the investor. We assume that the type of security issued is known, i.e., the functional form of the payoff, $f(x)$, is fixed. The innovator, however, has the discretion in determining the parameters associated with the contract that affect the allocation of the final cash flow between the issuer and the investor. Hence, the payoff of the security is represented by $f(\{a\}; x)$, with $\{a\}$ being the relevant parameters chosen by the innovator.[2]

At time $t = 1$, an imitator may decide sell the same product to the investor. The innovator will also decide whether to continue offering the same product to the investor. If both decide to sell the product, the price of the security is determined in a Cournot

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[1] One might argue that the framework is relatively limited to investigate the survivor of financial innovation. In fact, as McKelvey and Page (1986) shown that if there are sufficiently many public information of aggregate statistics, rational individuals start out with different priors beliefs will converge to a common posterior belief. Hence, if we study the survivor of financial innovation, we can not assume that there are sufficient learning opportunities and hence the issuers would maintain different beliefs. More time periods does not add insight about the survival of financial innovation.

[2] Henceforth, by the optimal security design in this paper we mean the optimal choices of the contract parameters, as suggested in Allen and Gale (1994), Harris and Raviv (1995).

equilibrium. The equilibrium price also depends on the realization of x in the first period through the conditional expectations of players as discussed later.

There are two marketable assets traded in this model. One is the new security with a one-period payoff of $f(x)$, another is the risk-free security, which is assumed to have a zero risk-free rate with no loss of generality in our analysis.

Both the innovator and the imitator are risk neutral. We assume that the innovator makes her decisions at time $t = 0$ in order to maximize her profits over two periods. The investor is risk averse. For simplicity without much impact on the intuition of our model, we assume that the investor is a myopic mean-variance optimizer and makes her allocation choices at both dates, $t = 0, 1$, by maximizing her expected utility of wealth period by period.

The Belief Structure

We assume that the fundamental distribution for x is normal, i.e., $x = \mu + \eta$ where μ is an unobservable mean and η represents shocks that are of zero mean and independently and identically normally distributed. The variance of η is σ_η^2. Each player correctly models the distribution of η, but has a different estimation of the expected value μ at time $t = 0$. In other words, different players have different priors of μ, which is believed to be normally distributed as

$$\mu_i \sim N(\alpha_{i0}, \sigma_{i0}^2), \tag{1}$$

where $i \in \{n, m, v\}$. We assume that $\sigma_{n0} \leq \sigma_{m0} \leq \sigma_{v0}$ to capture differential sophistication of players. In the ensuing analysis, $\mathbb{E}_i[\cdot]$ is the expectation taken over the posterior distribution of x for player $i \in \{n, m, v\}$.

At time $t = 1$, everyone observes the realization of x_1 and updates their expectation accordingly. The posteriors are then given by, respectively,

$$\mu_i | x_1 \sim N(\alpha_{i1}, \sigma_{i1}^2), \quad i \in \{n, m, v\} \tag{2}$$

where

$$\alpha_{i1} = \alpha_{i0} + \frac{\sigma_{i0}^2}{\sigma_{i0}^2 + \sigma_\eta^2} \left(x_1 - \alpha_{i0}\right), \qquad \sigma_{i1}^2 = \frac{\sigma_{i0}^2 \sigma_\eta^2}{\sigma_{i0}^2 + \sigma_\eta^2}. \tag{3}$$

Therefore, the expectation of player i of x conditional on observing x_1 is based on the posterior distribution given by equation (2) for $i \in \{n, m, v\}$.

The Cost Structure

We consider costs of issuing and marketing the security by both the innovator and the imitator. For the innovator, there is an initial fixed cost of developing the product incurred at time $t = 0$, which we denote as D. In addition, issuing N units of the security incurs a cost of $C_i(N)$, for $i \in \{n, m\}$. We assume that both cost functions $C_n(\cdot)$ and $C_m(\cdot)$ are increasing and (strictly) convex.[3] We also allow that the per-unit cost $C_i(N)/N$ is decreasing with respect to the number of unit N, consistent with the economy of scale of issuing securities.

For simplicity and tractability in the subsequent analysis, we assume a quadratic cost function $C_i(N) = \alpha_i N^2 + \beta_i N + \gamma_i$, where $\alpha_i, \beta_i, \gamma_i > 0$ for each $i \in \{n, m\}$. The quadratic cost structure satisfies the above assumptions when the volume, N, is bounded such that $\alpha_i N^2 \leq \gamma_i$.

2.1 Characterization of the Equilibrium

We derive a Bayesian-Nash equilibrium in this model. At time $t = 0$, the innovator determines the payoff structure $f(x)$ of the new security and its price p for the first period, taking into account of the potential competition from an imitator in the second period as well as the demand schedule from the investor that maximizes her expected utility in the first period. At time $t = 1$, there are potentially two suppliers of the security. The investor updates her demand schedule to maximize her expected utility in the second period. The innovator and the imitator compete in a Cournot game in determining their supply schedules, conditional on their belief structure at time $t = 1$. The equilibrium price at time $t = 1$ is determined by the market clearing condition.

Financial Equilibrium at Time $t = 1$

At $t = 1$, the investor's allocation problem is

$$\max_{\Phi_1(p_1)} \left\{ \mathbb{E}_v \left[W_{v2} | x_1 \right] - \frac{1}{2} \theta Var_v \left[W_{v2} | x_1 \right] \right\}, \tag{4}$$

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[3] As shown in Froot, Scharfstein, and Stein (1993), a convex cost structure follows from a costly reverification model of Townsend (1979).

where $W_{v2} = \Phi_1(p_1)f(x) + (W_{v1} - p_1\Phi_1(p_1))$ is the investor's wealth at $t = 2$, W_{v1} is her wealth at $t = 1$, p_1 is the security price at time one. The optimal demand schedule for the investor at time $t = 1$ is

$$\Phi_1(p_1) = \frac{(\mathbb{E}_v[f(x)|x_1] - p_1)^+}{\theta Var_v[f(x)|x_1]}. \tag{5}$$

For the security to be viable, i.e., $\Phi_1(p_1) > 0$, it is necessary that

$$p_1 < \mathbb{E}_v[f(x)|x_1].$$

The problems faced by the innovator and the imitator are similar:

$$\max_{\Phi_i} \mathbb{E}_i[\Phi_i(p_1)(p_1 - f(x)) - C_i(\Phi_i)|x_1], i \in \{n, m\}. \tag{6}$$

Once the financial equilibrium exists if and only if the market clears at an equilibrium price p_1, i.e.,

$$\Phi_n(p_1) + \Phi_m(p_1) = \Phi_1(p_1). \tag{7}$$

Hence, we have $p_1 = \Phi_1^{-1}(\Phi_n + \Phi_m) = G(\Phi_n + \Phi_m)$, where,

$$G(u) = \mathbb{E}_v[f(x)|x_1] - \theta Var_v[f(x)|x_1]u. \tag{8}$$

We model the competition between the innovator and the imitator through a Cournot equilibrium. Therefore, the first-order necessary conditions of their problems (6) with respect to Φ_n and Φ_m are, respectively,

$$\begin{aligned} p_1 - \mathbb{E}_n[f(x)|x_1] - C_n'(\Phi_n) + \Phi_n G'(\Phi_n + \Phi_m) &= 0, & (9) \\ p_1 - \mathbb{E}_m[f(x)|x_1] - C_m'(\Phi_m) + \Phi_m G'(\Phi_n + \Phi_m) &= 0, & (10) \end{aligned}$$

which jointly determine Φ_n and Φ_m, given the existence of such solutions.

The solutions are given by

$$\Phi_n = \frac{\phi_n}{\Delta}, \qquad \Phi_m = \frac{\phi_m}{\Delta} \tag{11}$$

where

$$\begin{aligned}\phi_n &= 2(\alpha_m + \theta Var_v[f(x)|x_1])(\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n)\\ &\quad -\theta Var_v[f(x)|x_1](\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m),\end{aligned}$$

$$\begin{aligned}\phi_m &= 2(\alpha_n + \theta Var_v[f(x)|x_1])(\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m)\\ &\quad -\theta Var_v[f(x)|x_1](\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n),\end{aligned}$$

and

$$\Delta = 4(\alpha_n + \theta Var_v[f(x)|x_1])(\alpha_m + \theta Var_v[f(x)|x_1]) - (\theta Var_v[f(x)|x_1])^2. \tag{12}$$

Therefore, the financial equilibrium at time $t = 1$ can be characterized by the following proposition:

Proposition 1 *The financial equilibrium at time $t = 1$ depends on the realization of x_1 as follows.*

(1) If $x_1 \in \{\phi_n > 0, \phi_m > 0\}$, then the optimal supplies from the innovator and the imitator are given by equation (11), the optimal demand is determined by equation (7), and the equilibrium price p_1 is given by:

$$\begin{aligned}p_1 := p[x_1] &= \frac{\theta Var_v[f(x)|x_1]}{\Delta}(2\alpha_m + \theta Var_v[f(x)|x_1])(\mathbb{E}_n[f(x)|x_1] + \beta_n) \\ &\quad + \frac{\theta Var_v[f(x)|x_1]}{\Delta}(2\alpha_n + \theta Var_v[f(x)|x_1])(\mathbb{E}_m[f(x)|x_1] + \beta_m) \\ &\quad + \left\{1 - \frac{\theta Var_v[f(x)|x_1]}{\Delta}(2\alpha_m + 2\alpha_n + 2\theta Var_v[f(x)|x_1])\right\}\mathbb{E}_v[f(x)|x_1].\end{aligned} \tag{13}$$

(2) If $x_1 \in \{\phi_n \leq 0, \phi_m > 0\}$, then the innovator does not issue the security at $t = 1$, but the imitator does. The equilibrium price p_1 is given by

$$p_1 := p_m[x_1] = q_m\mathbb{E}_v[f(x)|x_1] + (1 - q_m)(\mathbb{E}_m[f(x)|x_1] + \beta_m), \tag{14}$$

where

$$q_m = \frac{2\alpha_m + \theta Var_v[f(x)|x_1]}{2\alpha_m + 2\theta Var_v[f(x)|x_1]}.$$

The optimal demand is then

$$\Phi_1 = \frac{\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1] - \beta_m}{2\alpha_m + 2\theta Var_v[f(x)|x_1]}. \tag{15}$$

(3) *If $x_1 \in \{\phi_n > 0, \phi_m \leq 0\}$, then only the innovator issues the security at $t = 1$. The equilibrium price p_1 is given by*

$$p_1 := p_n[x_1] = q_n\mathbb{E}_v[f(x)|x_1] + (1 - q_n)(\mathbb{E}_n[f(x)|x_1] + \beta_n), \tag{16}$$

where

$$q_n = \frac{2\alpha_n + \theta Var_v[f(x)|x_1]}{2\alpha_n + 2\theta Var_v[f(x)|x_1]}.$$

The optimal demand is then

$$\Phi_1(p_1) = \frac{\mathbb{E}_v[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1] - \beta_n}{2\alpha_n + 2\theta Var_v[f(x)|x_1]}. \tag{17}$$

(4) *In other scenarios of x_1, with $\phi_n \leq 0, \phi_m \leq 0$, no market equilibrium exists in which the security is sold at $t = 1$.*

Note that $\phi_n \leq 0$ if and only if $\mathbb{E}_n[f(x)|x_1]+\beta_n \geq p_m[x_1]$. This indicates that $\mathbb{E}_n[f(x)|x_1]+\beta_n$ is the reservation price for the innovator to stay in the market. When the market price is below the reservation price, the innovator exits the market. Furthermore, there is no market equilibrium if $\mathbb{E}_n[f(x)|x_1] + \beta_n > \mathbb{E}_v[f(x)|x_1]$. Similarly, the reservation price of the imitator is $\mathbb{E}_m[f(x)|x_1] + \beta_m$. When this reservation price is greater than $p_n[x_1]$, then the imitator stays out of the market at time $t = 1$. If $\mathbb{E}_m[f(x)] + \beta_m < p_n[x_1]$ and $\mathbb{E}_n[f(x)] + \beta_n < p_m[x_1]$, then both the innovator and the imitator are willing to issue the security in a competitive environment, the market price is thus obtained in the first case of Proposition 1.

When the belief of the innovator is the same as that of the imitator, at $t = 1$, there are only two separate cases: either both issue the security or they all stay out of the market. The necessary and the sufficient condition for the issuance is $\mathbb{E}_n[f(x)|x_1] + \beta_n < \mathbb{E}_v[f(x)|x_1]$. This condition depends on the realization of x_1, and may not hold for some realization of x_1. When this happens, the market breaks down with no sellers. When beliefs are heterogeneous, the condition for an active market is more complicated, and we will examine in Section 3 the issue of a possible breakdown in the market for securities with different payoff structures of $f(x)$.

Financial Equilibrium at Time $t = 0$

At $t = 0$, the investor's allocation problem is

$$\max_{\Phi_0(p_0)} \left\{ \mathbb{E}_v \left[\Phi_0(p_0) f(x) + (W_{v0} - p_0 \Phi_0(p_0)) \right] - \frac{1}{2} \theta Var_v \left[\Phi_0(p_0) f(x) + (W_{v0} - p_0 \Phi(p_0)) \right] \right\}. \tag{18}$$

where W_{v0} is the initial wealth of the investor. The optimal demand schedule for the investor at time $t = 0$ is

$$\Phi_0(p_0, f(x)) = \frac{(\mathbb{E}_v[f(x)] - p_0)^+}{\theta Var_v[f(x)]}, \tag{19}$$

with the following feasibility condition on the equilibrium price, p_0:

$$p_0 < \mathbb{E}_v[f(x)]. \tag{20}$$

At $t = 0$, only the innovator offers the new security for sale who also determines its structural parameters $\{a\}$ in the payoff function $f(\{a\}; x)$ and its initial pricing p_0. The problem solved by the innovator is

$$\begin{aligned} \max_{\{p_0, \{a\}\}} \mathbb{E}_n \left[\Phi_0(p_0, f(\{a\}; x_1))(p_0 - f(\{a\}; x_1)) - D - C_0(\Phi_0(p_0, f(\{a\}; x_1))) \right] \\ + \frac{1}{1+\rho} \mathbb{E}_n \left[\Phi_n(p_1)(p_1 - f(\{a\}; x_2)) - C_n(\Phi_n) \right], \end{aligned}$$

where D is the innovation cost, $C_0(\Phi_0)$ is the issuing cost, and p_1 is the price at $t = 1$ determined earlier. x_1 and x_2 are the realizations of x at $t = 1$ and $t = 2$, respectively.

The following proposition characterizes the equilibrium price and volume of the security at $t = 0$.

Proposition 2 *The financial equilibrium at time $t = 0$ is characterized by the equilibrium price*

$$p_0 = \frac{2\alpha_n + \theta Var_v[f(x)]}{2(\alpha_n + \theta Var_v[f(x)])} \mathbb{E}_v[f(x)] + \frac{\theta Var_v[f(x)]}{2(\alpha_n + \theta Var_v[f(x)])} (\mathbb{E}_n[f(x)] + \beta_n), \tag{21}$$

and the volume

$$\Phi_0(p_1) = \frac{1}{2(\alpha_n + \theta Var_v[f(x)])} \{\mathbb{E}_v[f(x)] - (\mathbb{E}_n[f(x)] + \beta_n)\} \tag{22}$$

of the security in the market.

2.2 Analysis of the Model

In this subsection, we analyze the impact of heterogeneous beliefs among sellers of the security on the competition for market shares, the equilibrium price and the total volume. For simplicity, we assume that both the innovator and the imitator have the same cost structures, i.e., $\alpha_n = \alpha_m = \alpha$, and $\beta_n = \beta_m = \beta$.

A Benchmark Model

We start with a benchmark model with homogeneous beliefs in order to establish a base case for the relative advantage between the innovator and the imitator in market shares and profitability, as well as for the level of equilibrium price. In the benchmark model, we assume that the imitator has the same expectation as the innovator, and their priors have the same precision, too, i.e., $\sigma_{n0} = \sigma_{m0}$.

In this benchmark model, the price in the first period p_0 is determined in the same way as in Proposition 2. In the second period, because both the imitator and the innovator have the same cost and belief structures, each will half of the total demand. By Proposition 1, the price in the second period is then

$$p_1^b = \frac{2\theta Var_v[f(x)|x_1]}{2\alpha + 3\theta Var_v[f(x)|x_1]}(\mathbb{E}_n[f(x)|x_1] + \beta) + \frac{2\alpha + \theta Var_v[f(x)|x_1]}{2\alpha + 3\theta Var_v[f(x)|x_1]}\mathbb{E}_v[f(x)|x_1]. \tag{23}$$

Therefore, in the benchmark model, there is no advantage in market shares and profits in the second period for either seller, but the innovator will reap the first-mover advantage in the first period.

If there is no presence of the imitator, then the innovator will be a monopolist in both periods. The price in the second period is depicted in Case 3 of Proposition 1. The following corollary summarizes the effect of the presence of the imitator on the security price, issue volume and expected profit of the innovator.

Corollary 1 1. *The presence of the imitator lowers the security price.*

2. *The total volume of the security is higher in the presence of the imitator. The volume issued by the innovator declines with the entry of the imitator.*

3. *The expected profit of the innovator is reduced in the presence of an imitator.*

Effects of Belief Heterogeneity

When the innovator and the imitator have different beliefs, their supplies of the security will differ, and the price of the security will be affected as well. The following corollary describes the deviation of the security price, p_1, from p_1^b, the price in the benchmark case of homogeneous beliefs.

Corollary 2 *Holding the innovator's belief,* $\mathbb{E}_n[f(x)|x_1]$*, the same as in the benchmark case, and allowing the imitator's belief,* $\mathbb{E}_m[f(x)|x_1]$*, to change, the deviation of the security price,* p_1*, from* p_1^b *in (23) is*

$$p_1 - p_1^b = \frac{\theta \mathcal{V}[x_1]}{2\alpha + 3\theta \mathcal{V}[x_1]} (\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]). \tag{24}$$

where $\mathcal{V}[x_1] = Var_v[f(x)|x_1]$.

This result implies that the security price is higher than the benchmark price if the imitator has a higher valuation of the security payoff than the innovator, although the price increase is only a fraction of the difference in their valuations. The higher price will motivate the innovator to sell more of the security, holding her expectation constant, while reducing the demand from investors. Therefore, the difference in beliefs will affect the relative advantage of the sellers, as demonstrated in the following corollary.

Corollary 3 *Holding the innovator's belief,* $\mathbb{E}_n[f(x)|x_1]$*, the same as in the benchmark case, and allowing the imitator's belief,* $\mathbb{E}_m[f(x)|x_1]$*, to change.*

(1) If $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$*, then compared to their counterparts in the benchmark model of homogeneous beliefs, (i) the volume of the security issued by the innovator is lower; (ii) the total volume of the security issued is higher; and (iii) the innovator's expected profit is higher.*

(2) If $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$*, then compared to their counterparts in the benchmark model of homogeneous beliefs, (i) the volume of the security issued by the innovator is lower; (ii) the total volume of the security issued is higher. However, the shift in the innovator's expected profit is ambiguous.*

Corollary 3 indicates that the divergence of beliefs between the innovator and the imitator impacts the amount of security issuance both by the innovator and in aggregate. It also affects the profits made by the innovator depending on the direction of deviation of the imitator's belief from that of the innovator. The following corollary characterizes the relative advantage between the innovator and the imitator in the issuing volume and profits due to the belief heterogeneity.

Corollary 4 *Holding the innovator's belief,* $\mathbb{E}_n[f(x)|x_1]$*, the same as in the benchmark case, and allowing the imitator's belief,* $\mathbb{E}_m[f(x)|x_1]$*, to change.*

(1) If $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$*, then the innovator issues a larger amount of the security and expects a higher profit than the imitator.*

(2) If $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$*, then the innovator issues a smaller amount of the security and expects a lower profit than the imitator.*

When beliefs of the innovator and the imitator are divergent, the competition from the imitator reduces the amount of the security issued by the innovator from its monopolistic policy, but it increases the total volume of the security issuance. While the competition from the imitator will reduce the innovator's expected profit from its monopolistic level, if the imitator has a low valuation of the security payoff, $f(x)$, the expected profit of the innovator will be higher than that of the imitator.

3 The Vulnerability of Financial Innovation

By the vulnerability of financial innovation, we refer to the likelihood of non-issuance of the new derivative security, which represents a breakdown in the market for the security. In order to investigate this issue more closely, we consider several specific examples of derivative securities, which are building blocks of recent financial innovation. The first one is a forward contract with linear payoff structure $f(x) = ax$, where a is a positive percentage and x is the underlying variable. The second one is a call option with a payoff $\max\{x-L,0\}$. The next one, called a capped forward, has a payoff $\min\{x,K\}$. A capped forward is equivalent to a long position of a forward and a short position of a call option. The last one, called a spread, has the payoff $\max\{x-K,0\}-\max\{x-L,0\}$. A spread is a combination of longing a call option and writing a put option.

We study how heterogeneous beliefs, together with the market realization of x_1 at time one, affect the market for these securities and identify conditions that lead to a market breakdown. We then discuss the implications of our analysis for one prototypical structure of collateralized debt obligations (CDO), and explain how heterogeneous beliefs together with the adverse market realization of x help shed light on the recent credit crisis.

For simplicity we assume that both the innovator and the imitator have the same expectation of the unobservable mean μ but their precisions of such expectations are different. Specifically, $\alpha_{n0}=\alpha_{m0}$, and $\sigma_{n0}<\sigma_{m0}<\sigma_{v0}$. The heterogeneity in beliefs among the innovator and the imitator is thus represented by the divergence between σ_{m0} and σ_{n0}. In addition, we assume that the issuing cost is the same for both the innovator and the imitator.

3.1 A Forward Contract

We first consider a linear payoff structure $f(x)=ax$ where a is a positive percentage parameter. It is easy to see that $\mathbb{E}_i[f(x)]=a\alpha_{i0}$. We assume $a(\alpha_{v0}-\alpha_{n0})>\beta$ such that the innovator is willing to issue the contract at $t=0$.

We focus on the market in the second time period where the belief heterogeneity matters. The following proposition illustrates the effects of belief heterogeneity on seller participation and hence on the market equilibrium.

Proposition 3 *Suppose the security is a forward contract with payoff $f(x)=ax, a>0$. Denote*

$$q=\frac{2\alpha+\theta Var_v[f(x)|x_1]}{2\alpha+2\theta Var_v[f(x)|x_1]},\ g(x)=\frac{x^2}{x^2+\sigma_\eta^2}, \tag{25}$$

where

$$Var_v[f(x)|x_1]=a^2\frac{\sigma_{v0}^2\sigma_\eta^2}{\sigma_{v0}^2+\sigma_\eta^2}. \tag{26}$$

1. *The innovator will continue to issue the forward contract at* $t = 1$ *if and only if* $x_1 > A(\sigma_{m0})$*, where*

$$A(\sigma_{m0}) = \frac{qg(\sigma_{v0})\alpha_{v0} + (1-q)g(\sigma_{m0})\alpha_{n0} - g(\sigma_{n0})\alpha_{n0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a}{qg(\sigma_{v0}) + (1-q)g(\sigma_{m0}) - g(\sigma_{n0})}. \tag{27}$$

2. *If* $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) > g(\sigma_{m0})$*, then the imitator will enter the market and issue the forward contract to the investor, if* $x_1 > B(\sigma_{m0})$*, where*

$$B(\sigma_{m0}) = \frac{qg(\sigma_{v0})\alpha_{v0} + (1-q)g(\sigma_{n0})\alpha_{n0} - g(\sigma_{m0})\alpha_{n0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a}{qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) - g(\sigma_{m0})}. \tag{28}$$

   If $qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}) < g(\sigma_{m0})$*, then the imitator will enter the market and issue the forward contract to the investor, only if* $x_1 < B(\sigma_{m0})$*.*

We call the quantities, $A(\sigma_{m0})$ and $B(\sigma_{m0})$, *participation boundaries,* which are similar to the notion used in Person and Warther (1997) in their analysis of the boom and bust patterns of the adoption of financial innovation. Proposition 3 indicates that the innovator will continue to issue the forward contract at $t = 1$ if and only if the underlying variable for the forward contract is booming in the market, i.e., $x_1 > A(\sigma_{m0})$, whether the imitator enters the market or not. The innovator will stop issuing the contract at $t = 1$ if $x_1 \leq A(\sigma_{m0})$.

The participation decision of the imitator significantly depends on the precision of her prior, σ_{m0}. If the imitator's precision is close to that the innovator, *in the sense that,*

$$g(\sigma_{m0}) < qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}), \tag{29}$$

then the imitator enters the market if and only if $x_1 > B(\sigma_{m0})$. But when the imitator's precision is far greater than that of the innovator such that

$$g(\sigma_{m0}) > qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}), \tag{30}$$

then the imitator's entry decision actually negatively depend on the realized value of x_1. In this case, the imitator issues the forward only when $x_1 \leq B(\sigma_{m0})$.

Proposition 3 helps us characterize the market equilibrium at $t = 1$. First, if a satisfies $a(\alpha_{v0} - \alpha_{n0})(1 - g(\sigma_{v0})) > \beta$, then $A(\sigma_{m0}) > B(\sigma_{m0})$.[4] If the precisions of all players line up such that $g(\sigma_{m0}) < qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0})$, both the innovator and the imitator issue the financial contract to the investor if $x_1 > A(\sigma_{m0})$, but only the imitator issues the security if $B(\sigma_{m0}) < x_1 \leq A(\sigma_{m0})$. The market breaks down, i.e., there is no issuance of the forward contract at $t = 1$, if $x_1 \leq B(\sigma_{m0})$. If the dispersion among precisions is large such that $g(\sigma_{m0}) > qg(\sigma_{v0}) + (1 - q)g(\sigma_{n0})$, then the market breaks down if $B(\sigma_{m0}) < x_1 \leq A(\sigma_{m0})$. Moreover, only the innovator issues the contract if $x_1 > A(\sigma_{m0})$, and only imitator sells the contract in the market if $x_1 < B(\sigma_{m0})$.

Second, if the parameter a satisfies $a(\alpha_{v0} - \alpha_{n0})(1 - g(\sigma_{v0})) \leq \beta$, then $A(\sigma_{m0}) < B(\sigma_{m0})$. When the imitator's belief is similar to the innovator as indicated in (29), the market breaks down if $x_1 \leq A(\sigma_{m0})$, as neither the innovator nor the imitator issues the security. Remarkably, the market for the forward contract never breaks down when the imitator has a more uncertain belief as implied in (30). The intuition is as follows. Because they have different confidence on the unobservable mean of the underlying x, the imitator will enter the market even when the innovator stops issuing the security. It is also possible to see the joint presence of both the innovator and the imitator in the market.

Figure 1 displays the regions for market breakdown as σ_{m0} varies. In this figure, $\sigma_{n0} = 12\%, \sigma_{v0} = 17.5\%$. σ_{m0} is moved between 12% and 17%. $f(x) = ax, a = 0.8$. The cost function $C(x) = 0.01x^2 + 0.06x + 1$. Moreover, $\theta = 0.9, \alpha_{n0} = \alpha_{m0} = 1.2, \alpha_{v0} = 1.5$ and $\sigma_\eta = 20\%$. Hence $a(\sigma_{v0} - \sigma_{n0})(1 - g(\sigma_{v0})) = 0.1359 > \beta$. As shown, $A(\sigma_{m0})$ is increasing and $B(\sigma_{m0})$ is decreasing, both start with 0.6381. The boundary $A(\sigma_{m0})$ lies above on the boundary $B(\sigma_{m0})$. In this example, a critical level of σ_{m0} is 17.47%. When $\sigma_{m0} < 1.24\%$, the condition (29) is satisfied, and the imitator's participation decision is similar to that of the innovator. Hence, the market breaks down when $x_1 \leq B(\sigma_{m0})$. When $\sigma_{m0} > 17.47\%$, the imitator's participation in the market becomes in contrast to the decision of the innovator. The imitator decides to enter the market only when $x_1 < B(\sigma_{m0})$, and the innovator will not continue issuing the security in the second time period. There is no co-existence of the

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[4] Let

$$I(x) := \frac{qg(\sigma_{v0})\alpha_{v0} - q(\alpha_{v0} - \alpha_{n0}) + q\beta/a + \alpha_{n0}x}{qg(\sigma_{v0}) + x}.$$

Then $I(x)$ is increasing if and only if $a(\alpha_{v0} - \alpha_{n0})\frac{\sigma_\eta^2}{\sigma_{v0}^2 + \sigma_\eta^2} > \beta$. Moreover, $(1 - q)g(\sigma_{m0}) - g(\sigma_{n0}) > (1 - q)g(\sigma_{n0}) - g(\sigma_{m0})$. Hence, $a(\alpha_{v0} - \alpha_{n0})(1 - g(\sigma_{v0})) > \beta$ if and only if $A(\sigma_{m0}) > B(\sigma_{m0})$. By the same derivation we also see that $A(\sigma_{m0})$ is increasing while $B(\sigma_{m0})$ is decreasing with respect to σ_{m0}.

innovator and the imitator in the market in this case. The market breaks down when x_1 moves between $B(\sigma_{m0})$ and $A(\sigma_{m0})$.

If the percentage parameter a is small enough, say $a \leq 0.353$, then $a(\sigma_{v0} - \sigma_{n0})(1 - g(\sigma_{v0})) < \beta$. In this case, the market never breaks when $\sigma > 17.47\%$ for the specified forward contract. Figure 2 displays the market breakdown region for a forward contract with payoff $f(x) = 0.35x$.

3.2 Option-like Contracts

We consider several examples of option-like contracts. Let $f_1(x) = \max\{x - L, 0\}, f_2(x) = \min\{x, K\}, f_3(x) = \max\{x - K, 0\} - \max\{x - L, 0\}$ denote the payoffs of these option-like contracts, respectively. We examine the market for each security as a separate example and require

$$\mathbb{E}_n[f_j(x)] + \beta_n \leq \mathbb{E}_v[f_j(x)], j \in \{1, 2, 3\} \tag{31}$$

in order to maintain the viability of the market.

The Market for Call Option Contracts

We first consider the innovator's decision for the issuance of a call option. The following proposition characterizes the innovator's decision under different market conditions.

Proposition 4 *If an innovator issues a call option contract at $t = 0$, she will continue issuing the same product at $t = 1$ when the underlying market condition is strong, that is $x_1 >> 0$.*[5] *When the market condition is weak, i.e., $x_1 << 0$, she will stop the issuance of the call option contract. However, the innovator's decision at $t = 1$ is not monotonic in the underlying variable x_1 for a medium range of market realization, x_1.*

Proposition 4 says that, in a large movement of x_1, the innovator's decision is simple and robust, independent of the belief heterogeneity and the possibility of entry of an imitator. For instance, since the payoff to a deep in-the-money call option is similar to the payoff of

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[5] By $x >> 0$ we mean there exists a constant c such that $x > c$. It essentially means that for sufficiently large x. Similarly $x << 0$ denotes sufficiently small x.

a forward contract when $x_1 >> 0$, the innovator's decision is similar to that in the case of a forward contract. However, the non-linearity of the payoff and belief heterogeneity makes the participation boundary very complex when x_1 is in a moderate range, so the innovator decision is not clear-cut.

Proposition 5 *When the imitator's belief is such that σ_{m0} satisfies (29), she will enter the market for call option contracts if $x_1 >> 0$, but will not when $x_1 << 0$. The entry decision for a medium range of x_1 is indeterminate.*

When the imitator has a far different belief from the innovator, in that σ_{m0} satisfies (30), she will not enter the market when either $x_1 << 0$ or $x_1 >> 0$. Again, the entry decision for a medium range of x_1 is indeterminate.

The intuition behind this proposition is straightforward. When the imitator's belief is not too different from the innovator, such that (29) holds, her decision of entry is similar to that for the innovator to continue. For $x_1 >> 0$, both the innovator and the imitator will issue the call option. On the other hand, when $x_1 << 0$, the market breaks down because there will be no seller in this market.

When the imitator's belief is far different from the innovator's belief, in the sense that

$$g(\sigma_{m0}) > qg(\sigma_{v0}) + (1-q)g(\sigma_{n0}),$$

the imitator's entry decision is different. The imitator does not enter the market even in a very strong market when $x_1 >> 0$. This is because, with a high uncertainty σ_{m0}, the imitator has a high estimation of the payoff from the option contract, and hence of her liability, and the market price is not high enough to make it profitable in expectation for the imitator. So she shies away from the market.

Like the forward contract market, is this possible that the innovator exits from the market while the imitator enters? Proposition 5 indicates that it is only possible for a medium level of x_1. In fact, for a medium level of x_1, the market for call option contracts is in sharp contrast to the market for forward contracts. Proposition 3 implies that the market for forward contracts is resilient. The condition for the non-issuance of call option contracts is more sensitive to the movement of x_1 than that of forward contracts, making the market of call option contracts more vulnerable than the market for forward contracts.

The Market for Capped Forward Contracts

The payoff to a capped forward contract may be expressed as $x - \max\{x - K, 0\}$. Following the discussion above about call option contracts, we can derive the condition for a breakdown in the market for capped forward market contracts as stated in the next proposition.

Proposition 6 *Under a strong market condition at $t = 1$, i.e., $x_1 >> 0$, the market for capped forward contracts breaks down. Under a weak market condition at $t = 1$, i.e., $x_1 << 0$, the innovator will exit the market, but the imitator will enter the market if the imitator's belief is far different from the innovator, in that (30) holds. Otherwise, the market breaks down with $x_1 << 0$ as well.*

This proposition characterizes the entry/exit decisions of the innovator and the imitator in the capped forward market with a large movement in x_1. First, there is no issuer if the market is very strong. Intuitively, a capped forward is similar to a zero-coupon debt in a very strong market situation. The risk and return profile of the zero-coupon bond is then not attractive enough for the issuers. Second, because of its payoff structure, the capped forward is correlated with the call option. By Proposition 4 and 5, the market for the call option breaks down for $x_1 << 0$ when both sellers have similar beliefs, such that (29). Similarly, the capped forward market breaks down for $x_1 << 0$ under the same condition.

The Market for Spread Contracts

Finally, we examine the market for spread contracts at $t = 1$. The next proposition characterizes the condition for a breakdown in the market with a large movement in x_1.

Proposition 7 *The market for spread contracts breaks down at $t = 1$ when there is a large shift in the market situation, that is either $x_1 >> 0$, or $x_1 << 0$.*

This proposition says that with an extreme movement of x_1, either $x_1 >> 0$ and $x_1 << 0$, the innovator would stop issuing the contract and the imitator will not enter the market. Note that in the spread market, as long as $x_1 << 0$, the imitator does not issue the security regardless her belief, unlike in the capped forward market. The proposition implies that the spread market is only available for a moderate range of x_1 at $t = 1$, which is consistent with the payoff profile of the contract.

Summary

The participation zones for the innovator as well as the imitator in these option markets are summarized in Table 1. The table shows that if both the innovator and the imitator have similar beliefs, their participation decisions are also similar, especially conditional on large realizations of x_1. When their beliefs diverge, the belief heterogeneity has important and varied effects on their decisions in these markets, especially those for the option-like contracts.

3.3 Implications for Asset Securitization

Our analysis above has implications for a complex financial innovation: asset securitization. In essence, asset securitization is to pool underlying assets and then issue a prioritized structure of claims, known as *tranches*, against these collateral pools. A prototypical asset securitization in structured finance is the collateralized debt obligation (CDO). There are three prioritized tranches in a typical CDO structure. The tranche with the least priority and bearing the first brunt of losses is called the *equity tranche*, the tranche with the highest priority is the *senior tranche*, and the tranche with a middle priority is *mazzanine tranche*. In the following discussion of the implications of our analysis for the vulnerability of CDO tranches, we abstract from practical institutional intricacies in issuing, monitoring and managing CDO securities.

In its barest form, the payoff to the equity tranche of a CDO structure is represented by a call option, $max\{x-(F-K),0\}$, where x is the value of the underlying collateral pool, F is its face value, and K is the detachment point designating the amount of losses born by the equity tranche.[6] The payoff to the senior tranche is represented by a capped forward contract, $min\{x, F-L\}$, where L is the second detachment point designating the maximum level of losses before the senior tranche will be hit. The payoff to the mezzanine tranche will have a cash flow of $max\{x-(F-L),0\}-max\{x-(F-K),0\}$, similar to that of a spread contract. The detailed correspondence between CDO tranches and option contracts is laid

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[6] In this simple model, the assumption of a normal distribution for x may admit a negative value. This assumption does not affect the intuition of our discussion, and in all likelihood the probability of a negative x is very small.

out in the appendix. In contrast, a forward contract with the payoff $f(x) = ax, a > 0$ can be viewed as a security on the total collateral pool.[7]

Because different CDO tranches are usually sold to separate groups of investors with different risk appetite and investment restrictions, so it is useful to consider different tranches as being transacted in segmented markets. It is in this sense we derive implications of our analysis for the vulnerability of CDO tranches.

As our analysis indicates, when the market condition is poor, i.e, $x_1 << 0$, the innovator stops selling any of the option-like contracts. Hence, the innovator will not be attempted to issue any tranche of the CDO. With this poor market situation, the imitator will not attempt to issue CDO tranches either, unless she has a dispersed prior with a large σ_{m0}. In that case, the imitator may issue the senior tranche, but she will have to keep the mezzanine and equity tranches after constructing a CDO structure. Therefore, with a poor realization of the value of the collateral pool, x_1, the CDO market essentially freezes up with barely any trade.

When the market realization of the underlying collateral value is strong, i.e., $x_1 >> 0$, Proposition 4 implies that the innovator, and perhaps the imitator as well, will keep issuing the equity tranche (the call option), while optimally retaining the senior tranche as well as the mezzanine tranche. Although it is outside of the model itself, it is conceivable that the issuers may be tempted to sell the senior and the mezzanine tranches to investors at lower prices, or higher yields, as long as the potential losses can be offset by the gains from selling the equity tranche and from the benefits of not holding any of these tranches in their inventories. This is reminiscent of the situation during the boom of the CDO market.

The discussion above implies that the CDO market is vulnerable to the extreme movement in the underlying asset market. However, even when the movement in the underlying asset market is moderate, the resilience of the CDO market is affected in a complicated way by the non-linear payoff structure with implicit leverage that can shift rapidly with the changing market condition and by the diversity of beliefs among market participants. For instance, even in a homogeneous belief environment, the innovator and the imitator will issue the equity tranche if and only if

$$G(\mu_n(K, x_1), g(\sigma_{n0})) < G(\mu_v(K, x_1), g(\sigma_{v0})) - \frac{\beta}{\sigma_\eta} \tag{32}$$

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[7] A fractional claim to the underlying pool, in structured finance, is known as a path-through security. We do not go into the details of this type of securitization.

where

$$G(\mu,\sigma^2) = \mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \sqrt{1+\sigma^2}n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$$

and $\mu_i(K, x_1) = \frac{\alpha_{i0}-K+g(\sigma_{i0})(x_1-\alpha_{i0})}{\sigma_\eta}, i \in \{m, n, v\}$.[8] Meanwhile, the necessary and sufficient condition for the innovator and the imitator to issue the senior tranche is

$$G(-\mu_n(L,x_1), g(\sigma_{n0})) > G(-\mu_v(L,x_1), g(\sigma_{v0})) + \frac{\beta}{\sigma_\eta}. \tag{33}$$

The nonlinearity of these conditions illustrates that the participation decision of an issuer of the equity tranche and the senior tranche depends on the realization of x_1 in a non-monotonic way. The heterogeneity in beliefs further increases the complexity of the issuing decision and hence the vulnerability of the security.

For a forward contract or a pass-through security of the collateral pool, the market equilibrium is rather straightforward and robust. Once a critical level of x_1, the participation boundary, is reached, the participation of market players is stable and independent of the level of x_1. Therefore, the market for the pass-through security is resilient. In contrast, for the tranche securities, the market equilibrium sensitively depends on the realization of x_1. It is possible that even a small movement of x_1 could cause the market to freeze up and break down, and then resume as the underlying asset market condition improves. Moreover, either the innovator or the imitator decides to participate in or exist from the market in a disparate fashion as x_1 changes. Therefore, the market for the tranched securities can be rather fickle and vulnerable to the volatility in the underlying asset market.

4 Conclusion

We have presented a dynamic equilibrium model to analyze the effect of heterogeneous beliefs in the presence of imitation on the pricing and sustainability of financial innovation. We show that both volume and price of the new security after it is introduced are sensitive to the differing beliefs of participating players. We identify conditions for the innovator to continue issuing the new security and for the imitator to enter the market, and discuss scenarios when there is no transaction in the market. We argue that with competition from an imitator, the

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[8] It follows from the proof of Proposition 4 in Appendix A.

first-mover advantage can be affected by the relative optimism in the imitator's belief about the underlying asset value with respect to that of the innovator.

Our analysis of several specific types of contracts, i.e., forward- and option-like securities that are building blocks of financial innovation, illustrates the differing vulnerability of these contracts under vary market scenarios. For instance, we show that under an adverse market condition, the market for call option contracts may be unstable as the supply of the security can dry up quickly. In general, the market for forward-like contracts is more resilient to the underlying market movement compared to the market for option-like contracts. Our analysis sheds new light on the impact of heterogeneous beliefs and competition on the pricing and trading of complex financial securities, such as trenched CDO securities, and helps us assess the efficacy of these securities for sharing and managing risk.

Appendix

A. Proofs

In the following proofs, we denote $\mathcal{E}_i \equiv \mathbb{E}_i\left[f(x)\right] + \beta_i$ for $i \in \{m, n, v\}$, with $\beta_v = 0$. Moreover, $\mathcal{V}$ denotes $Var_v[f(x)]$. Similarly, we denote $\mathcal{E}_i[x_1] \equiv \mathbb{E}_i\left[f(x)|x_1\right] + \beta_i$ and $\mathcal{V}[x_1] \equiv Var_v[f(x)|x_1]$ at time $t = 1$ conditional on $x = x_1$.

Proof of Proposition 1

After solving $\Phi_n(p_1)$ and $\Phi_m(p_1)$ in a Cournot equilibrium, and assuming both of $\Phi_n(p_1)$ and $\Phi_m(p_1)$ are positive, then the equilibrium price p_1 follows from equation (5) and (7).

If one of $\Phi_n(p_1)$ and $\Phi_m(p_1)$ is negative, say $\Phi_m \leq 0$, then by the previous derivation, there is no imitator in the market. In this case, the total demand $\Phi_1(p_1)$ is determined by equation (5), and the total supply is determined by

$$\max_{\Phi} \mathbb{E}_n\left[\Phi(p_1 - f(x)) - C_n(\Phi)|x_1\right].$$

The optimal solution of the last problem is

$$\Phi^* = \frac{p_1 - \mathcal{E}_n[x_1]}{2\alpha_n}.$$

In equilibrium, $\Phi_1(p_1) = \Phi^*$. Then the market price p_1 equals to $p_n[x_1]$. The demand $\Phi_1(p_1)$ follows easily. Other situations are similar and omitted. $\square$

Proof of Proposition 2

Given a set of parameters $\{a\}$ in the functional form $f(\{a\}; x)$ of the contract, by Proposition 1, the market price p_1 depends on the parameters $\{a\}$ only. Hence, the market price $p_0 = p_0(a)$, given $\{a\}$ is determined by

$$\max_{p_0} \mathbb{E}_n\left[\Phi_0(p_0, f(\{a\}; x_1))(p_0 - f(\{a\}; x_1)) - D - C_0(\Phi_0(p_0, f(\{a\}; x_1)))\right].$$

The solution of the last optimization problem is similar to case (3) of Proposition 1. Then we can derive the formula of p_0 as required. ☐

Proof of Corollary 1

Let $p_n[x_1], \Phi_n[x_1], \mathbb{E}_n[x_1]$ denote the security price, the market shares and the expected profit of the innovator, when only innovator issues the security at time one, for short. Then

$$p_n[x_1] = \frac{\theta\mathcal{V}[x_1]}{2\alpha + 2\theta\mathcal{V}[x_1]}\mathcal{E}_n[x_1] + \frac{2\alpha + \theta\mathcal{V}[x_1]}{2\alpha + 2\theta\mathcal{V}[x_1]}\mathcal{E}_v[x_1]. \tag{A-1}$$

Therefore, we have

$$p_n[x_1] - p_1^b \quad = \quad \frac{\theta\mathcal{V}[x_1](2\alpha + 2\theta\mathcal{V}[x_1])}{(2\alpha + 2\theta\mathcal{V}[x_1])(2\alpha + 3\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}. \tag{A-2}$$

Since $\mathcal{E}_v[x_1] > \mathcal{E}_n[x_1]$ in the benchmark model, we have $p_1 < p_n[x_1]$. Moreover, the total volume difference is

$$\Phi_1(p_1^b) - \Phi_n[x_1] = -\frac{1}{\theta\mathcal{V}[x_1]}(p_1^b - p_n[x_1]) > 0. \tag{A-3}$$

As for the market shares of the innovator, by tedious calculation, we have

$$\Phi_n(p_1^b) - \Phi_n[x_1] = -\frac{\theta\mathcal{V}[x_1](2\alpha + \theta\mathcal{V}[x_1])}{\Delta(2\alpha + 2\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}. \tag{A-4}$$

Hence we have proved that $\Phi_n(p_1^b) < \Phi_n[x_1] < \Phi_1(p_1^b)$.

Write W as $\Phi_n(p_1^b) - \Phi_n[x_1]$. Hence W is negative, and $p_1^b - p_n[x_1] = (2\alpha + \theta\mathcal{V}[x_1])W$. Therefore we have

$$\begin{aligned}
\mathbb{E}_n^b - \mathbb{E}_n[x_1] \quad &= \quad W \times \left\{p_1^b + \Phi_n[x_1](2\alpha + \theta\mathcal{V}[x_1]) - \mathcal{E}_n[x_1] - \alpha(\Phi_n^b + \Phi_n[x_1])\right\} \\
&= \quad W\frac{4\alpha + 5\theta\mathcal{V}[x_1])}{2(2\alpha + 3\theta\mathcal{V}[x_1])}\left\{\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1]\right\}.
\end{aligned}$$

where $\mathbb{E}_n^b$ is the expected profit of the innovator in the benchmark model. Therefore, $\mathbb{E}_n^b \leq \mathbb{E}_n[x_1]$. ☐

Proof of Corollary 2

It follows from Proposition 1 and equation (22). $\square$

Proof of Corollary 3

By Proposition 1 and Proposition 2, the total supply (and demand) volume deviation is

$$\Phi_1(p_1) - \Phi_1^b(p_1) = \frac{\mathbb{E}_n[f(x)|x_1] - \mathbb{E}_m[f(x)|x_1]}{2\alpha + 3\theta\mathcal{V}[x_1]}. \tag{A-5}$$

The volume deviation of the securities issued by the innovator is

$$\Phi_n(p_1) - \Phi_n^b(p_1) = \frac{\theta\mathcal{V}[x_1]}{\Delta}\left(\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]\right). \tag{A-6}$$

We now study the effects of the expected profit. We use $\mathbb{E}_n$ and $\mathbb{E}_n^b$ to denote the expected profit of the innovator in a heterogeneous belief model and a benchmark model, respectively. It is straightforward to derive that

$$\begin{aligned}\mathbb{E}_n - \mathbb{E}_n^b \;=\;& \{\Phi_n(p_1) - \Phi_n^b(p_1)\} \times \{p_1 + (2\alpha + \theta\mathcal{V}[x_1])\Phi_n^b(p_1) \\ & -\mathcal{E}_n[x_1] - \alpha(\Phi_n(p_1) + \Phi_n^b(p_1))\}\end{aligned} \tag{A-7}$$

Tedious calculation implies that

$$\begin{aligned}M \;:=\;& p_1 + (2\alpha + \theta\mathcal{V}[x_1])\Phi_n^b(p_1) - \mathcal{E}_n[x_1] - \alpha(\Phi_n(p_1) + \Phi_n^b(p_1)) \\ =\;& \frac{1}{\Delta}\{\theta\mathcal{V}[x_1](\alpha + \theta\mathcal{V}[x_1])\mathcal{E}_m[x_1] + 2(\alpha + \theta\mathcal{V}[x_1])(2\alpha + \theta\mathcal{V}[x_1])\mathcal{E}_v[x_1] - (\Delta - \alpha\theta\mathcal{V}[x_1])\mathcal{E}_n[x_1]\}.\end{aligned}$$

We first show that, in a market with both innovator and imitator as sellers,

$$\mathcal{E}_n[x_1] + \mathcal{E}_m[x_1] < 2\mathcal{E}_v[x_1]. \tag{A-8}$$

In fact, by Proposition 1, because of the presence of both innovator and the imitator, we have $\phi_n > 0, \phi_m > 0$. Hence

$$\begin{aligned}\mathcal{E}_m[x_1] &< q\mathcal{E}_v[x_1] + (1-q)\mathcal{E}_n[x_1], \\ \mathcal{E}_n[x_1] &< q\mathcal{E}_v[x_1] + (1-q)\mathcal{E}_m[x_1].\end{aligned} \tag{A-9}$$

To sum up the last two inequalities, we obtain $\mathcal{E}_m[x_1] + \mathcal{E}_n[x_1] < 2\mathcal{E}_v[x_1]$. Then we have

$$M \geq \frac{2(\alpha + \theta\mathcal{V}[x_1])^2}{\Delta}\{\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1]\}. \tag{A-10}$$

If $\mathbb{E}_m[f(x)|x_1] > \mathbb{E}_n[f(x)|x_1]$, we have $\mathbb{E}_n - \mathbb{E}_n^b \geq \frac{2\theta\mathcal{V}[x_1](\alpha+\theta\mathcal{V}[x_1])^2}{\Delta^2}(\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1])^2$, and $\mathbb{E}_n = \mathbb{E}_n^b$ if $\mathbb{E}_m[f(x)|x_1] = \mathbb{E}_n[f(x)|x_1]$.

On the other hand, if $\mathbb{E}_m[f(x)|x_1] < \mathbb{E}_n[f(x)|x_1]$, we see $\mathbb{E}_n - \mathbb{E}_n^b \leq \frac{2\theta\mathcal{V}[x_1](\alpha+\theta\mathcal{V}[x_1])^2}{\Delta^2}(\mathcal{E}_m[x_1] - \mathcal{E}_n[x_1])^2$. However, $\mathbb{E}_n - \mathbb{E}_n^b$ might be positive or negative, hence the advantage of the expected profit of the innovator is ambiguous. ☐

Proof of Corollary 4

First, we have

$$\Phi_n(p_1) - \Phi_m(p_1) = \frac{1}{2\alpha + \theta\mathcal{V}[x_1]}\{\mathbb{E}_m[f(x)|x_1] - \mathbb{E}_n[f(x)|x_1]\}. \tag{A-11}$$

By using the last equation, it is easy to see that $\mathbb{E}_n - \mathbb{E}_m$ is $\Phi_n(p_1) - \Phi_m(p_1)$ times

$$N := p_1 - \mathcal{E}_m + (\alpha + \theta\mathcal{V}[x_1])\Phi_n(p_1) - \alpha\Phi_m(p_1).$$

It is tedious to check that,

$$N = \frac{\alpha + 2\theta\mathcal{V}[x_1]}{2\alpha + 3\theta\mathcal{V}[x_1]}\{2\mathcal{E}_v[x_1] - \mathcal{E}_n[x_1] - \mathcal{E}_m[x_1]\}. \tag{A-12}$$

Then $N > 0$ by equation (A-8). ☐

Proof of Proposition 3

We first note that for $f(x) = ax, i \in \{m, n, v\}$,

(1) $\mathbb{E}_i[f(x)|x_1] = a\alpha_{i1}, \mathbb{E}_i[f(x)] = a\alpha_{i0}$,

(2) $\mathcal{V}[x_1] = a^2\sigma_{i1}^2, \mathcal{V} = a^2\sigma_{i0}^2$.

The above result hold in the unconditional situation case (in the first time period).

Now consider the case when the innovator will continue issue the security. By using the above formula (1) - (2), $\mathcal{E}_n[x_1] \geq p_m[x_1]$ if and only if

$$\alpha_{n1} \geq q\alpha_{v1} + (1-q)\alpha_{m1} - q\beta/a.$$

Note that $g(\cdot)$ is increasing, and $\sigma_{n0} \leq \sigma_{m0} < \sigma_{v0}$. Then $g(\sigma_{n0}) < qg(\sigma_{v0}) + (1-q)g(\sigma_{m0})$. Hence $\mathcal{E}_n[x_1] \geq p_m[x_1]$ holds only if $x_1 \leq A(\sigma_{m0})$. Similarly, $\mathcal{E}_m[x_1] \geq p_n[x_1]$ if and only if

$$\begin{aligned} \alpha_{m0} + g(\sigma_{m0})(x_1 - \alpha_{m0}) \quad &\geq \quad q\{\alpha_{v0} + g(\sigma_{v0})(x_1 - \alpha_{v0})\} \\ &\quad +(1-q)\{\alpha_{n0} + g(\sigma_{n0})(x_1 - \alpha_{n0})\} - q\frac{\beta}{a} \end{aligned} \tag{A-13}$$

If σ_{m0} satisfies formula (29), the above inequality (A-13) holds if and only if $x_1 \leq B(\sigma_{m0})$. Hence the imitator does not entre the market only if $x_1 \leq B(\sigma_{m0})$. Because $A(\sigma_{m0}) > B(\sigma_{m0})$, the first part of this proposition is proved.

If σ_{m0} satisfies formula (30), by the same derivation, we see that the imitator does not issue the security if and only if $x_1 \geq B(\sigma_{m0})$. Therefore the second part of this proposition is proved. □

Lemmas

To prove our results for option-like contracts, we need the following lemmas.

Lemma 1 *Given a normal distributed random variable ζ with mean μ and variance σ^2. Then we have*

(1) $E[n(\zeta)] = \frac{1}{\sqrt{1+\sigma^2}} n(\frac{\mu}{\sqrt{1+\sigma^2}})$,

(2) $E[\zeta n(\zeta)] = \frac{\mu}{(1+\sigma)^{3/2}} n(\frac{\mu}{\sqrt{1+\sigma^2}})$,

(3) $E[N(\zeta)] = N(\frac{\mu}{\sqrt{1+\sigma^2}})$

(4) $E[\zeta N(\zeta)] = \mu N(\frac{\mu}{\sqrt{1+\sigma^2}}) + \frac{\sigma^2}{\sqrt{1+\sigma^2}} n(\frac{\mu}{\sqrt{1+\sigma^2}})$,

where $n(\cdot)$ and $N(\cdot)$ is the normal density function and the cumulative normal distribution of a standard normal variable.

Proof: The proof is standard and omitted. □

Lemma 2 *Given a normal distributed random variable ζ with mean μ and variance σ^2. Then*

$$\frac{\partial\{E_i[\zeta N(\zeta)] + E_i[n(\zeta)]\}}{\partial\mu} = N(\frac{\mu}{\sqrt{1+\sigma^2}}) \tag{A-14}$$

and

$$\frac{\partial\{E_i[\zeta N(\zeta)] + E_i[n(\zeta)]\}}{\partial\sigma} = \frac{\sigma}{\sqrt{1+\sigma^2}} n(\frac{\mu}{\sqrt{1+\sigma^2}}). \tag{A-15}$$

Proof: Lemma 2 follows from Lemma 1 easily. □

Lemma 3 *Assume that $f(x) = (x-L)^+, L > 0$. Then*

$$\mathbb{E}_i[f(x)|x_1] = \sigma_\eta\{\mu N(\frac{\mu}{\sqrt{1+\sigma^2}}) + \sqrt{1+\sigma^2} n(\frac{\mu}{\sqrt{1+\sigma^2}})\} \tag{A-16}$$

where $\mu := \frac{\alpha_{i1}-L}{\sigma_\eta}$ and $\sigma^2 := \frac{\sigma_{i0}^2}{\sigma_{i0}^2+\sigma_\eta^2}$. Moreover $\mathbb{E}_i[f(x)]$ is calculated similarly in which μ and σ are replaced by $\frac{\alpha_{i0}-L}{\sigma_\eta}$ and σ_{i0}^2, respectively.

Proof: . For a normal distributed variable $\eta \sim N(0, \sigma_\eta^2)$, it is well known that

$$\mathbb{E}[(\eta - A)^+] = \sigma_\eta n(\frac{-A}{\sigma_\eta}) - AN(\frac{-A}{\sigma_\eta}). \tag{A-17}$$

Then by using the law of iterated expectations, we have

$$\begin{aligned} \mathbb{E}_i[(x-L)^+|x_1] &= \mathbb{E}_i[\mathbb{E}_i[(\eta - (L-\mu))^+|x_1, \mu]|x_1] \\ &= \sigma_\eta \mathbb{E}_i[n(\zeta) + \zeta N(\zeta)] \end{aligned}$$

where $\zeta = \frac{\mu - L}{\sigma_\eta}$ is normal distributed with mean $\frac{\alpha_{i1}-L}{\sigma_\eta}$ and variance $\frac{\sigma_{i1}^2}{\sigma_{i0}^2}$. Then this lemma follows from Lemma 1. □

Lemma 4 *Let $F(x) = x\sum_{i=1}^m a_i N(b_i + c_i x), c_i > 0$. Then*

$$\lim_{x\to-\infty} F(x) = 0, \tag{A-18}$$

and

$$\lim_{x\to+\infty} F(x) = \begin{cases} \infty, & \sum_{i=1}^m a_i > 0 \\ -\infty, & \sum_{i=1}^m a_i < 0 \\ 0, & \sum_{i=1}^m a_i = 0 \end{cases} \tag{A-19}$$

Proof: It is easy to see that $\lim_{x\to-\infty} xN(b+cx) = 0, \lim_{x\to\infty} xN(b+cx) = \infty, c > 0$ and $\lim_{x\to\infty} x\{N(b_1+c_1x) - N(b_2+c_2x)\} = 0$. Hence $\lim_{x\to-\infty} F(x) = 0$. Moreover, since

$$F(x) = \sum_i a_i x N(b_m + c_m x) + \sum_{i=1}^m a_i x \left\{N(b_i + c_i x) - N(b_m + c_m x)\right\},$$

then the limit of $F(x)$ when $x \to \infty$ is solved as desired.

If x is replaced by $-x$, we have the following limit results: Put $L(x) = x\sum_{i=1}^m a_i N\left(b_i - c_i x\right), c_i > 0$. Then

$$\lim_{x\to\infty} L(x) = 0,$$

and

$$\lim_{x\to-\infty} L(x) = \begin{cases} -\infty, & \sum_{i=1}^m a_i > 0 \\ +\infty, & \sum_{i=1}^m a_i < 0 \\ 0, & \sum_{i=1}^m a_i = 0 \end{cases}$$

We will need both the limits of $F(x)$ and $L(x)$ when $x \to \infty$ or $x \to -\infty$ in the following proofs below. □

Proof of Proposition 4

By Proposition 1, the innovator continue the issuance if and only if $\mathcal{E}_n(x_1) < p_m[x_1]$, or equivalently,

$$\begin{aligned} H(x_1) \;&:=\; qG(\mu_v(L, x_1), g(\sigma_{v0})) + (1-q)G(\mu_m(L, x_1), g(\sigma_{m0})) \\ &\quad -G(\mu_n(L, x_1), g(\sigma_{n0})) - q\frac{\beta}{\sigma_\eta} \end{aligned}$$

is strictly positive. By Lemma 4, and using the fact that $qg(\sigma_{v0}) + (1-q)g(\sigma_{m0}) > g(\sigma_{n0})$, we have

$$\lim_{x_1 \to \infty} H(x_1) = \infty. \tag{A-20}$$

Hence $H(x_1) > 0$ for $x_1 >> 0$. Since $\beta > 0$, then

$$\lim_{x_1 \to -\infty} H(x_1) = -q\frac{\beta}{\sigma_\eta} < 0. \tag{A-21}$$

Therefore, $H(x_1) < 0$ for $x_1 << 0$. ☐

Proof of Proposition 5

By Proposition 4, the imitator will enter the market in the second time period if and only if

$$\begin{aligned} L(x_1) \quad := \quad & qG(\mu_v(L, x_1), g(\sigma_{v0})) + (1-q)G(\mu_n(L, x_1), g(\sigma_{n0})) \\ & -G(\mu_m(L, x_1), g(\sigma_{m0})) - q\frac{\beta}{\sigma_\eta} \end{aligned}$$

is strictly positive. Since $\lim_{x \to -\infty} L(x_1) = -q\frac{\beta}{\sigma_\eta} < 0$, then there exists no imitator when $x_1 << 0$. For the low volatility, by Lemma 4, we have $L(x_1) > 0$ for $x_1 >> 0$; and for high volatility, $L(x_1) < 0$ for $x_1 << 0$. ☐

Proof of Proposition 6

Using the formula

$$G(\mu, \sigma^2) - G(-\mu, \sigma^2) = \mu,$$

$\mathbb{E}_i[f_2(x)|x_1]$ can be written as $K - \sigma_\eta G(-\mu_i(K, x_1), g(\sigma_{i0}))$. Therefore, the innovator will continue the issuance in the second time period if and only if

$$\begin{aligned} K(x_1) \quad := \quad & G(-\mu_n(K, x_1), g(\sigma_{n0})) - q\frac{\beta}{\sigma_\eta} \\ & -qG(-\mu_v(K, x_1), g(\sigma_{v0})) - (1-q)G(-\mu_m(K, x_1), g(\sigma_{m0})) \end{aligned}$$

is positive. First, $\lim_{x_1 \to \infty} K(x_1) = -q\frac{\beta}{\sigma_\eta} < 0$, then no issuance from the innovator when $x_1 >> 0$. By the discussion after Lemma 4, we see that $\lim_{x_1 \to -\infty} K(x_1) = -\infty$. Hence the innovator will exits from the market too when $x_1 << 0$. The proof for the imitator is similar. $\square$

Proof of Proposition 7

Note that

$$\mathbb{E}_i[f_3(x)|x_1] \;=\; \sigma_\eta\{G(\mu_i(K,x_1),g(\sigma_{i0})) - G(\mu_i(L,x_1),g(\sigma_{i0}))\}.$$

Hence, the innovator will continue to issue the mezzanine tranche if and only if

$$\begin{aligned} M(x_1) \;:=\; & q\{G(\mu_v(K,x_1),g(\sigma_{v0})) - G(\mu_v(L,x_1),g(\sigma_{v0}))\} \\ & +(1-q)\{G(\mu_m(K,x_1),g(\sigma_{m0})) - G(\mu_m(L,x_1),g(\sigma_{m0}))\} \\ & -\{G(\mu_n(K,x_1),g(\sigma_{n0})) - G(\mu_n(L,x_1),g(\sigma_{n0}))\} - q\frac{\beta}{\sigma_\eta} \end{aligned}$$

is positive. Clearly $\lim_{x_1 \to -\infty} M(x_1) = -q\frac{\beta}{\sigma_\eta} < 0$. Then there is no innovator for $x_1 << 0$. By Lemma , we see that

$$\lim_{x_1 \to \infty} M(x_1) = -q\frac{\beta}{\sigma_\eta}. \tag{A-22}$$

Hence there is no innovator neither for $x_1 << 0$. The proof for the imitator is similar. $\square$

B. Correspondence between CDO Tranches and Option Contracts

Suppose the loss of the collateral pool is y and the face value of the pool is F, then the value of the pool is $x = F - y$.

The loss of the equity tranche is $min\{y, K\}$, and its payoff is thus $K - min\{y, K\} = max\{x - (F - K), 0\}$.

The loss of the mezzanine tranche is $max\{y - K, 0\} - max\{y - L, 0\}$, the payoff is then $L - K - max\{y - K, 0\} + max\{y - L, 0\}$, which is $max\{x - (F - L), 0\} - max\{x - (F - K), 0\}$.

The loss of the senior tranche is $max\{y - L, 0\}$, so the payoff is $F - L - max\{y - L, 0\} = min\{x, F - L\}$.

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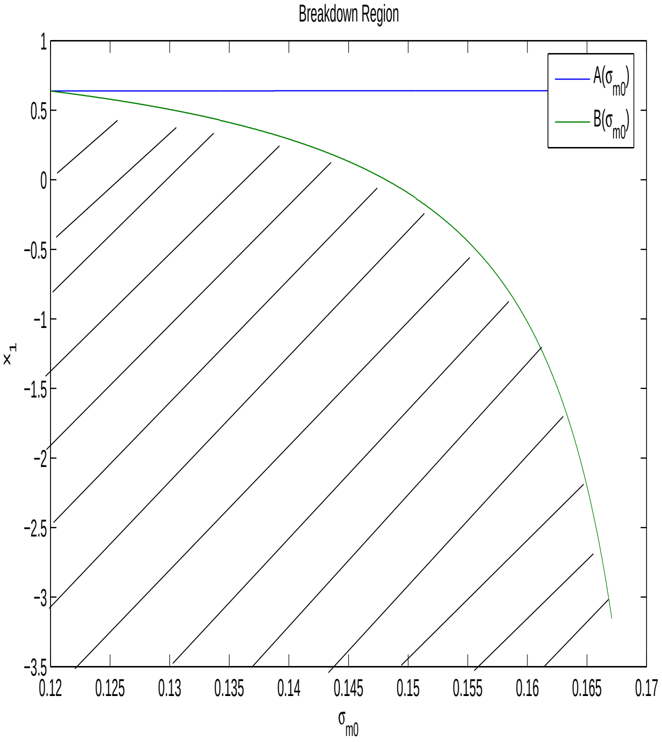

Figure 1: This figure displays the market breakdown when σ_{m0} changes. In this graph, $\sigma_{n0} = 12\%, \sigma_{v0} = 17.5\%$. Other parameters are: $\alpha_{n0} = 1.2, \alpha_{v0} = 1.5, \theta = 0.9, C(x) = 0.01x^2 + 0.06x + 1, f(x) = 0.8x$

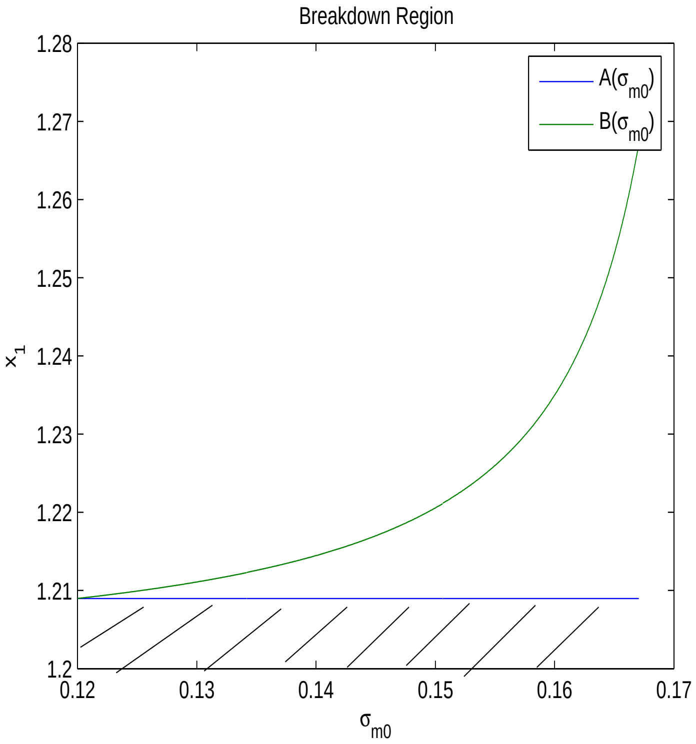

Figure 2: This figure displays the market breakdown when σ_{m0} changes. In this graph, $\sigma_{n0} = 12\%, \sigma_{v0} = 17.5\%$. Other parameters are: $\alpha_{n0} = 1.2, \alpha_{v0} = 1.5, \theta = 0.9, C(x) = 0.01x^2 + 0.06x + 1, f(x) = 0.35x$

Table 1: Participation Zones for Sellers of Financial Innovation

This table reports zones of x_1 for the innovator or the imitator to participate in the market at $t = 1$. "m" denotes under some medium level of x_1. We consider the following examples of financial innovation: forward, call option, capped forward and spread option.

Security	Innovator	Imitator	
		similar beliefs per (29)	dissimilar beliefs per (30)
Forward	$x_1 > A(\sigma_{m0})$	$x_1 > B(\sigma_{m0})$	$x_1 < B(\sigma_{m0})$
Call Option	$x_1 >> 0$	$x_1 >> 0$	m
Spread Option	m	m	m
Capped Forward	m	m	$x_1 << 0$